BEAM PHENOMENA AT THE INTERACTION POINT

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Introduction

Two major issues of the beam-beam phenomena in linear colliders are the deformation of the bunch (disruption) and the synchrotron radiation (beamstrahlung) due to the field created by the on-coming beam. In this note we shall report some results of the study on these problems with an emphasis on flat beams.

These problems can be studied in two separate steps: first the disruption and then the beams-trahlung. The energy loss due to the beamstrahlung may change the process of the luminosity enhancement but this effect can be ignored since we are only interested in the case when the average energy loss is small. The following topics will be discussed in this report.

1. pinch enhancement of the luminosity.
2. kink instability and luminosity reduction due to the displacement of the beam.
3. disruption angle.
4. energy spectrum of electron after collision.
5. behavior of electrons with large energy loss.

The notation in this paper is:

E_0	beam energy.
γ	E_0 in units of the rest mass.
N	number of particles in a bunch.
$\sigma_{x,y,z}$	r. m. s. beam size.
$\beta_{x,y}$	beta function at the collision point.
$R = \sigma_x/\sigma_y$	aspect ratio. Assume $\sigma_x \geq \sigma_y$.
α	fine structure constant.
r_e	classical electron radius.
$D_{x(y)} = \frac{2Nr_e\sigma_z}{\gamma\sigma_{x(y)}(\sigma_x+\sigma_y)}$	disruption parameter.
$A_{x(y)} = \sigma_z/\beta_{x(y)}$	
N_γ	average number of beamstralung photons per electron.
ξ	(critical energy)/(initial energy)
δ	average relative energy loss.
$L_{00} = \frac{f_{rep}N^2}{4\pi\sigma_x\sigma_y}$	geometrical luminosity, f_{rep} being the repetition rate.
L_0	geometrical luminosity with the variation of beta taken into account.
L	luminosity with disruption.

The computer simulation was done using the code ABEL (Analysis of Beam-beam Effects in Linear colliders) described in [1] but improved considerably since then. Some results given in this report are still preliminary. They will be refined in later papers but the qualitative feature will not change.

Luminosity Enhancement

Our primary interest is the enhancement of the luminosity due to the pinch effect. The detail has been discussed in [2] for round beams and will be given in [3] for flat beams. As was pointed out in [2], the luminosity is infinite if the initial beam is parallel and the computation is perfectly accurate. This is because a parallel beam can be focused to a point. Thus, we introduced a parameter $A_{x(y)} = \sigma_z/\beta_{x(y)}$ which is proportional to the emittance for given beam size $\sigma_{x(y)}$. The computed enhancement factor $H = L/L_0$ for flat beams is plotted in Fig. 1 as a function of D_y and A_y. Here, we used

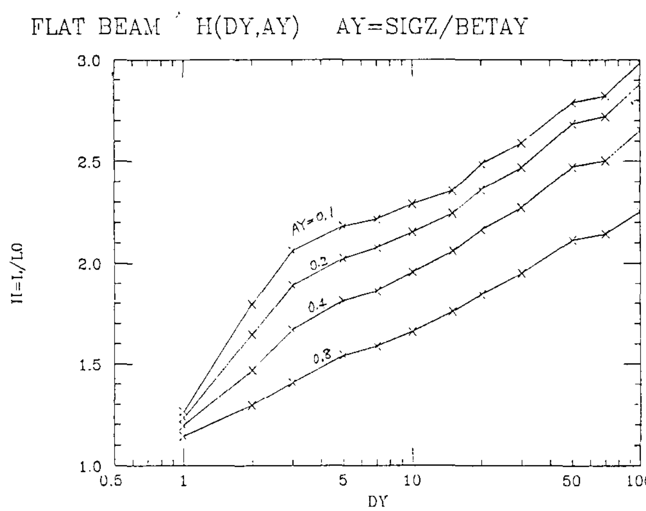


Figure 1: Luminosity enhancement factor for flat beams.

the distribution function uniform in x and Gaussian in y and z (UGG), instead of three-dimensional Gaussian distribution (GGG), for easiness of computation. The enhancement factor of GGG distribution for given D_y is a superposition of UGG for the disruption parameter between 0 and $\sqrt{6/\pi}D_y$. The enhancement factor for round beams is shown in Fig. 2.

In both cases, H is monotonically increasing as a function of D (or D_y) at least up to D =100. This result is

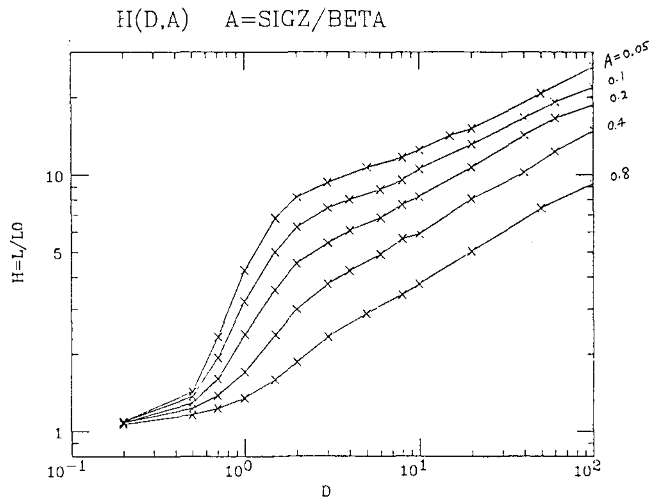


Figure 2: Luminosity enhancement factor for round beams.

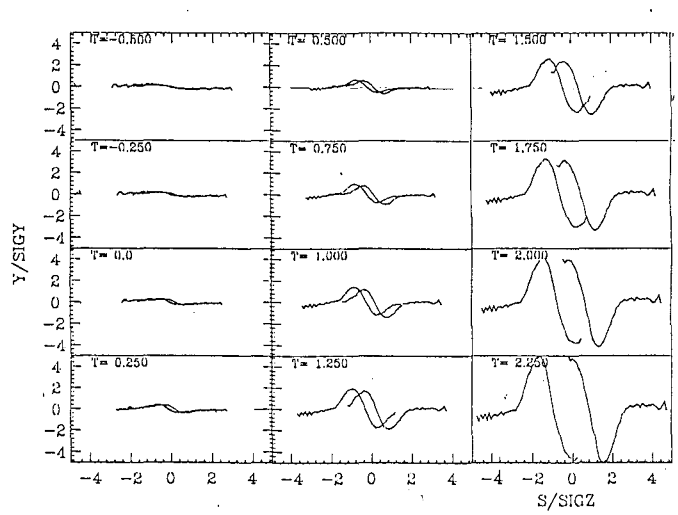


Figure 3: An example of kink instability.

qualitatively the same as that found by Fawley and Lee [4] but in contradiction to Holebeek [5] and to Solyak [6]. Our simulation was done in the following manner. For the flat beam case, the collision has an exact up-down ($+y$ and $-y$) symmetry. Any deviation from the symmetry comes from computation errors. In our code the initial condition is generated by random numbers, which statistically causes an asymmetry of the order $1/\sqrt{N_p}$, N_p being the number of macro particles. This asymmetry is enhanced during the collision due to the beam-beam force when the disruption parameter is large. To minimize the computing errors, the particle distribution function is symmetrized at every time step so that the beam-beam force has the up-down symmetry. In the round beam case, only the radial force is computed. This process eliminates the possible instability triggered by computing errors. The actual beam has more or less asymmetry but our principle is that the asymmetry in the simulation should be introduced intentionally not by random errors so that we can know the relation between the degree of asymmetry and the luminosity reduction. The efffect of initial beam displacement will be discussed in the next section.

By comparing Figs. 1 and 2 one finds that the enhancement factor for flatbeams is not close to the square root of that for round beam when D is large. (Empirically, about cubic root.) This is because the horizontal focusing can enhance the vertical pinch effect (and vice versa) in the round beam case.

Kink Instability

If one of the beam is displaced vertically by some reason, this offset triggers a vertical oscillation and, when D is large, the oscillation is enhanced by the beam-beam force. This phenomena is known as 'kink instability'. Fig. 3 shows an example. The bunch is sliced longitudinally and the vertical coordinate y of the center-of-mass of each slice (in units of σ_y) is plotted against the longitudinal coordinate s (in units of σ_z). Each graph corresponds to a different time t, which is written at the top-left corner in units of σ_z/c (downwards from top-left). The initial offset is $0.2\sigma_y$ (full) and the disruption parameter is $D_y = 20$.

For uniform beam and small amplitude oscillation, one gets an equation of motion of fluid dynamics (flat beam version of the equation given in [7];

$$\left[\frac{\partial}{\partial t} \pm \frac{\partial}{\partial s}\right]^2 y_\pm = -\omega_0^2 (y_\pm - y_\mp), \quad \omega_0^2 = \frac{\sqrt{2\pi}}{6}\frac{D_y}{\sigma_z^2}, \quad (1)$$

where $y_\pm$ is the y coordinate of $e^+$and e^-beams. The most unstable solution is

$$y_\pm = \text{const.} \times \exp\left[\pm i(\frac{\sqrt{3}}{2}\omega_0 s - \frac{\pi}{6}) + \frac{1}{2}\omega_0 t\right]. \quad (2)$$

This solution is in reasonable agreement with Fig. 3 in various points such as the phase difference $\pi/3$ between $e^-$and e^+, the growth rate and the standing wave nature.

However, this instability is not always a harmful effect, because, in the initial phase of the instability, the beams attract each other, which prevents the otherwise rapid fall off of the luminosity for large initial offset. Fig. 4 shows the luminosity enhancement factor as a function of the offset Δ_y (in units of σ_y) for various values of D_y. The dotted line is the geometrical enhancement factor without beam-beam force which is written as $\exp(-\Delta_y^2/4\sigma_y^2)$. UGG distribution is used and $A_y = 0.2$ for all. The up-down symmetry is not enforced except at the point $\Delta_y = 0$.

One finds the best value of D_y is between 5 and 10 in the sense that a high luminosity is kept up to large offsets. For these values of D_y, H is still above unity even at $\Delta_y = 3\sigma_y$. Above this region of D_y the beam breakup is serious and below this region the attraction is not strong enough.

The same data as in Fig. 4 is plotted in Fig. 5 in a different way where the horizontal axis is D_y and each curve corresponds to different Δ_y. (The region of large D_y and small Δ_y is not very accurate because of the sensitivity to computing errors.) Now, one sees a saturation and decrease of H as a function of D_y unless $\Delta_y = 0$, as in the simulation in [5] and [6]. For practical application it will

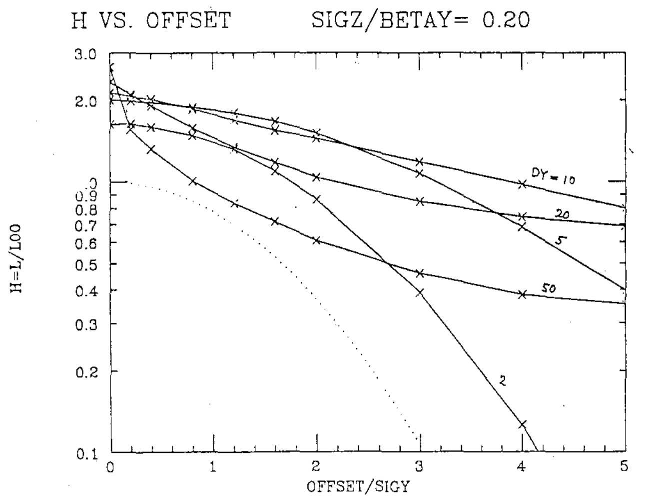


Figure 4: Luminosity reduction due to offset. Flat beams.

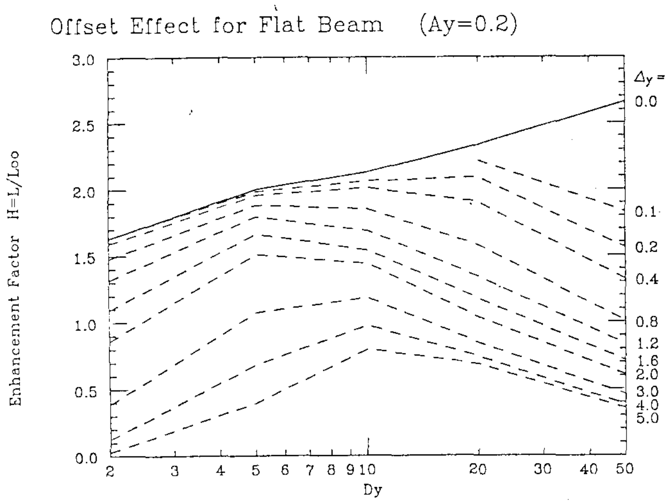


Figure 5: Luminosity reduction due to offset. Flat beams.

be safer to adopt the curve for Δ_y =0.2 or 0.4, instead of Δ_y =0, as the enhancement factor of head-on collision.

Fig. 6 shows the offset effect for round beams but it is still preliminary, for the large D region has not yet been investigated because it requires a very accurate computation (and, therefore, long computing time and many macro particles).

Disruption Angle

The information of the final direction of the electron trajectory after collision is necessary for designing the interaction region, especially for the aperture of the final quadrupole magnets. If the disruption parameter is very small, the transverse location of a particle during collision is nearly constant. Then we can estimate the disruption angle θ_x and θ_y as functions of the initial transverse coordinates x_0 and y_0. For very flat Gaussian beams we have

$$\theta_x = -\sqrt{\frac{\pi}{2}} D_x \frac{\sigma_x}{\sigma_z} \mathcal{I}m \left[\frac{i}{\pi} \int_{-\infty}^{+\infty} \frac{\exp(-t^2)dt}{x_0/\sqrt{2}\sigma_x - t - i0} \right] \quad (3)$$

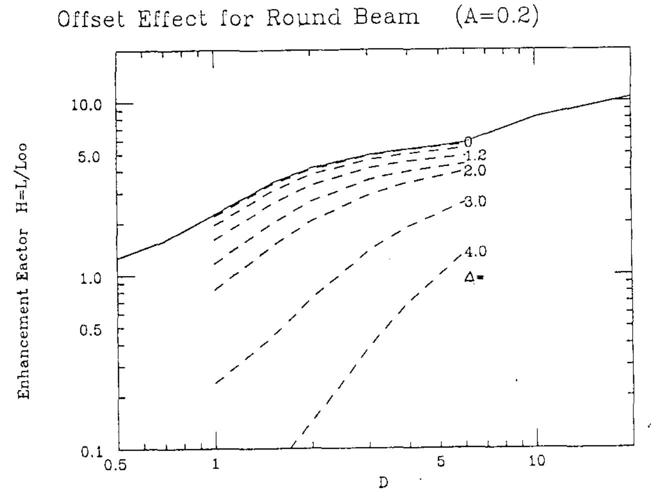


Figure 6: Luminosity reduction due to offset. Round beams.

$$\theta_y = -\sqrt{\frac{\pi}{2}} D_y \frac{\sigma_y}{\sigma_z} \left[\frac{2}{\sqrt{\pi}} \int_0^{y_0/\sqrt{2}\sigma_y} e^{-t^2} dt \right] e^{-x_0^2/2\sigma_x^2} \quad (4)$$

where the quantities in the square brackets can be expressed by the complex error function $w(x_0/\sqrt{2}\sigma_x)$ and the real error function $\mathrm{Erf}(y_0/\sqrt{2}\sigma_y)$. Here the emittance is ignored. One finds that the maximum and r. m. s. of the disruption angle is

$$\begin{aligned}
\theta_{x,max} &= 0.765 D_x \frac{\sigma_x}{\sigma_z} \qquad (x_0 = 1.31\sigma_x) && (5) \\
\theta_{y,max} &= \sqrt{\pi/2} D_y \frac{\sigma_y}{\sigma_z} \quad (x_0 = 0,\ y_0 = \infty) && (6) \\
\theta_{x,rms} &= \sqrt{\pi/(6\sqrt{3})} D_x \frac{\sigma_x}{\sigma_z} && (7) \\
\theta_{y,rms} &= \sqrt{\pi/(6\sqrt{3})} D_y \frac{\sigma_y}{\sigma_z} = \theta_{x,rms} && (8)
\end{aligned}$$

(Rigorously speaking, for flat beams with large but finite R, θ_y takes maximum near $y_0 \sim \sigma_x$ and then decreases but this is not important.) The distribution functions of θ_x and θ_y are shown in Fig. 7. the singularities at $\theta_x = \theta_{x,max}$ and $\theta_y = 0$ are not so sharp like this for actual beams because of the finite emittance, various errors and the disruption but the qualitative difference between horizontal and vertical angles is still seen in the simulations for not small disruption parameters.

We need a computer simulation for the vertical disruption angle for finite D_y. (We consider the case of small D_x only.) Fig. 8 shows the maximum(solid) and r.m.s.(dashed) vertical disruption angle normalised by $D_y\sigma_y/\sigma_z$. The four curves corrrspond to A_y = 0.1, 0.2, 0.4, 0.8 respectively. The dependence on A_y is not so significant as in the enhancement factor except for small D_y region where the finite emittance appears as it is initially. (The distribution of initial $\sigma_{x'}$ is cut at 2.5 standard deviation.)

The simulation results can roughly be fitted by

$$\theta_{y,rms} \sim \sqrt{\frac{\pi}{6\sqrt{3}}} \frac{\sigma_y}{\sigma_z} \frac{D_y}{[1 + (0.5D_y)^5]^{1/6}} \quad (9)$$

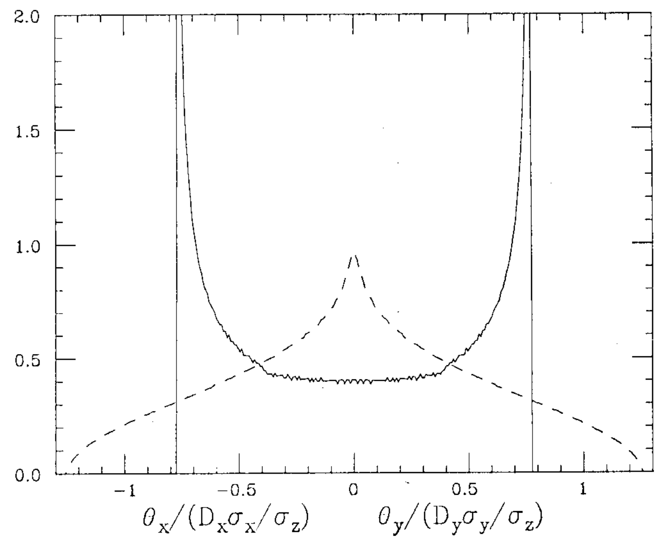


Figure 7: Distribution of the disruption angle for small disruption parameters. Flat beam.

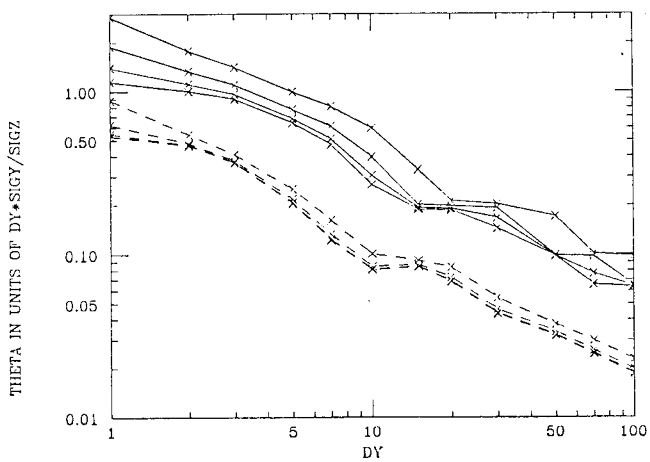


Figure 8: Maximum and r. m. s. vertical disruption angle. Flat beam.

and $\theta_{y,max} \sim 2.5\theta_{y,rms}$. Here the contribution of the initial emittance ($= A_y\sigma_y/\sigma_z$ for $\theta_{y,rms}$) is not included. The reason that the angle does not linearly increase as D_y is that the trajectory is bent back and oscillates when D_y is large.

So far, the collision is assumed to be head-on. For flat beams, the disruption angle in the presense of vertical offset is also important in determining the aperture of the final quads. The deflection angle of the center-of-mass of the bunch can be written in the form

$$\Theta_y = \frac{1}{2}\frac{\sigma_y}{\sigma_z}D_y F(D_y, \frac{\Delta_y}{\sigma_y}) \qquad (10)$$

where the weak dependence on A_y is ignored. The function F is given by [8]

$$F(D_y, \frac{\Delta_y}{\sigma_y}) = \int_0^{\Delta_y/\sigma_y} e^{-y^2/4}dy \qquad (11)$$

for small D_y. Fig. 9 shows F computed by simulations. UGG distribution is used here.

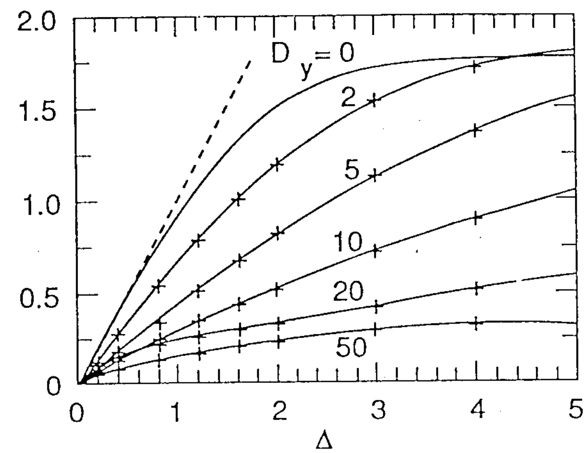


Figure 9: Center-of-mass deflection angle. Function F(D_y,Δ_y/σ_y).

Roughly speaking, the maximum disruption angle in the presense of offsets is the sum of the center-of-mass deflection angle Θ_y and the maximum angle in the absense of offsets, $\theta_{y,max}$.

Energy Spectrum of Electron

The energy spectrum is important in two points; the spread of the e^+e^- center-of-mass energy is the main reason to avoid large energy loss collision and the electron with large energy loss may be bent by a large angle by the beam-beam force to cause a background. For the first point the spectrum near the initial energy is important and the low energy tail for the second point.

The energy spectrum of radiation can be characterized by the parameter*

$$\xi = \frac{(\text{critical energy})}{(\text{initial energy})} = \frac{3}{2}\frac{r_e\gamma^2}{\alpha\rho} \qquad (12)$$

where ρ is the instantaneous curvature radius. Its typical value during the collision is

$$\xi_1 = \frac{r_e^2\gamma N}{\alpha\sigma_z}\frac{2}{\sigma_x+\sigma_y}. \qquad (13)$$

The average of ξ is a bit smaller than eq. (13) (by a factor about 2/3) but we adopt eq. (13) for the better description of the low energy electron tail which comes from beamstrahlung with large ξ.

The number of emitted photons per electron is

$$N_\gamma = N_{cl}U_0(\xi_1) \text{ with } N_{cl} = 1.06\alpha r_e N\frac{2}{\sigma_x+\sigma_y} \qquad (14)$$

*The notation $\Upsilon = \frac{2}{3}\xi$ is used in literatures but in some cases comfused with ξ. Our notation is after Sokolov and Ternov.

where N_{cl} is the number of photons computed by the classical formula and $U_0(\xi)$ is the ratio of quantum-theoretical number of photons to classical. A simple formula for U_0 is

$$U_0(\xi) = \frac{1 - 0.598\xi + 1.061\xi^{5/3}}{1 + 0.922\xi^2} \qquad (0 \le \xi < \infty), \quad (15)$$

whose relative error is within 0.7 percent.

We have developed an approximate formula for the energy spectrum of electrons after collision. The detail will be given in [9]. Here, we quote the results only. The distribution function $\psi(\varepsilon)$ ($\varepsilon = E/E_0$) normalized as $\int \psi(\varepsilon)d\varepsilon = 1$ can be written as

$$\psi(\varepsilon) \simeq e^{-N_\gamma}\left[\delta(\varepsilon - 1) + \frac{e^{-y}}{1-\varepsilon}h(N_1 y^{1/3})\right] \qquad (16)$$

with

$$\begin{aligned} h(x) &= \frac{1}{2\pi i}\int_{\lambda - i\infty}^{\lambda + i\infty} \exp\left(xp^{-1/3} + p\right)dp \quad (\lambda > 0) \\ &= \sum_{n=1}^{\infty} \frac{x^n}{n!\,\Gamma(n/3)} \end{aligned} \qquad (17)$$

and

$$y = \frac{1}{\xi_1}\left(\frac{1}{\varepsilon} - 1\right)$$

$$N_1 = \frac{1}{1+\xi_1 y}N_{cl} + \frac{\xi_1 y}{1+\xi_1 y}N_\gamma. \qquad (18)$$

(This formula does not exactly satisfy the normalization condition except for $\xi_1 \to 0$ which leads to $N_1 = N_\gamma = N_{cl}$.) The function $h(x)$ can be estimated very accurately by

$$h(x) = \sqrt{\frac{3}{8\pi}}\left[\frac{\sqrt{x/3}}{1 + 0.53x^{-5/6}}\right]^{3/4} \exp\left[4\left(\frac{x}{3}\right)^{3/4}\right] \qquad (19)$$

with the relative error less than 2% for any x. Fig.10 compares eq. (16) with the simulation results using the parameters for CLIC[10] and those by R. Palmer[11] (wavelength 17mm). (The parameters are summarized in Table.1.) The histogram is the simulation and the crosses are eq. (16). The agreement is excellent.

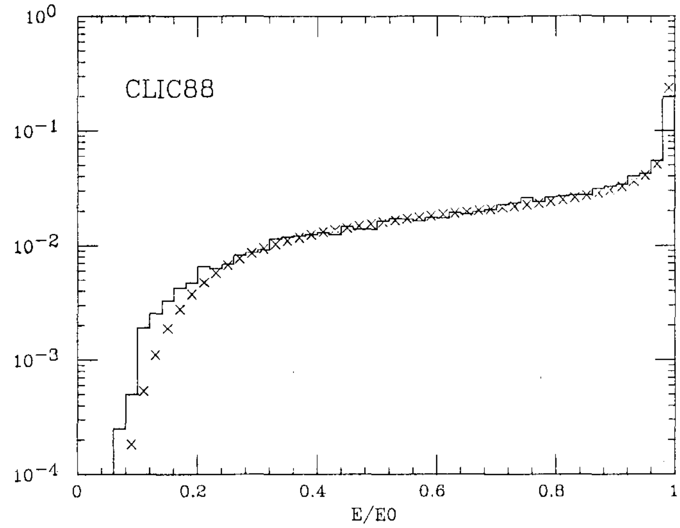
The center-of-mass energy spectrum is approximately calculated by the following way using $\psi(\varepsilon)$. The average energy spectrum of one of the beam during collision is

$$\langle \psi(\varepsilon) \rangle = \int_0^1 d\tau \psi(\varepsilon, \tau) \qquad (20)$$

where $\psi(\varepsilon, \tau)$ is given by $\psi(\varepsilon)$ but with N_γ and N_{cl} replaced by τN_γ and τN_{cl}, respectively. Then the center-of-mass energy spectrum function $\Psi(x)$ ($x = S/4E_0^2, S =$ center-of-mass energy squared) is

$$\Psi(x) = \int_x^1 \langle \psi(\varepsilon) \rangle \left\langle \psi\left(\frac{x}{\varepsilon}\right)\right\rangle \frac{d\varepsilon}{\varepsilon}. \qquad (21)$$

This expression is compared with simulation in Fig. 11 using the same parameter sets as in Fig. 10.



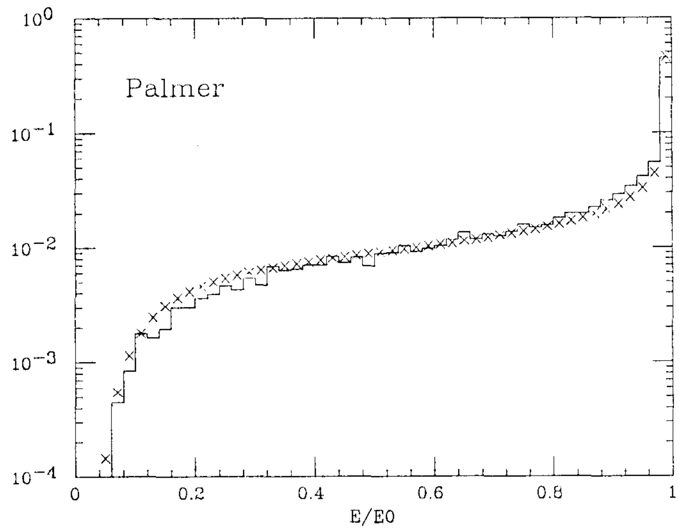


Figure 10: Electron energy spectrum for CLIC and Palmer's parameter sets.

Deflection of Low Energy Particles

The particles which lost a large fraction of its initial energy is of our concern because they may be bent by a large angle and cause backgrounds. Consider an electron which emitted a hard photon at some time during the collision and became of energy εE_0 ($\varepsilon \ll 1$). The effective disruption for this particle becomes D_x/ε and D_y/ε. One might think that the eqs. (5) to (9) apply by replacing D by D/ε. However, the collision of a single particle on a beam with the disruption parameter D/ε is different from the collision between two beams with D/ε, although the qualitative feature is the same; i.e., the disruption angle increases linearly in D for small D and more slowly for large D.

A simulation was done by putting low energy test particles during the middle of collision. The maximum disruption angle for given ε can be written very crudely as

$$\theta_{max} \sim \frac{\sigma}{\sigma_z}\frac{D/\varepsilon}{\sqrt{1 + (0.75D/\varepsilon)^{4/3}}} \qquad (\varepsilon \ll 1) \qquad (22)$$

Table 1: Parameters used in the simulation

		CLIC	Palmer's
	E_0	1 TeV	0.5 TeV
	N	5×10^9	8×10^9
	σ_z	$200\mu m$	$26\mu m$
	σ_x	60 nm	190 nm
	σ_y	12 nm	1 nm
	ϵ_x	1.53×10^{-12} m	2.58×10^{-12} m
	ϵ_y	0.51×10^{-12} m	0.0233×10^{-12} m
	D_x	0.667	0.033
	D_y	3.333	6.27
	A_x	0.085	0.0002
	A_y	0.71	0.60
*	L/L_{00}	1.84	1.61
*	δ	0.25	0.15
*	N_γ	3.0	1.33
	ξ_1	1.48	3.43
*	$L_{\gamma\gamma}/L$	2.17	0.31
*	$L_{e^-\gamma}/L$	1.32	0.53

* quantities computed by simulations.

where $D = D_x(D_y)$ and $\sigma = \sigma_x(\sigma_y)$ for horizontal (vertical) angle.

Now, the problem is how small ε we have to care. Since the number of photons N_γ for linear colliders in the near future is of order unity, the spectrum function $\psi(\varepsilon)$ given in eq. (16) is always dominated by the factor e^{-y} at the tail $y \gg 1$ (in a logarithmic sense). Therefore, if we allow n particles among N to create backgrounds, the minimum ε we have to care is approximately given by $y = \log(N/n)$ or

$$\varepsilon_{min} = 1/\left(1 + \xi_1 \log(N/n)\right). \tag{23}$$

We can compute the largest disruption angle by eq.(22). Since the dependence on n is only logarithmic, we may put $n = 1$. For example, $\varepsilon_{min} = 0.03$ (0.013), $\theta_{x,max} =$ 1.0 (10.) mrad and $\theta_{y,max} = 0.4$ (0.4) mrad for CLIC (Palmer's) parameter set. For CLIC parameter set $\theta_{x,max}$ is considerably larger than the maximum crossing angle for negligible luminosity reduction, $\sigma_x/\sigma_z = 0.3$ mrad.

In determining the vertical aperture of the final quads, the information of $\theta_{y,max}$ may not be enough, because vertical beam offset due to errors can cause larger deflection angle.

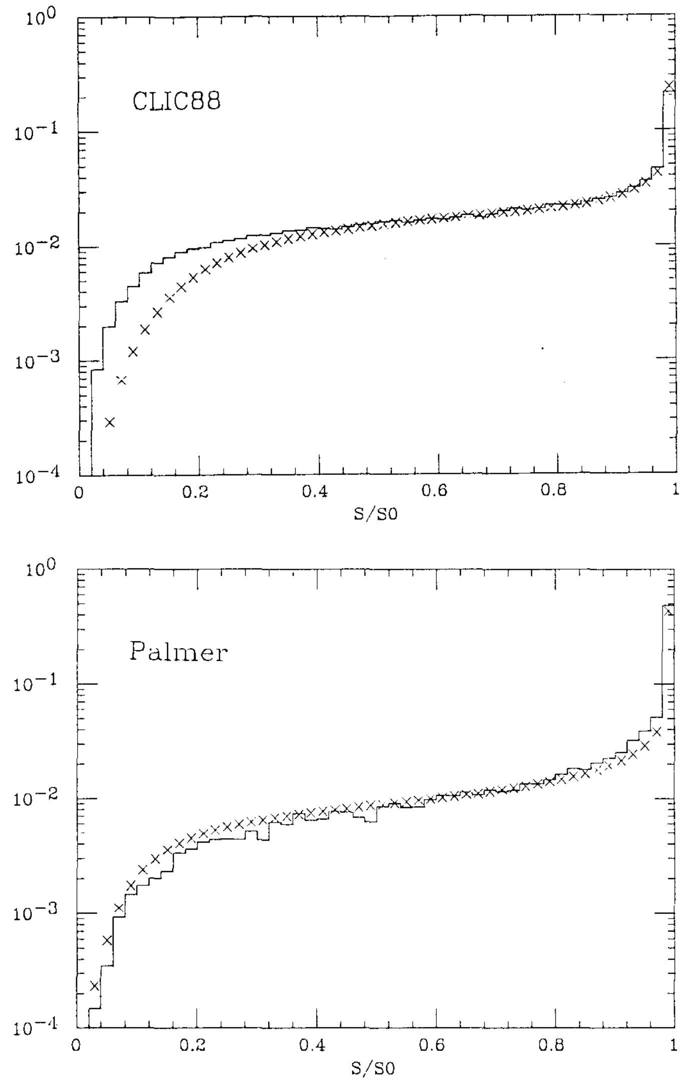


Figure 11: Center-of-mass energy spectrum for CLIC and Palmer's parameter sets.

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