RF PARAMETERS AND SCALING RELATIONS FOR FOUR CHAMBER RFQ CAVITIES.

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ABSTRACT

The comparison (in 2D approximation, without taking into account end plates) of RF parameters for four chamber RFQ cavities, differing in chamber shape, is done. Using lumped parameters expressions for RF characteristics estimations are given. It is shown that per unit length capacity between electrodes depends only on the shape of the electrode and don't depend on dimension. Dependence of cavity characteristics from working frequency is considered.

INTRODUCTION

RFQ [1] cavities becomes now the necessary part of modern linear ion accelerators. Features of the structure for particle beams are now well known. There are several methods to form quadruple field distribution. Four chamber H-cavity is widely used. Differing in design, four chamber cavities have some general properties. In this paper attempt to analyze this properties is done.

DEFINITIONS OF VALUES

Numerical calculation of the RFQ cavity properties with end plates and electrode modulation in account is essentially 3D problem. But main RF characteristics of the regular part of the cavity can be estimated with using usual 2D codes, SUPERFISH [2] or MULTIMODE [3]. For consideration, let's define main parameters. Taking into account 2D approximation, we shall use all quantities divided on length of the cavity:

a) RF power dissipation

$$P_s = \frac{R_s}{2} \int_{\Gamma} H_z^2 \, d\Gamma, \qquad (1)$$

b) stored energy

$$W_0 = \frac{\mu_0}{2} \int_S H_z^2 \, dS, \qquad (2)$$

c) capacity of the electrodes

$$C = \frac{2 W_0}{U^2}, \qquad (3)$$

d) shunt resistance (for transversal focusing field)

$$R_t = \frac{U^2}{2 P_s}, \qquad (4)$$

Voltage between electrodes and quality factor don't depend on the length

$$U_0 = \omega \mu_0 \int_S H_z \, dS, \qquad (5)$$

$$Q = \frac{\omega W_0}{P_s}, \qquad (6)$$

Here R_s is surface resistance, Γ,S are boundary and square of the chamber. To avoid misunderstanding, we shall use all quantities, mentioned above, for one chamber, so integration in (1),(2),(5) is inside one chamber. For the whole cavity, taking into account identity of all chambers, we easy find

$$P_s^{\Sigma} = 4 P_s, \quad W_0^{\Sigma} = 4 W_0, \quad U^{\Sigma} = U, \quad C^{\Sigma} = 4 C,$$
$$R_t^{\Sigma} = R_t/4, \quad Q^{\Sigma} = Q$$

All quantities, mentioned above, can be calculated during numerical simulation. But H-cavity may be considered as a device with lumped parameters. Plots of lines H_z=const on fig.1 for different shapes of chambers are shown. In 2D approximation lines H_z=const coincides with lines of the electric field. Number of lines at each plot equals 9 and H_z between lines changes at 0.1 H_{zmax}. One can see that without dependence on chamber shape electric field is concentrated in the small space between electrodes and in main part of the chamber magnetic field is approximately uniform. It allows successfully use definitions of the electrode capacitance and the chamber inductance.

DEPENDENCE OF THE CAVITY CHARACTERISTICS FROM DIMENSIONS

RF power needed to have the voltage given is defined by shunt resistance. At the same time ratio R_t/Q is defined by electrodes capacitance:

$$R_{t/Q} = \frac{U^2}{2\omega W_0} = \frac{1}{\omega C} \qquad (7)$$

Next statement is valid: for electrodes without modulation per unit length capacitance between electrodes depends only on shape and placement of electrodes and don't depend on dimensions. Suppose we have several infinitely long parallel conductors which are arbitrary placed in XY plane. To find capacitance between i-th and j-th conductor we keep i-th conductor with potential Uo and calculate charge induced at j-th one. This conductor configuration generates distribution $U_1(x,y)$ of the potential in XY plane. Capacitance per unit length is:

$$\frac{dC}{dl} = \frac{\int \sigma \, d\Gamma}{U_0} = \frac{\int (E_n \, n) d\Gamma}{U_0} = -\frac{\int \mathrm{grad}(U_1(x,y)) \, d\Gamma}{U_0} \quad (8)$$

where Uo - voltage between conductors, σ - surface charge density, E_n - electric field at the surface of the conductor. Let's change cross dimensions in ν times without changing in potentials at the conductors. If so, distribution $U_2=U_1(x/\nu,y/\nu)$ also will be the solution of the Laplas problem and dC in (8) will be the same, because changing in $d\Gamma$ will be compensated by inverse changing in grad(U_2). This property is a straight consequence of the longitudinal homogeneity of the problem. For electrodes with simple shape of the cross-section analytical relation for capacitance may be found. In this paper numerical method by using (3), which seems more flexible, was applied. For proposed in Ref. [4] electrodes with half- circular cross-section and radius of circular be equal to radius of the aperture capacitance is C = (31.3 ± 0.5) pf/m. Comparison of electrodes with different shape one can find, for example, in Ref. [5]. To have working frequency with capacitance C given it is needed to adjust inductance $\mathcal{L}$. In assumption that H_z is uniform in main part of the chamber estimation for inductance:

$$\mathcal{L} = \mu_0 \, S \quad (9)$$

and next relations are valid:

$$U=\mu_0 \, \omega \, H_{zmax} S, \quad I=H_{zmax}, \quad P_s = \frac{R_s \, H^2_{zmax} \, L}{2},$$

$$I= U \, C \, \omega, \quad (10)$$

where L is the length of the chamber perimeter, I is equivalent current. Taking into account these relations, we can find estimations for Q and R:

$$Q = \frac{1}{R_s} \sqrt{\frac{\mu}{C}} \, \frac{\sqrt{S}}{L} \quad (11)$$

$$R_t = \frac{1}{R_s} \, \frac{\mu_0 \, S}{C \, L}$$

In formulas (11) there is geometrical formfactor - $\sqrt{S}/L$ - ratio of chamber square to chamber perimeter. For reasonable shapes of the chambers, which are shown in fig. 1 , this formfactor changes slightly and has maximum value $\sqrt{S}/L$ =0.28 for circular. It explains more high value of Q for variant, shown in fig. 1 . In table 1 calculated by MULTIMODE code values Q fnd R_t for chambers, shown in fig.1, are given. The results presented confirm the conclusion that there are no large increase in quality factor with optimization of the chamber shape. It is difficult to say, that any reasonable shape has large features in their radiotechnical characteristics and technological aspects come at first position. Results, which are presented in table 1, confirm relation (7): for the electrode shape given R_t is directly proportional to Q.

RFQ cavity must provide quadruple focusing and RF power needed to adjust gradient G of the focusing field is

$$P_s = \frac{G^2 \, a^4}{8 \, R_t} \quad (12)$$

Usually RFQ cavity operates with maximum possible electric field E_s at the surface of the electrodes. RF power needed is

$$P_s = \frac{E_s^2 \, a^2}{8 \, \alpha^2 R_t}, \quad (13)$$

$$\alpha = \frac{2 \, E_s}{G \, a} = \frac{a \, E_s}{U}, \quad (14)$$

where a is aperture radius, α - ratio of maximum electric field to middle one in the aperture. For electrodes proposed in Ref. [5] α =1.4. Relations (12),(13) are usual for systems with quadruple field distribution. Results of this chapter shows, that to decrease RF power needed it is necessary to take minimal possible aperture and electrodes with small value of the capacitance. Large improvement in R with chamber shape optimization seems not be possible.

WORKING PARAMETERS VS FREQUENCY

To estimate RF parameters of the cavity at frequency, if the same parameters are known at frequency for similar cavity, it is helpful to know there dependence vs frequency.

From relations (6) - (8) we find:

$$Q = f^{-1/2}, \; R_t = f^{-3/2}. \quad (15)$$

Such unusual dependence of R_t vs f is consequence of definition (4). To find maximum electric field avaluable at a given frequency one usually use Kilpatric relation. If so, for RF power in (13) we have dependence:

$$P_s = a^2 \, f^{5/2} \quad (16)$$

Taking into account that length of the chamber perimeter decreases as f^{-1}, we see that heat loading per unit square of the camber surface. rises as $f^{7/2}$. With increasing of working frequency RFQ cavity cooling becomes severe problem.

Table 1 Calculated values of Q, R_t and frequency shifts caused by boundary displacement at 1 mm inside cavity. Working frequency is 198.2 MHz.

N	Q	R_t,kOm	df1	df2	df3	df4	df5	R, H cm
1	12300	315	-37.0	-4.7	-1.0	2.9	3.3	19.3
2	13270	340	-37 . 0	-4.7	-1.0	2.1	3.0	16.1
3	13500	346	-37.0	-4.7	-1.0	2.0	2.9	17.6
4	13800	354	-36.5	-4.6	-0.97	2.3	2.4	16.05
5	13830	355	-37.0	-4.6	-1.0	1.5	3.3	14.05
6	15200	390	-35.7	-4.5	-1.0	2.1	3.9	7.12

SENSITIVITY OF THE WORKING FREQUENCY TO SMALL DEVIATIONS IN CHAMBER DIMENSIONS

For usual cavity dimensionles coefficient

$$D_i = \frac{df}{dx_i}\frac{x_i}{f} \qquad (17)$$

where x_i - i - th dimension of the cavity , which depend only on cavity shape and don't depend on frequency, describes sensitivity of the working frequency to small deviation's in dimensions. Peculiarity of H- cavity is that, such description isn't valid for region near axis. Small deviations in electrode dimensions destroy geometrical similarity and capacitance becomes depending on dimensions. So, for dimension of the electrodes Di is inverse proportional to absolute values of dimensions. In table 1 frequency changes (in MHz) caused by displacement of boundary parts inside the cavity at 1 mm are given. Working frequency is 198.2 MHz radius of the aperture is equal to 6.536 mm. The parts of boundary are labeled at fig. 1c. Variants in the table 1 are in accordance with shown in fig. 1.

CONCLUSION

Some properties of the four chamber cavities, which are radiotechnical realization of the RFQ structure, are considered. It is shown that per unit length capacity between electrodes depends only on the shape of the electrode and don't depend on dimension. Some expressions for RF characteristics estimations are given. The author wish to thank I.V. Gonin for the help in preparation of this work and L.V. Kravchuk for helpful discussion.

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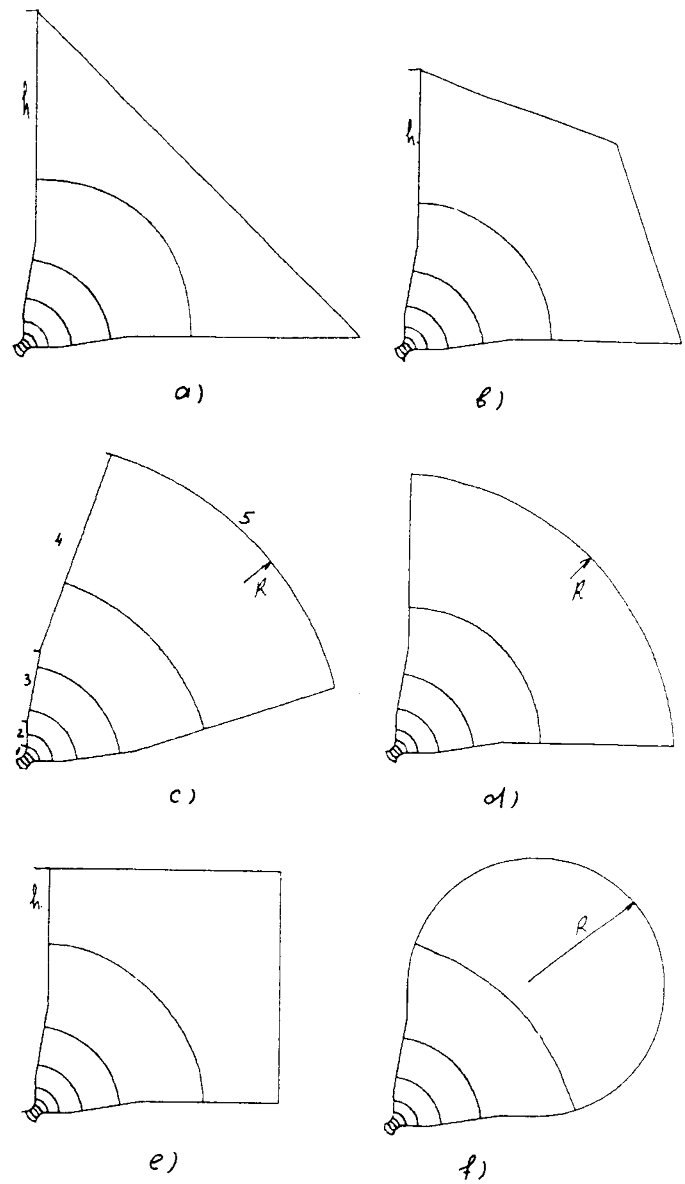

Fig. 1. Electric field distribution for different shapes of chamber.