TRANSVERSE RESISTIVE-WALL INSTABILITY OF A BUNCHED ELECTRON BEAM IN A WIGGLER*

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Abstract

The transverse motion of a beam traversing a narrow beam pipe is modified by resistive-wall effects. Depending on the strength of the focusing force from the alternating wiggler field, the effect ranges from a modification of the oscillation to a growth in transverse displacement with the length of pipe. This transverse effect saturates after a number of bunches have passed. The saturated transverse effect depends only on the pipe radius b (it increases as $1/b^2$), but is independent of the thickness τ and conductivity σ of the pipe. However, τ and σ affect the time needed to attain saturation.

Introduction

When a charge travels in a smooth pipe of small radius, it will generate a wakefield if the pipe is not perfectly conducting. The transverse force is zero immediately behind the bunch because of the cancellation of the electric force of the image charge and the magnetic force from the induced current. A finite conductivity allows the induced magnetic field to penetrate into the metal pipe with the result that the magnetic force decays much more slowly than the electric force. This gives rise to a net wakefield force on the later bunches. This force increases rapidly as the radius of the pipe decreases. In future free-electron lasers (FELs), because of efficiency requirements and limitations on achievable magnetic field, an electron beam is required to travel in a pipe several millimeters in diameter over a length of 10 mm or more in the wiggler. Therefore, the question arises as to whether transverse resistive-wall effects of the electron beam could compromise the performance of the wiggler.

Estimates of the effects of the transverse resistive-wall instability were done previously with formulae derived by Caporaso *et al.*[1] These formulae were derived for a dc beam and with an induced magnetic field decreasing as the square root of time, a dependence valid only for a limited time. For an FEL injected with an rf linac that has a bunched beam and a *pulse* of long duration, these results are not appropriate. Neil and Whittum[2] have recently analyzed the case of a bunched beam. They investigated the problem in the frequency domain and used the dominant mode in the expression for the wakefield. Their analysis is equivalent to the case where the first bunch is displaced off-axis and the subsequent bunches follow on-axis.

In this paper, an analysis of transverse resistive-wall instability of a bunched electron beam in a wiggler is carried out. The full expression for the wakefield is used and a complete solution is obtained analytically. The steady-state solution is discussed. Because of space limitations, proofs of the equations are not given. These details can be found elsewhere.[3]

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Transverse Resistive-Wall Wakefield of a Bunched Beam Moving in a Circular Pipe

The transverse wakefield induced by a beam of relativistic particles off-axis has been investigated by Bodner *et al.*[4] When a dc beam current I established at time $t = 0$ is traveling at a distance ξ off-axis in a pipe of inner radius b, outer radius d, thickness $\tau(= d - b)$, and conductivity σ, the wakefield consists of a magnetic field B_y given (in cgs units) as

$$B_y(t) = \frac{8I\xi}{cb^2} \sum_{i=1}^{\infty} \frac{\exp(-t/T_i)}{C_i} \quad , \tag{1}$$

where

$$T_i = \frac{4\pi\sigma b^2}{c^2 y_i^2} \quad , \tag{2}$$

$$C_i = y_i^2 \left(\left[\frac{J_2(y_i)}{J_0(y_i d/b)} \right]^2 - 1 \right) \quad , \tag{3}$$

and y_i's are zeros of the function

$$J_0(yd/b)N_2(y) - J_2(y)N_0(yd/b).$$

Here J_i's and N_i's are Bessel functions of the first and second kind, respectively.

The expression for the magnetic field at time t behind a (delta-function) bunch with charge q is found from Eq. (1) to be

$$B_y(t) = -\frac{8q\xi}{cb^2} \sum_{i=1}^{\infty} \frac{\exp(-t/T_i)}{C_i T_i}. \tag{4}$$

The magnetic field of a bunched beam traversing the beam pipe with a fixed time interval Δ between any two bunches will be a sum of the fields of the individual bunches. All the bunches taken together form a *pulse*. In Eq. (4), the longest decay time is T_1; for a *thin-wall* approximation (small τ/b) it is given by

$$T_1 \approx \frac{2\pi\sigma b\tau}{c^2}. \tag{5}$$

The decay time T_1 is usually referred to as the *diffusion time* and has the value of 0.5 μs for the parameters in Table I. These parameters are related to parameters used for a proposed XUV FEL.[5]

TABLE I. RELEVANT PARAMETER VALUES

b	0.18 cm
d	0.198 cm
σ	$2 \times 10^{16} s^{-1}$ (for titanium)
B	0.75 tesla
pulse length	300 μs
energy	500 MeV
pipe length	800 cm
bunch separation	6.8 ns
average current	300 mA

An approximate formula for the magnetic field behind a (delta-function) bunch, valid for short times, can be derived from Eq. (4) as

$$B_y(t) = -\frac{2q\xi}{\pi b^3(\sigma t)^{1/2}}. \qquad (6)$$

The variation of B_y with t as given by the exact formula [Eq. (4)] is compared with the approximate formula [Eq. (6)] in Fig. 1. The approximate formula is a good approximation to the exact formula up to one *diffusion time*. After one *diffusion time*, the magnetic field drops off rapidly as an exponential function of time because it has diffused through the pipe wall. One *diffusion time* is much shorter than the value of our *pulse length* of 300 μs (see Table I), which is typical of an rf-based FEL. Therefore, the exact formula should be used for the analysis of such rf-based FELs.

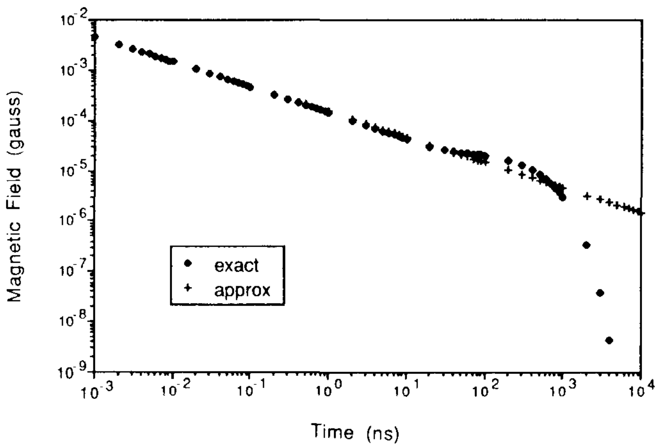

Fig. 1. Variation of the transverse resistive-wall wakefield B with time t elapsed since the passage of a bunch. The results obtained by using the exact formula [Eq. (4)] are compared with those obtained by using the approximate formula [Eq. (6)].

Transverse Resistive-Wall Instability of a Bunched Beam

The beam is considered to be a series of bunches traveling with speed c. The transverse displacement from the axis of the Kth bunch is denoted by $\xi(z = ct, K)$. Employing the Lorentz force equation and Eq. (4), the equation of motion for $\xi(t, K)$ is found to be

$$\frac{d^2\xi(t,K)}{dt^2} + \omega_0^2\xi(t,K) = \sum_{i=1}^{\infty} G_i \sum_{l=0}^{K-1} \exp(-(K-l)\Delta/T_i)\xi(t,l), \qquad (7)$$

where

$$G_i = \frac{8cqv}{m\gamma c^2 b^2 C_i T_i}. \qquad (8)$$

Here ω_0 is the frequency of the betatron motion caused by the alternating wiggler field, γ is the usual relativistic factor, and v is the z-component of beam velocity. The sum over l represents the sum of the interactions between the Kth bunch and the wakefields of all bunches ahead of it.

Equation (7) has been solved analytically using Laplace transforms.[3] In this paper, the analytic solution is given only for the simple case where the first mode is assumed to dominate (i.e., only the first term in the sum over i is considered). This assumption is valid for the parameter values given in Table I. A complete solution is obtained after assuming that the initial transverse velocity is zero for all bunches and after considerable manipulation.[3] For $K \geq 1$:

$$\bar{\xi}(t,K) = \sum_{k=1}^{K} \xi(0, K-k)\exp(-k\Delta/T_1)$$
$$\sum_{n=1}^{k} \frac{G_1^n C_{k-n}^n \pi^{\frac{1}{2}}}{2^{n+\frac{1}{2}}\omega_0^{2n} n!}(\omega_0 t)^{n+\frac{1}{2}} J_{n-\frac{1}{2}}(\omega_0 t) \quad , \qquad (9)$$

where

$$\bar{\xi}(t,K) = \xi(t,K) - \xi(0,K)\cos(\omega_0 t) \qquad (10)$$

and

$$C_l^n = (-1)^l \frac{(-n)(-n-1)\dots(-n-l+1)}{l!}. \qquad (11)$$

The displacement $\bar{\xi}(t,K)$ is zero for $K = 0$.

Equation (7) was also solved numerically using a computer program. The initial displacement $\xi(0,j)$ was assumed to be equal to 1.0 for all j. The result for I = 300 mA is shown in Fig. 2, which shows the behavior of $\xi(t,K)$ as a function of $K\Delta$ at the end of the pipe (ct = 800 cm). Results obtained using Eq. (9) confirm both the approach to saturation and the saturation value.[3] The unbounded growth seen by using the approximate formula [Eq. (6)] does not exist in actuality because the magnetic field decays exponentially for times longer than one *diffusion time*. The adverse effects of resistive-wall instability on the operation of the FEL are, therefore, limited.

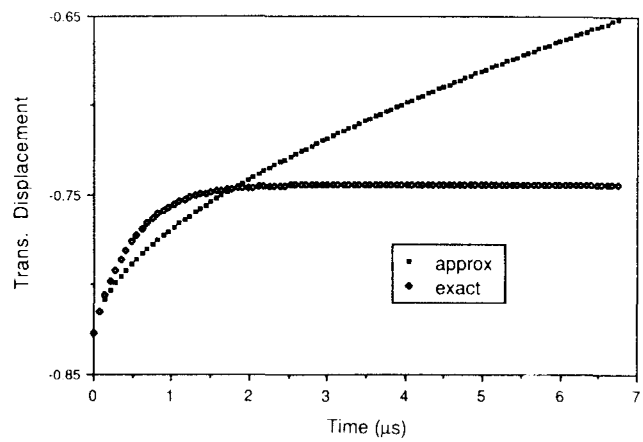

Fig. 2. The transverse displacement of the beam at the end of the wiggler obtained by numerically solving Eq. (7). The bunches travel along the beam pipe and execute betatron oscillations with frequencies modified by resistive-wall effects. The amplitude of all oscillations is the same and is normalized to unity. Results obtained by using the approximate formula are also shown.

A simpler expression for the saturation value $\xi(t,\infty)$ can be derived. In the limit $K \to \infty$, all $\xi(t,K)$ in Eq. (7) can be replaced by $\xi(t,\infty)$ to give

$$\frac{d^2\xi(t,\infty)}{dt^2} + (\omega_0^2 - \Omega^2)\xi(t,\infty) = 0 \quad , \qquad (12)$$

where

$$\Omega^2 = \sum_{i=1}^{\infty} G_i \sum_{j=1}^{\infty} \exp(-j\Delta/T_i). \qquad (13)$$

The quantity Ω^2 represents the defocusing effects of the resistive-wall instability. For $\Omega^2 < \omega_0^2$, Eq. (12) is just a betatron oscillation equation with reduced focusing. The betatron frequency has changed from ω_0 to $\omega_0(1-\Omega^2/\omega_0^2)^{\frac{1}{2}}$. For $\Omega^2 > \omega_0^2$, the defocusing effects become so large that $\xi(t,\infty)$ grows exponentially with time. It can be shown[3] that

$$\Omega^2 = \frac{2cI\nu}{m\gamma c^2 b^2}. \qquad (14)$$

Thus, Ω^2 is independent of the conductivity σ and thickness τ of the pipe. For the parameters given in Table I, ω_0 and Ω have values of $9.53 \times 10^7 s^{-1}$ and $3.16 \times 10^7 s^{-1}$, respectively.

As a final remark, we draw attention to the fact that the *time* required to achieve a steady-state solution *does* depend on τ and σ of the pipe. It was seen earlier that the magnetic field diffuses out through the wall on a time scale of one *diffusion time* (T_1). Therefore, one would expect $\xi(t, K)$ to attain $\xi(t, \infty)$ within a few T_1's. Using $5T_1$ for specificity, we obtain the number of bunches K_∞ required to reach a steady-state solution to be

$$K_\infty \approx \frac{10\pi\sigma b\tau}{\Delta c^2}. \tag{15}$$

For the parameters given in Table I, we estimate the value of K_∞ to be approximately 330, in agreement with the numerical result (Fig. 2).

References

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