MAGNETIZATION CURRENT MODELS FOR FERRITE LOADED LIA CAVITIES

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Abstract

The calculation of magnetization currents in ferrite-loaded Linear Induction Accelerator (LIA) cavities is essential to the design of efficient, low current devices of this type. The magnetization currents in such accelerators represent a significant fraction of the total load current, and an accurate estimate of their magnitude must be made before the pulsed power drivers can be specified. The ability to model the magnetization of ferrite is particularly important for applications of LIAs that require precise accelerating voltages, such as free electron lasers. The calculation of magnetization currents is complicated by the nonlinear, time-dependent properties of the ferrite, as well as by radial and displacement current effects associated with the geometry of the cavity. A simple ferrite model, which includes saturation, rate of magnetization, and radial variations within the ferrite, is described below and used to define a nonlinear circuit element. This element can be used in circuit simulations to account for displacement currents. Examples of its use and comparisons with measurements are presented below.

Ferrite Models

Simple Ferrite Model

The object of a ferrite model is to calculate the magnetization current for a particular type and geometry of ferrite as a function of time in response to an applied voltage pulse. In many cases the voltage pulse is known, either from measurements or because it is the desired design goal. In these cases, a simple calculational model can be developed if displacement currents are neglected and if a simple description of the ferrite properties is assumed.

In an axisymmetric geometry and neglecting displacement currents, Ampere's law and Faraday's law can be written as

$$2\pi r H = I, \qquad (1)$$

$$dV/dr = l \dot{B}, \qquad (2)$$

respectively. If we assume that the magnetization of the ferrite is described by an equation of the form

$$H = \dot{B}^{\alpha} f(B), \qquad (3)$$

where f is an arbitrary function of the magnetic induction that includes saturation, then we may use Equation (1) to eliminate H in Equation (3) and solve the resulting equation for $\dot{B}$ to obtain

$$\dot{B} = \{I/[2\pi r f(B)]\}^{1/\alpha}, \qquad (4)$$

provided that α is not zero. This may be substituted in Equation (2) and integrated to obtain

$$V = l \left\{ \frac{1}{2}\pi \int_a^b \frac{l dr}{rf(b)} \right\}^{1/\alpha} I^{1/\alpha}. \qquad (5)$$

Equations (4) and (5) can be solved simultaneously by assuming an initial radial distribution for B, using Equation (5) to obtain I, advancing B in time with Equation (4) and repeating the process throughout the applied voltage pulse.

The form chosen for Equation (3) is arbitrary. It permits an accurate description of ferrite magnetization at a fixed rate, describes the rate dependence in a manner that is at least qualitatively correct, and permits a solution of the problem. Both α and the function f (B) must be determined experimentally for a given ferrite at a magnetization rate appropriate to the intended application. Hysteresis has not been included because it is assumed that only the first, constant polarity portion of the applied voltage pulse is of interest. Note that if α is not zero, Equation (5) describes the ferrite as a nonlinear resistor. If α is zero, then Equation (3) must be differentiated with respect to time to obtain an expression for $\dot{B}$, and the corresponding version of Equation (5) would describe the ferrite as a nonlinear inductor.

Circuit Models with Ferrite

The simple ferrite model suffers from two significant limitations; the neglect of displacement currents, and the necessity of specifying the applied voltage pulse in detail. These limitations can be removed by developing an equivalent component for use in a circuit model. Equation (5) suggests that a nonlinear resistor of the form

$$R = g(B)^{1/\alpha - 1} \qquad (6)$$

could be used for this purpose. However, it is not obvious that this form can correctly account for effects associated with the radial or rate dependencies of magnetization.

The simple ferrite model was used to determine whether Equation (6) can be used to describe an equivalent component. A series of calculations were performed for annular rings of ferrite with constant cross section but differing mean radii.

The function

$$Q(B) = V(r/I)^{1/\alpha} \qquad (7)$$

was calculated for five values of the mean radius r over a 2:1 range. The resulting calculated functions differed by less than ± 3%. This form for Q (B) was chosen because only the ratio (I/r) appears in Equation (1) and the exponent is that of Equation (5). Two calculations were performed at differing rates of magnetization for a core whose outer radius was twice its inner radius. The rates differed by a factor of two and the resulting calculated functions differed by less than ± 2%. Thus it appears reasonable to model the behavior of the ferrite, in the absence of displacement currents, as a nonlinear resistor of the form given in Equation (6).

Circuit simulations were performed which included displacement currents by considering a ferrite core to be represented by a network of components as shown in Figure 1. The core was considered to be segmented either axially or radially, as appropriate, and each segment was represented by a section of transmission line based on the saturated permeability and permittivity of the ferrite in series with a nonlinear resistor. Displacement currents flow in the transmission lines, while the nonlinear resistor supports the applied voltage prior to saturation and accounts for the energy dissipated in the ferrite. The nonlinear properties of the resistor were determined by applying the simple ferrite model to a single core segment with an appropriate, constant applied voltage, and calculating the function Q (B) as defined in Equation (7). The magnetic induction was calculated during each simulation for each segment by employing Equation (2) and assuming that $\dot{B}$ is constant throughout each segment.

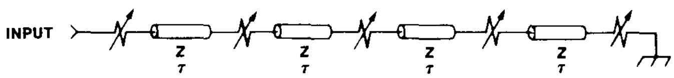

Figure 1. Equivalent circuit for four-section ferrite model. Transmission line impedance and length obtained from geometry and saturated ferrite characteristics. First and last nonlinear resistors one-half as large as others. Resistor characteristics obtained as described in text.

Comparisons with Measurements

Simple Ferrite Model

The results of a single pulsed measurement are meaningless unless the ferrite properites are known from other measurements made on the same material at similar magnetization rates. The data base used to support the development of these models includes measurements made at Physics International[1] on three samples of TDK PE-11B and measurements reported by Turner[2,3] on an accelerator cell designed for the ETA-II LIA at the Lawrence Livermore National Laboratory. The ETA-II cell also uses PE-11B ferrite. The parameters for these cores are listed in Table 1.

Table 1. Ferrite core parameters.

Core	b (m)	a (m)	l (m)	Aspect Ratio b:a:l	Bsat (T)	Brem (T)
W163	0.051	0.037	0.025	1.4:1:1	0.41	0.36
W157	0.051	0.025	0.025	2:1:1	0.46	0.20
W193	0.250	0.126	0.030	2.0:1:0.24	0.42	0.28
ETA-II	0.178	0.102	0.102	1.8:1:1	0.34	0.19*
Proposed	0.31	0.09	0.24	3.4:1:2.7	0.41	0.36

*Needed to match saturation in data.

The data from Shot W163 were taken as representing the intrinsic properties of the ferrite because this sample possessed the most benign geometry and coincidentally had the largest remanence field. The B-H loop from the material was inferred from the measured V-I characteristics by assuming: (1) that the magnetic field intensity is uniform throughout the sample with the value calculated at the mean radius of the core, (2) the magnetic induction was similarly uniform, and (3) an appropriate value for α was chosen for Equation (3), which describes the magnetization of the ferrite. Unfortunately, none of the measurements explicitly addresses the dependence of ferrite properties on the rate of magnetization, because there were no measurements taken at different voltages on the same sample.

The measured V-I characteristics as well as the current profile calculated with the simple ferrite model described above are shown in Figure 2. The agreement between the measurements and the model indicates only that the assumption of uniform average fields in the sample can be used to obtain the B-H loop. The arbitrary choice of $\alpha = 0.5$ is not validated by Figure 2 since the inclusion of α in both the inferrence of the B-H loop and the simple ferrite model is self-compensating.

The simple ferrite model was also used to analyze the data from Shot W157 with the results shown in Figure 3. This sample has a more radical geometry and a smaller remanence field. The remanence field was measured by driving the core through a complete cycle around the B-H loop. The remanence field was accounted for in the simple ferrite model by offsetting the initial value for the magnetic induction. This does not correctly describe the initial response of the ferrite, but as Figure 3 shows, it does reasonably describe the measurement.

The simple ferrite model was also used with the data from Shot W193 with less impressive results, as shown in Figure 4. This core had the same radical aspect ratio as that of the core used in Shot W157 but was relatively much thinner. The value of α was varied over the range from 0.1 to 1.5. The "best" agreement was obtained for $\alpha = 0.5$ but the differences were not great and the selection was largely subjective. The best support for the choice of $\alpha = 0.5$ is described below.

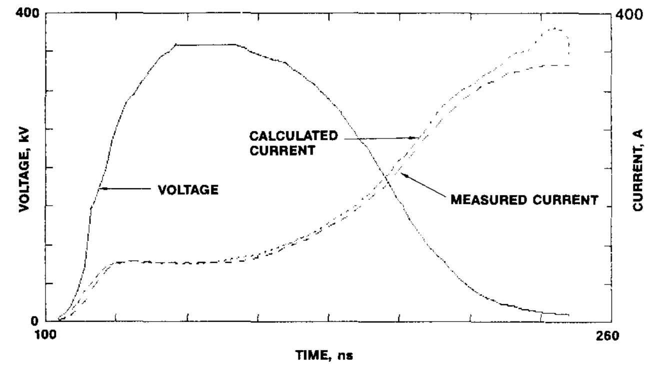

Figure 2. V-I characteristics and calculated current from simple ferrite model for core tested in Shot W163. Core dimensions listed in Table 1. $\alpha = 0.5$.

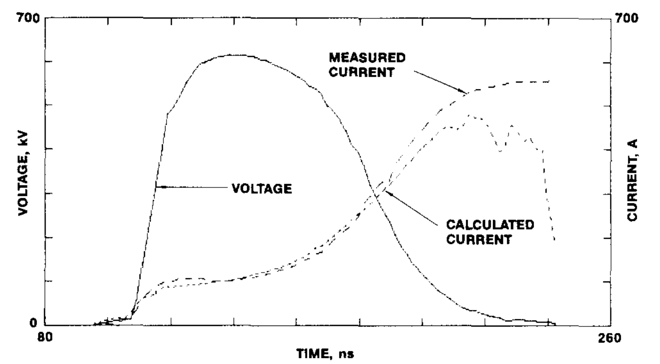

Figure 3. V-I characteristics and calculated current from simple ferrite model for core tested in Shot W157. Core dimensions listed in Table 1. $\alpha = 0.5$.

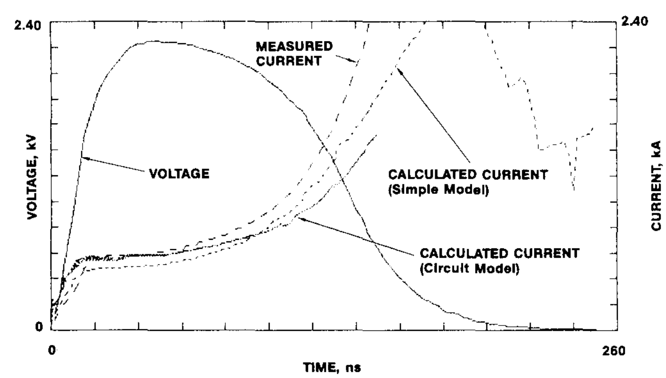

Figure 4. V-I characteristics and calculated current from both the simple ferrite model and circuit model for core tested in Shot W193. Circuit model assumes axial displacement currents with $\varepsilon = 12$. Core dimensions listed in Table 1. $\alpha = 0.5$.

Circuit Model

An attempt to apply the simple ferrite model to the ETA-II cell data was not successful, even after adjusting the intrinsic B-H loop to account for the difference reported for the saturation induction, and for the value of remanence required to produce the saturation of the core observed in the measurements. The omission of displacement currents was suspected to account for the discrepancy, so the circuit model technique described above was employed with the assumption that the displacement currents flow radially in the ETA-II cell. The results of the circuit simulation are shown in Figure 5, together with the measured voltage and current profiles and the results of the simple ferrite model. In the circuit simulations, the measured voltage pulse was applied to the ferrite core circuit from a zero impedance voltage source. The inclusion of displacement currents substantially increases the total current drawn by the core. A hypothetical reduction of displacement currents by an increase in the transmission line impedance used in the circuit model duplicated the results of the simple ferrite model.

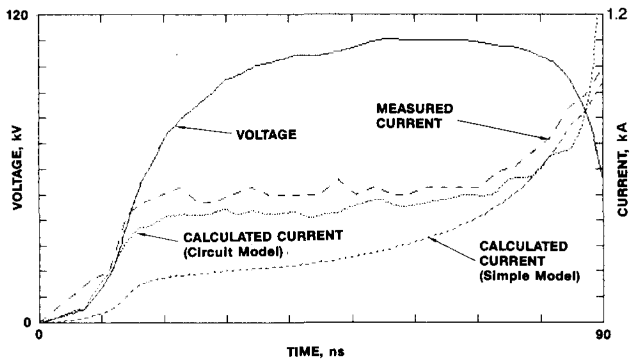

Figure 5. V-I characteristics and calculated current from both the simple ferrite and circuit model for ETA-II cell taken from References 2 and 3. Circuit model assumes radial displacement currents with $\varepsilon = 12$. Ferrite dimensions listed in Table 1. $\alpha = 0.5$.

A set of circuit simulations was performed for the ETA-II cell with different values of α. The results are shown in Figure 6. The value of $\alpha = 0.5$ produces the best agreement with the strikingly flat measured current profile.

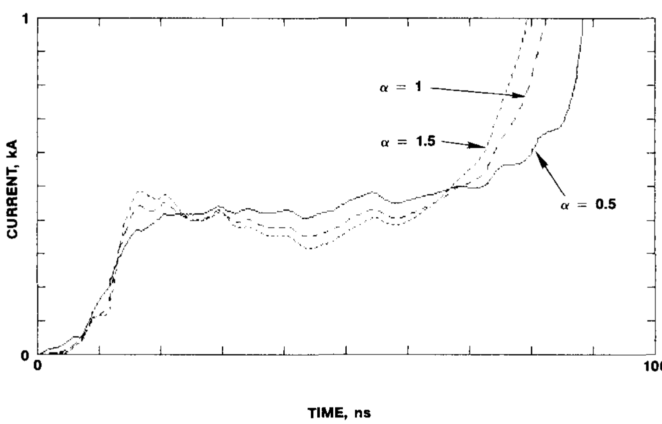

Figure 6. Calculated current from the circuit model for ETA-II cell for three values of α.

The circuit model was also applied to the data for Shot W193. In this case, the displacement currents were assumed to flow axially due to the thickness of the core. The results of the circuit simulation are shown in Figure 4. The inclusion of displacement currents provides some improvement between the calculations and measurement, but less than desired. The discrepancy at the end of the pulse reveals the difficulty in modeling the saturation of the cores. Very small errors in the calculation of the magnetic induction during the inference of the intrinsic B-H loop or the model calculation result in substantial errors in current. Similarly, small instantaneous or accumulated errors in the measurement of the applied voltage can result in the same effect. This problem is exacerbated by the use of zero impedance voltage sources in the calculations, which is necessary for the simple ferrite model.

LIA Cell Design

The circuit model can be applied to the design of LIA cells provided that properties of the ferrite to be used are known. If displacement currents are to be used in the model, then the direction of the currents must also be known. The circuit model for the ferrite can be combined with the equivalent circuit for the driving source, the equivalent circuit for the remainder of the cell, and the beam current load to any desired sophistication.

As an example of the design approach, an equivalent circuit for a proposed LIA cell was analyzed. It included a driving circuit with constant impedance, the feed rod that connects the external source to the accelerating gap, the capacitance of the gap, the inductance experienced by the beam propagating through the bore of the cell, and the beam current. This equivalent circuit is shown in Figure 7. The source impedance was chosen to give the desired 250 kV accelerating voltage with a beam current of 1 kA. The feed rod impedance and gap capacitance were determined from calculations performed with an electrostatic code. The source voltage and beam current profiles were chosen to have sine squared shapes during the 0 to 100% risetimes of 20 ns. Obviously, more stages in the pulse forming system could have been included to give a more realistic equivalent source. Displacement currents were included and assumed to flow radially.

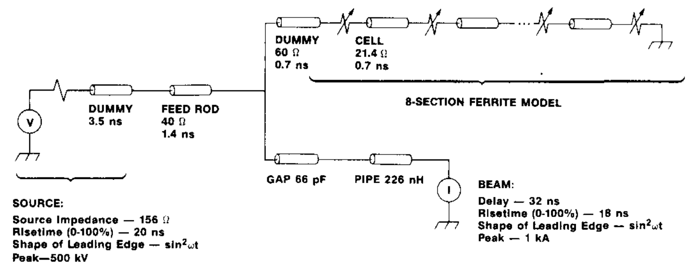

Figure 7. Equivalent circuit for proposed LIA cell. Nonlinear resistor characteristics defined as described in text. Dummy transmission line elements inserted to satisfy circuit topology constraints of circuit analysis code.

The results of the circuit simulation are shown in Figure 8. The current flow into the ferrite model is shown together with the beam current and the accelerating voltage experienced by the beam. The timing of the beam current relative to the source voltage was selected to produce a relatively flat accelerating voltage in spite of the substantial current drawn by the ferrite and gap capacitance early in the pulse.

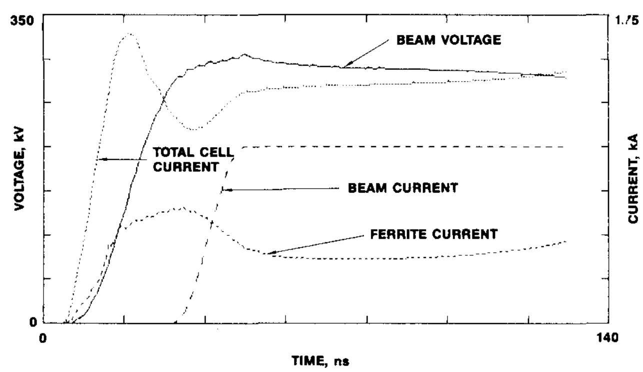

Figure 8. Calculated response of proposed LIA cell obtained with the circuit model as shown in Figure 7. Ferrite dimensions listed in Table 1. Radial displacement current flow and $\alpha = 0.5$ assumed.

The circuit model described above can be an effective technique for the design of LIA cells provided that sufficient information is available concerning the properties of the ferrite to be used at the magnetization rates of interest. The variation of the properties with ferrite composition and manufacturing process require specific and careful measurements of these properties for the circuit model to be useful.

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2. W. C. Turner et al., "Impedance Characteristics of Inducion Accelerator Coils," in **Proceedings of Seventh International Conference on High Power Particle Beams,** Karlsruhe, W. Germany 1988.

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