# Design Study for a 500 GeV Linear Collider

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### Abstract

The feasibility of a 500 GeV<sub>c.m.</sub> linear collider has been studied, which is based lmost entirely on conventional rf-technology. The basic components are S-band travelling wave, constant-gradient accelerating structures and 130 MW klystrons. 3 GeV damping rings are used to produce extremely small emittances in both planes which are in the same range than these of the next generation synchrotron light sources. In both cases, the envisaged methods concerning stability and feedback give rise to the assumption, that the proposed values can be reached today. Also very strong focussing and a dedicated chromatic correction scheme near the interaction region is necessary to reach spot sizes that have not been produced yet. We present a status report of the investigations that have recently been started. Already at this early stage of our studies it seems that such a collider is technically feasible and could indeed be built in this decade.

# Introduction

Linear  $e^+-e^-$  colliders will open up an energy range for particle physics, which circular machines cannot reach for reasons of size and economy. The very first step in this new energy regime which only will be accessible to linear  $e^+-e^-$  colliders is generally considered to be between center of mass energies of 200 GeV and 500 GeV. This is the mass-range where the missing 6<sup>th</sup> quark, the top quark, is expected to be produced and where p-p-colliders have difficulties to search for the Higgs particle.

Linear colliders are in a uniquely difficult position in so far, that the very first machine of this new kind must already be of enormous size to be of interest for particle physics. There is no slow evolution possible.

The risks in building a new project in the billion dollar range using new and untried technologies are great. New ideas [1] like two-beam accelerators, wake field accelerators, very high gradient and very high frequency accelerators might eventually mature to the point where the construction of a large collider becomes feasible, but the required time for the necessary R + D-work may be much longer than often optimistically assumed. The question is therefore of interest, what a linear collider based on present-day technology might look like and how it compares with the more daring studies presently under way elsewhere. To avoid misconceptions it must be pointed out that any linear collider with energies and luminosities of interest for high energy physics requires extensions of known technologies beyond what is available today: Damping rings must produce beams with smaller emittances than ever before, the requirements on emittance control during acceleration have to be much tighter than in the past and the final focus with linear beam size demagnifications of three orders of magnitude also

• This Study is a summary of the work of groups from Deutsches Elektronensynchrotron **DESY** and Technische Hochschule Darmstadt **THD**. requires extensive studies. While these are formidable problems, their solutions do not seem to be out of reach. Development work on existing facilities such as the one by the international collaboration on the SLAC final focus test facility FFTF might produce the required solutions in time.

In our study we assume basically a linear collider using present SLAC technology: Two S-band linacs, each of which 5 times as long as the SLAC-linac, direct their beams against each other. Each linac uses an accelerating gradient of 17 MeV/m, which may not be too far from the gradient of a cost opimized machine in which costs of total rf-power and accelerator length are balanced. In order to improve power efficiency we will use 2 µsec long current pulses with many bunches (172). The number of particles in each bunch is limited to 7.10<sup>9</sup>, such that the total energy spread produced by the fundamental and higher order mode losses of  $\Delta E/E = \pm 0.7 \%$  is still within the acceptance of the final focus. This bunch charge together with the number of bunches within the rf-pulse represent an avarage pulse current of 100 mA, not unlike the currents already achieved in the SLAC-linac. The individual bunch charge is considerably smaller than that which is stably accelerated in the SLAC-linac, but multibunch instabilities will have to be carefully examined.

The R + D-work which we assume to be going on before such a study reaches the proposal stage may result in S-band klystrons of 130 MW and 3µsec pulse duration, a modest improvement on the 130 MW, 1µsec-klystrons already available today. Other R + D-work should concentrate on economizing linac technology. Klystron modulators, the largest single cost item, might be built with hard tube pulsers, thereby avoiding the expenses for the costly pulse transformers and the low inductance pulse capacitors. A special study of a hopefully cheaper hard tube pulser is on the way.

A parameter list of the main parameters is given in table 1.

# **Damping Rings**

The magnetic lattice for a 650 m circumference damping ring at an energy of 3.15 GeV has been designed [2].



Figure 1: Unit cell of the damping ring arc structure.

General Parameters		
Anargy	CeV	$250 \pm 250$
luminosity (incl. crossing angle	uc v	200   200
no ehancement from disruption)	$(cm^2 sec)^{-1}$	$24.10^{33}$
active length	(cm sec)	29411
repetition rate	H <sub>7</sub>	50
number of particles per bunch	112	7.109
Particle Production and Damping Bings		
damping ring energy GeV 315		
damping time	msec	3.10
ting singumforence	maec	650
ring circumerence	111	410/4 10-8
invariant emittance $\gamma \epsilon_{x,y}$		410/4 • 10
energy spread	70	0.112
ri-voltage		5.0
ri-irequency	MHZ	500
bunch length $\sigma_s$	mm	3.0
wiggler peak heid	1	2.0
wiggler period length	m	0.20
wiggler total length	m	84
dynamic acceptance	m	4.10-0
Main Linac		
wave length	m	0.10
average shunt impedance	$M\Omega/m$	53.6
attenuation	neper	0.57
structure length	m	6
group velocity	% of c	4.1 - 1.3
filling time	μsec	0.825
maximum energy width (peak)	%	$\pm 0.7$
klystron power	MW	130
number of klystrons		2451
structures per klystron		2
klystron efficiency	%	45
total rf-pulse length	usec	2.8
zero current energy	GeV	536
mean power	MW	94
rf-peak power	MW	271 000
average pulse current	mA	100
average pulse current		100
current pulse length	$\mu sec$	179
number of bunches per pulse		112
bunch to bunch distance	m	3.2
bunch length (rins)	mm	0.2
Final Focus and Interaction		
$\beta$ -function at IP $\beta^*_{x,y}$	mm	3, 0.3
beam dimension at IP $\sigma^*_{x,y}$	nm	169, 5.48
aspect ratio		30.8
crossing angle	mrad	$\pm 0.8$
disruption parameter $D_x$ , $D_y$		0.53, 16.4
luminosity enhancement		1.8
maximum disruption angle	mrad	0.77, 0.20
dilution parameter		0.38
critical energy of beamstrahl.	GeV	78
mean number of beamstrahl.		
photons per particle		1.1
critical radiation parameter		0.208
mean fractional energy loss		0.06
mean fract. reduct. c.m. energy		0.021
momentum acceptance	%	$\pm 0.8$

Table 1: Table of general parameters

A normalized emittance of  $\gamma \epsilon_x = 4.1 \cdot 10^{-6}$  m and a transverse damping time of  $\tau_{x,y} = 3.8$  msec are achieved. The layout is based on a modification of a Chasman-Green lattice and includes wiggler sections with a total active length of 84 m assuming a peak field of 2 T in the wiggler magnets (Fig. 1). The lattice is optimized for a local compensation of the nonlinear distortions caused by the sextupoles required for chromaticity correction. As particle tracking results show, this leads to a lower sensitivity of the dynamic aperture to errors than with other designs (e.g. a FODO-lattice). Both a sufficient injection acceptance (about  $4 \cdot 10^{-6}$  m including the wiggler nonlinearities) and the desired small emittance coupling ratio ( $\epsilon_y / \epsilon_x = 1\%$ ) can be achieved with a tolerance limit of 0.1 mm (rms) for transverse magnet position errors. An investigation of coherent (especially multibunch-) instability problems is on the way.

# High Voltage Pulser

The high voltage pulsers are the most expensive item of the S-band linear collider. Thus some investigations have been started to develop a hard tube pulser as an alternative to the commonly used systems employing a high voltage, short pulse transformer.

In collaboration with SLAC it is planned to set up a hard tube pulser next year as a first step. Although these klystron pulsers seem to be the "least interesting" portion of a linear collider, much attention must be paid to these and they will certainly be one of the key issues of R & D work in the next year.

#### Accelerating Structures

The design of the structure is based on the well known features of constant-gradient travelling wave tubes with a phase advance of  $2\pi/3$  per cell [3]. As a preliminary choice an attenuation of 0.57 neper is chosen. The maximum structure-length is  $\approx 12$  m and given by the peak power limit of one klystron. In order to have a moderate variation in group velocity (4.1 to 1.3 % of c) and shunt impedance, the length of one disk loaded structure is chosen to be 6 m. In this case the required peak power per tube is 56 MW to reach a loaded gradient of 17 MeV per meter. Using S-band frequency, conservative assumptions for the Q-value (13000) and for the shunt impedance (53.6 M\Omega/m) are made for the calculations.

Two possibilities are investigated: - The 130 MW Klystrons will feed two of the forward-wave structures. - The second possibility requires two 6 m structures of different type one forward-wave and one backward-wave structure with one coupler in the middle of the tube. This idea may have some advantages, for example, concerning transverse multi-bunch instabilities driven by dipole modes, which will have completely different frequencies in the two tubes. The transverse kicks in the coupling cells due to phase- and amplitude-asymmetries are avoided by feeding the coupler cell from both sides.

#### **Transient Beam Loading**

The current pulse length (2  $\mu$ sec) in this type of collider is much longer than one filling time (T<sub>f</sub> = 0.825  $\mu$ sec) of the assumed 6 m long rf-structure. Starting with beam injection exactly after one filling time T<sub>f</sub> causes a bunch to bunch energy variation during the following second filling time due to the transient beam loading. The voltage as a function of time for one tube is shown in Fig. 2. For a constant gradient structure the full energy spread is given by (1):

$$\Delta V = -\frac{R'_{s} \cdot i \cdot L}{2} \left[ 1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}} \right] , \qquad (1)$$

 $R'_s =$ shunt-impedance per meter

i = the average current in one pulse

L = structure length

 $\tau$  = attanuation parameter ,



Figure 2: Voltage as a function of time for one 6 m long travelling wave tube.

and approximately 6.7% in this case, which is far beyond the acceptance of the final focus and therefore must be compensated.

Two possible schemes have been investigated. First, injecting the bunch train after the loaded voltage  $V_0$  has been achieved and before the structure is completely filled, which is also the proposed method for the other linear collider schemes, reduces the energy spread to 5.2%. With the second scheme, the bunches being accelerated during the second filling time, will have different phases with respect to the crest of the wave. This method enables an almost exact compensation of the energy spread due to phase variation from bunch to bunch. Let  $V_{max}$  be the unloaded voltage, the phase angle of the first bunch is given simply by :

$$\frac{V_0}{V_{\text{max}}} = \cos(\phi_0), \qquad (2)$$
  
with:  $\phi = 0$  be the crest of the wave  
and  $\phi_0 = 21^0$  be the phase angle of bunch # 1

Starting with the unloaded Voltage  $V_{max}$  (at t'=0 and t'=T\_f+t)), the energy gain of the following bunches is given with :

$$V(t) = V_{\max} \cdot \cos (\phi_0 + \Delta \phi(t')) \\ - \frac{R_s i L}{2} \left[ 1 - \frac{\omega}{Q} e^{-2\tau} \cdot t' - e^{-\omega/Q \cdot t'} \right] \quad (3)$$
  
with:  $\phi_0 + \Delta \phi(t') = 0 \text{ for } t' = T_f$ 

To provide the phase variation as a function of time, a conventional bunch compressor is used as a **'bunch to bunch'**compressor, in order to transform a given energy spread into a distance variation according to :

$$\frac{\Delta\phi(\mathbf{t}')}{2\pi} = \frac{\Delta \mathbf{s}}{\lambda} = \frac{1}{\lambda} \cdot \text{const}_{\text{comp}} \cdot \frac{\Delta \mathbf{p}(\mathbf{t}')}{\mathbf{p}_{0}}$$
(4)

with a typical compressor parameter of  $const_{comp} = 4.7 / (\frac{\Delta p}{p_0})$  [mm/%] as for example presented in [4].

The required 'bunch to bunch' energy spread is in the range of 1.2% at 3.15 GeV, the damping ring energy. The energy variation before the 'bunch to bunch'-compressor can be produced by short low-Q cavities to make the desired fast voltage control possible, analogous to the transient beam loading  $\Delta V(t)$  as a function of time. First calculations show, that the energy spread in the main linac can be reduced to less than 0.3%. The possibility of making the first

stage bunch compression and the 'bunch to bunch' compression for energy compensation in one single compressor is still investigated.

#### Single–Bunch Instabilities

Each of the 172 bunches in one pulse has to traverse a large number of acceleration gaps before they reach their final energy of 250 GeV. An energy spread along the bunch is induced by longitudinal wake field effects. Additionally transverse wake fields will be excited if the bunch moves off axis through an acceleration structure due to various forms of jitter. In the SLC the beam developes a corkscrew-like tail. This process of single bunch *beam breakup* is extensively studied analytically and numerically in [5]. Transverse wake fields scale roughly with the square root of the rms-bunch length. Since our bunch length ( $\sigma = 0.2 \text{ mm}$ ) is five times smaller than used in the SLC and since our bunch population is only  $7 \times 10^9$  electrons (or positrons) per bunch, transverse wake fields effects will be small by design for the  $2 \times 250$  GeV linear collider.

So far only estimations of this effect have been performed. The single bunch *beam breakup* can be estimated in the smooth focussing approximation using a two particle model. The equations of motion are for the head particle :

$$\frac{d}{ds}\left(\gamma \frac{d}{ds}x_1\right) + \gamma k^2 x_1 = 0 \tag{5}$$

and for the tail particle

$$\frac{d}{ds}\left(\gamma \frac{d}{ds}x_2\right) + \gamma k^2 x_2 = \frac{e}{m_0 c^2} \frac{q}{2} x_1 W_\perp . \tag{6}$$

 $x_1, x_2$  are the transverse position of the head and the tail particle,  $\gamma$  the normalized energy,  $\lambda_{\beta} = 2\pi/k$  the local wave length of the betatron oszillations,  $q = e7 \times 10^9$  the total charge in the bunch and  $W_{\perp}$  the transverse wake potential at the tail.

These equations can be solved assuming that the acceleration is adiabatic. We obtain for the relative deviation of the tail :

$$\left\|\frac{x_2 - x_1}{x_1}\right\| = \frac{\lambda_\beta}{4\pi} \frac{q}{2} e W_\perp \frac{1}{g} \ln\left(1 + \frac{g}{E_0}s\right) . \tag{7}$$

For a gradient g of 17 MeV/m, an initial energy  $E_0 = 3.15$  GeV, and  $\lambda_{\beta} = 100$  m we have

$$\left\|\frac{x_2 - x_1}{x_1}\right\| = 91\% \tag{8}$$

after s = 15 km. The dipol wake potential  $W_{\perp} = 0.79 \times 10^{15}$  V/(m<sup>2</sup>C) was scaled from TBCI - results [6] for a

 $\sigma = 3 \,\mathrm{mm}$  long bunch. Numerical calculations will provide more accurate results for the 0.2 mm long bunches. So far no BNS-damping was invoked, which certainly can reduce this effect significantly.

The total (peak to peak) energy spread within one bunch due to longitudinal wake fields was found to be

$$\frac{\Delta E}{E} = 1.2 \% . \tag{9}$$

Due to the fact that the bunch length is very short compared to the rf wave length this energy spread cannot (and needs not to) be compensated by positioning the bunch on the rf wave scope as proposed for the SLC. These estimates for the transverse and longitudinal single bunch beam dynamics demonstrate that the *beam breakup* effects seem to be controlable. Further studies with discrete focussing elements with the program LTRACK [5, 7] will give more detailed results and are under way.

#### Multibunch-Instabilities

Long current pulses with 172 bunches per pulse and a population of  $7 \cdot 10^9$  particles per bunch can excite strong transverse electromagnetic fields in the structure and may limit the current due to *beam breakup*. This instability is driven by the dipole modes of the structure, which are resonantly excited by an off axis beam. Regenerative *beam breakup* will be significantly reduced by the constant-gradient structure, in which the dipole mode frequencies differe from cell to cell due to the tapering of the iris. Cumulative *beam breakup* is probably the most severe problem which is currently investigated by particle tracking.

In order avoid the use of expensive damped structures [8] where the dipole modes are coupled out by slotted irises, more simple structures are investigated. One obvious method is, to randomly destribute the dipole mode frequencies in different sections of the linac over the entire length of 15 km. This can be achieved by using slightly different geometries for the cavities, that have to be tuned for the accelerating  $2\pi/3$  mode only. The use of a combined forward- and backward wave structure may be another possibility which is still investigated. This leads to two rather different passbands of the HEM-modes and can further improve the situation. At present the strength of the cumulative *beam breakup* is being investigated with tracking programs and the effects of various potential remedies are analyzed

#### **Ground Motion - Orbit Distortions**

In order to minimize the influence of transverse wake fields, strong quadrupole focussing is required. We assume that the focussing strength may be decreased with increasing energy thus avoiding extraordinarily long quadrupole magnets in the high energy end of the linac. With the average beta  $\bar{\beta}$  scaling roughly as  $\bar{\beta} = 0.09 \text{ m} \cdot \sqrt{\gamma}$  we obtain a magnet filling factor of less than 0.05 everywhere and a total of N  $\approx 1000$  quadrupole magnets. It can be shown that for any scaling law  $\bar{\beta} \approx \gamma^{\epsilon}$  with  $0 \leq \epsilon \leq 1$  the rms orbit distortion  $\sigma_t$  due to rms quadrupole motion  $\sigma_{q,t}$  is estimated at [9] :

$$\sigma_{\mathbf{t}} = 2 \sigma_{\mathbf{q},\mathbf{t}} \sqrt{N}, \qquad \mathbf{t} = \mathbf{x}, \mathbf{y} . \tag{10}$$

As seen from long term measurements of ground motion in the HERA tunnel [10] we have to expect  $\sigma_{(q,y)} \approx 0.1 \,\mu\text{m}$ during working hours (see Figure 3) resulting in  $\sigma_y \approx 6 \,\mu\text{m}$ . This would be more than twice the vertical beam size. If compared with measurements taken at numerous different places in Germany [11], this amount of vertical ground motion is not particularly high, but quite typical for ground noise close to big human settlements. Since the dominant frequency component is around 5 Hz, the 50 Hz repetition rate does not allow effective beam stabilization by means of beam position measurement and feedback. To achieve the desired vertical quadrupole stability of  $\leq 20 \,\text{nm}$  for frequencies  $\geq 1 \,\text{Hz}$ , we propose continuous measurement of quadrupole motion by geophones and local compensation of each orbit kick by the dipole correction magnet attached to each



Figure 3: Long term measurement of rms-ground motion in the HERA tunnel during 1 week. The cut-off frequency of the ground motion detector is 1 Hz. The averaging time for the rms-values was one minute. The broken line represents our tolerance of uncorrelated rms vertical quadrupole motion. For the final focus quadrupoles considerably tighter tolerances will be specified.

quadrupole magnet. Alternatively, passive damping of quadrupole supports is considered.

#### **Final Focus Lattice**

Investigations of a final focus system with  $\beta_x^* = 3 \text{ mm}$ and  $\beta_y^* = 0.3 \text{ mm}$  at the interaction point have been performed. The layout is based on the CLIC lattice design by Napoly and Zotter [12] (see Fig. 4). First results show that for 250 GeV particle energy the momentum bandwidth



of the system can be increased to at least  $\Delta p/p = \pm 0.8\%$  (originally  $\pm 0.41\%$ ) by introducing additional sextupoles for chromaticity correction (see fig. 5).

The emittance growth caused by synchrotron radiation and sextupole nonlinearities is negligible for the ideal lattice. Studies of tolerance limits as well as practical correction algorithms have not been performed yet.

#### Beam-Beam effects

Due to our large aspect ratio, the disruption parameters are much different in both transverse planes [13]:

$$D_x = 0.53$$
,  $D_y = 16.4$ .

In order to allow the beam to freely pass the final focus quadrupoles of the opposite beam, we have chosen a finite



Figure 5: Relative change of the  $\beta$ -functions at the interaction point with momentum deviation before (a) and after (b) optimization with additional sextupoles (solid line: $\beta_{\mathbf{x}}^{\star}$ , dashed line: $\beta_{\mathbf{y}}^{\star}$ ).

horizontal crossing angle of  $\pm 0.8$  mrad. This is considerably larger than the maximum disruption angles as estimated from [14]:

> $\theta_{\rm y} = 0.20 \, {\rm mrad}$  $\theta_{\mathbf{x}} = 0.77 \,\mathrm{mrad}$ ,

However, this estimation does not consider the effect of the finite crossing angle and the rare beamstrahlung events with photon energies much larger than the critical energy  $u_c$  of beamstrahlung. The latter has been treated in [15] to show that, with our parameters, less than one electron per interaction will experience a kick  $\geq 2\theta_x$ . Theory predicts a luminosity pinch enhancement  $H_D$  due to mutual focusing of the beams during interaction. Using the formula of Chen and Yokoya [16] we expect  $H_D \simeq 2$ . However, to be conservative, we do not include this effect in our luminosity estimation nor do we make use of it to relax orbit jitter tolerances.

The critical radiation parameter is:

$$\Upsilon = \frac{2}{3} \frac{\mathbf{u}_c}{\mathbf{E}_0} = 2 \frac{\lambda_c}{2\pi} \frac{\mathbf{r}_e \, \mathbf{N} \, \gamma}{\sigma_s \left(\sigma_x + \sigma_y\right)} = 0.208 \tag{11}$$
with :

 $\lambda_{\rm c}/2\,\pi$  = Compton wavelength of the electron

- $r_e = Classical electron radius$ N = Number of particles per bunch

The critical beamstrahlung is formed within a distance  $l_r \approx$  $1\mu m$  which is much smaller than the bunch length. Thus it should be insensitive to details of the particle distribution function. The mean relative energy loss is estimated to be [17]:

$$<\epsilon>=rac{1}{3\sqrt{\pi}}rac{\mathbf{r_e}\sigma_{\mathbf{s}}\Upsilon^2}{(\lambda_c/2\,\pi)^2\gamma}pprox 0.06$$
 (12)

This formula holds for  $\Upsilon \ll 1$  and no pinch effect. While the first condition is satisfied quite well, we do expect some pinch. But since it will occur in the vertical plane mainly  $(D_x = 0.53)$  and because of  $l_r \ll \sigma_s$ , its influence on  $\Upsilon$  and  $<\epsilon>$  is expected to be small ( $\sigma_y \ll \sigma_x$  in equation (11)).

#### Summary

The aim of our study is, to use where ever possible conventional technology in order to reduce the immanent risks of the first linear collider. It seems from our recently started investigations that many components may in fact be taken "off the shelf" and that such a collider of 500 GeV center of mass energy could indeed be built before the end of this century.

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#### References

- [1] S. Turner, editor. Proceedings of the ECFA-CAS-IN2P3-IRF-EPS Workshop. New Developments in Particle Accelerator Techniques, Orsay, Frankreich, Juli 1987. ECFA 87/110. CERN 87-11, October 1987.
- [2] A Study of Low-Emittance Damping Ring Lattices. R. Brinkmann, DESY-M/90-09, 1990.
- [3] Structure Design for a 500 GeV S-Band Linear Collider, P. Hahne, N. Holtkamp, R. Klatt, T. Weiland, this conference.
- [4] Bunch Compression for the TLC, S.A. Kheifets, R.D. Ruth, T.H. Figuth, SLAC-PUB-5034, 1989.
- [5] Wakefield Effects in a Linear Collider, K.L.F. Bane, SLAC-PUB-4169, 1986.
- [6] T. Weiland, Transverse beam cavity interaction part 1: short range forces, NIM 212 (1983), p. 13-21.
- [7] A. Chao, R. Cooper, Particle Accelerator 13, 1, 1983. SLAC-PUB-4169, 1986.
- [8] Damped Acceleration Cavities, R.B. Palmer, SLAC-PUB-4542, 1988.
- [9] J. Roßbach, private communication.
- [10] Measurement of Slow Closed-Orbit Motion in the HERA Electron Ring in Correlation with Ground Motion, W. Decking, K. Flöttmann, J. Roßbach, Proc. EPAC Conf, Nice, 1990.
- [11] Systematische Untersuchung der kurzperiodischen seismischen Bodenunruhe in der Bundesrepublik Deutschland, M. Steinwachs, Geologisches Jahrbuch, Reihe E (Geophysik), 3, 1974.
- [12] Progress Report and the CLIC Final Focus Design, O. Napoly, B. Zotter, CERN, CLIC-Note 107, 1989.
- [13] Überblick über dieParameterderStrahl - Strahl Wechselwirkung am  $2 \times 250 \text{ GeV}$ -Linear Collider, J. Roßbach, DESY-M-VM 90-02.
- [14] Approximate Formulae for TeV Linear Colliders, R.B. Palmer, SLAC, AAS-30, 1987.
- [15] Electron Energy Spectrum and Maximum Disruption Angle under Multi-Photon Beamstrahlung, K. Yokoya, P. Chen, SLAC-PUB-4935, 1989.
- [16] R.B. Palmer, SLAC-PUB-5195, 1990.
- [17] An Introduction to Beamstrahlung and Disruption, P. Chen, Lecture Notes in Physics, 296, Springer, 1988.