ELECTRICAL CHARACTERISTICS OF A SHORT RFQ RESONATOR[1]

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Abstract

Electrical characteristics of a short RFQ resonator of the 'four rod' type have been studied by carrying out measurements on models and numerical simulations using the MAFIA codes. An empirical formula is obtained for the capacitance of vane-like electrodes in a four-rod RFQ resonator. It is shown that the electrode supports could account for a significant part of the total capacitance. This additional capacitance may change the circuit symmetry and give rise to a dipole component. This effect can be compensated by appropriate modifications of the support structure. The beam offset due to a dipole component is estimated.

An Improved Formula for Interelectrode Capacitance

The total capacitance is an important design parameter for any resonator. For a 4-rod RFQ structure[1], an estimate for interelectrode capacitance C is given in reference 2. Our measurements show that the formula in reference 2 *overestimates* C by about 10% for unmodulated structure ($m = 1$), and *underestimates* C by about 10% for large modulation ($m \sim 3$). If we compare a cross section of the modulated structure with that of the "equivalent" unmodulated structure with aperture $r_0 = a/\sqrt{\chi}$ it will be seen that the overall transverse dimension of the actual structure is larger. An unmodulated 4-rod structure with suitable "vane-like" extensions would be closer to the actual structure. We have measured capacitances for unmodulated 4-rod structures with and without an extension to arrive at an improved empirical formula.

The measurements were carried out using a digital LCR meter, model HP4271A. The electrodes were held in place by fixing them to a pair of plexiglass plates. It was found that typically, the fixtures contributed about 2 pF. The values reported in this work are corrected for the effect of these fixtures.

For the unmodulated structure, a fixed radius of curvature, $\rho = 1.59$ cm and a fixed length $l = 50.8$ cm were used. The measurements were carried out for $a = 2.2$, 3.5 and 4.8 cm. The extension length h was varied between 0 to 15 cm. The modulated structure had $\rho = 1.75$ cm, $l = 39.4$ cm, $a = 2$ cm and $m = 3.2$. The average value of χ (over four cells) is 0.26, giving $r_0 = a/\sqrt{\chi} = 4$ cm.

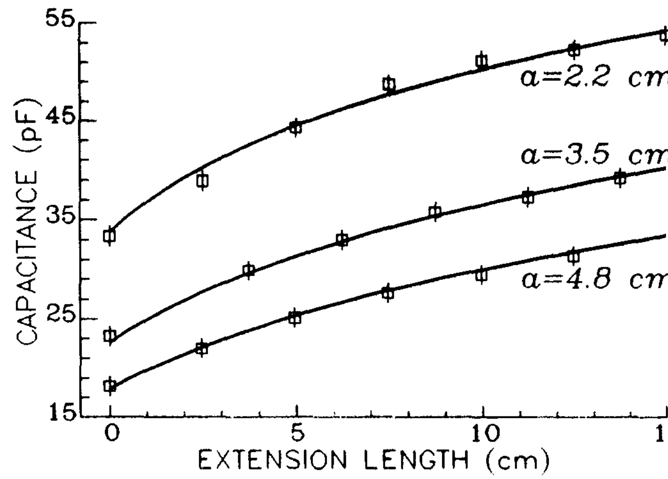

Figure 1: *Capacitance of unmodulated 4-rod structures with a=2.2, 3.5 and 4.8 cm as a function of extension length h. The solid lines are given by eq.(1).*

The variation of capacitance with the extension length h is shown in Fig.1 for the unmodulated structure. The experimental values are shown by squares and the error bars represent typical uncertainities of ± 1 pF in the measurements. The solid lines in Fig.1 represent estimates from an improved capacitance formula given by

$$\frac{C}{l} = \frac{39.37}{cosh^{-1}\left(\frac{r_0/\rho+1}{\sqrt{2}}\right)} + \frac{31.05}{r_0/\rho - \sqrt{2}+1} + 25.28\, ln\left(1+\frac{h}{a+\rho}\right) \quad pF/m \qquad \ldots(1)$$

The first two terms on the right hand side of eq.(1) have the same form as in reference 1 with somewhat different constants, while the last term is based on an approximate analytic estimation of capacitance increase due to the extensions. For the modulated structure, the appropriate value of h is $ma - r_0$. For the $m = 3.2$ structure studied, eq.(1) gives a value of $C = 22.01$ pF, which matches exactly with the measured value of 22 pF for the modulated structure.

In a real 4-rod RFQ, the electrodes must be held in place by some support structure. This produces additional capacitance, which could be significant, particularly for short RFQs. We have measured this effect with a support in the form of a rectangular frame.

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For the unmodulated structure, the capacitance increases from 33.4 pF ($a = 2.2$ cm) without supports to 42.2 pF with supports. In the case of modulated structure, these values are 22 pF and 30.2 pF respectively. Thus the supports do add significantly to the total capacitance, and can not be neglected.

Beam Offset due to Electrical Unbalance

Dipole field components in the RFQ are the result of deviation from the basic quadrupolar symmetry which can be expressed as a voltage asymmetry or an equivalent dipole field superimposed on the quadrupole field.

The equivalent circuit of the short, four rod, RFQ resonator is shown in Figure 2. The inductive path to a pair of electrodes has an inductance $L/2$ for the common part, then L_1 and L_2 inductive arms leading to the two electrodes. The inductive unbalance δ is defined as $\delta = 2(L_1 - L_2)/(L_1 + L_2)$. The parasitic capacitance between the support arms and the electrodes results in the capacitive unbalance which is described by the parameter $\epsilon = C'/C$, where $C - C'$ is the basic inter-electrode capacitance and C' is the single arm to electrode parasitic capacitance. These unbalances can be controlled by the proper design of the resonator[2]. We define $l = (L_1 + L_2)/2$ and $\alpha = \frac{l}{2(L+l)}$. The solution of the equations of this circuit to first order in δ and ϵ results in frequencies of the two lowest modes

$$\omega_0 = \frac{1}{2\sqrt{(L+l)C}}, \quad \omega_1 = \omega_0/\sqrt{\alpha} \tag{2}$$

Furthermore, the voltage unbalance $\Delta V/V$ at the fundamental frequency ω_0 is given by

$$\frac{\Delta V}{V} = \frac{\alpha}{1-\alpha}(\delta - \epsilon) \tag{3}$$

Thus one may eliminate the unbalance by cancelling the capacitive term ϵ against the inductive term δ. From the measured values of capacitance for the unmodulated structure described earlier in this paper,

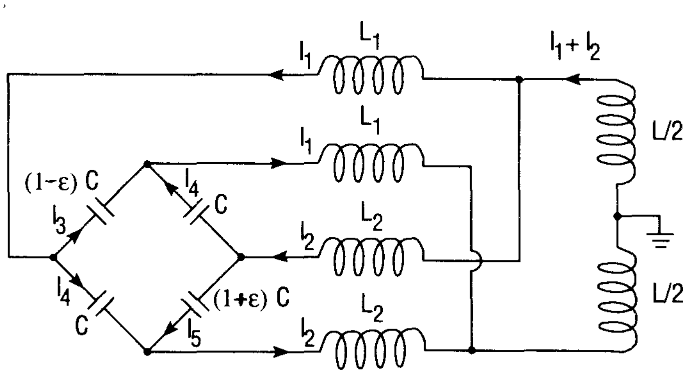

Figure 2: *The equivalent circuit of a short, four rod RFQ resonator*

we have $4(C - C') = 33.4$pF, and $4C = 42.2$pF. Under the simplifying assumptions of $L \approx L_1 \approx L_2$, eq.(3) gives a voltage unbalance of about 7%. Unbalance measured experimentally by bead-pulling is about 3%. The discrepancy in the estimate could be due to use of a simplified lumped circuit equivalent, as well as neglecting higher order terms in δ and ϵ.

By introducing a few simplifying assumptions we can estimate the effect of the dipole field on the beam. Let us use a model linac RFQ with smooth focussing and neglect acceleration. The dipole field is taken conservatively to be continuous and the result of a constant voltage asymmetry. The beam will establish a neutral axis offset from the geometrical beam axis. We wish to estimate the neutral axis offset relative to the beam size.

The transverse motion in a focussing channel is given by $x'' + k^2 x = 0$ where $k^2 = qV/(Mv_0^2 r_0^2)$. M, q and v_0 are the particle's mass, charge and velocity. Let the voltages of an electrode pair be $V + \Delta V$ and $V - \Delta V$. The motion in the dipole field is given by $x'' = qE_d/(Mv_0^2)$ We can express the dipole field as a function of the longitudinal coordinate z by

$$E_d(z) = \left(\frac{\Delta V}{a}\right) \frac{2\cos(kz + \Phi_s)}{(m+1) - (m-1)\sin kz} \tag{4}$$

where the aperture variation between a and ma is approximated as sinusoidal. Φ_s is the synchronous particle phase. The average dipole field $\overline{E_d}$ is obtained by integrating over the RFQ period,

$$\overline{E_d} = \left(\frac{\Delta V}{a}\right) \frac{\sin\Phi_s}{\sqrt{m}} \frac{\sqrt{m}-1}{\sqrt{m}+1} \tag{5}$$

This average dipole field in the presence of the quadrupole focussing channel results in a neutral axis offset x_0 given by

$$x_0 = r_0 \left(\frac{\Delta V}{V}\right) \frac{m+1}{2\sqrt{m}} \frac{\sqrt{m}-1}{\sqrt{m}+1} \sin\Phi_s \tag{6}$$

This expression for x_0 can be used as a guide to the severity of the dipole effects. As long as x_0 is significantly smaller than the beam radius σ_x, the dipole effects can be neglected. The figure of merit in terms of the RFQ and beam parameters is:

$$\frac{x_0}{\sigma_x} = \left(\frac{qV}{Mc^2}\right)^{1/4} \sqrt{\frac{r_0}{\epsilon_n}} \frac{\Delta V}{V} \frac{m+1}{2\sqrt{m}} \frac{\sqrt{m}-1}{\sqrt{m}+1} \sin\Phi_s \tag{7}$$

where ϵ_n is the normalized emittance. For a typical set of values[2] $\epsilon_n = 0.1\pi$ mm.mrad, $V = 420$ KV, $r_0 = 4$ cm, $m = 4$, $\Phi_s = 25_0$, and Pb^{35+} beam, we have $(x_0/\sigma_x) = 5.85(\Delta V/V)$.

Numerical Simulations by MAFIA Codes

The electrical characteristics of the RFQ depend on the RF field distribution in the resonator. A three

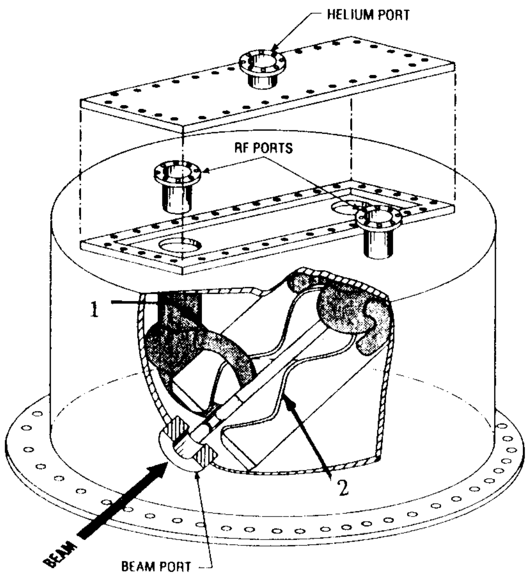

Figure 3: *A schematic view of the prototype superconducting RFQ resonator*

dimensional electromagnetic field was calculated using MAFIA[4] codes for the short RFQ structure.

A simpler problem was first run on a VAX computer with 42,735 mesh points to simulate an aluminum model which is the unmodulated 4-rods structure with a pair of rectangular frame supports. For our superconducting RFQ prototype resonator design, a real structure with *modulated* electrodes ($m = 4$) and a cylindrical tank (see Fig.3) was simulated and run on a vectorized IBM-3090 with up to 298,125 mesh points. It is necessary to have certain mesh density and large memory space for a good solution of this problem.

The results of both MAFIA simulations and comparisons with measurements and approximate expressions are presented in Table 1. Fig.4 is from MAFIA postprocessor plot, showing the electric field distribution in the region between a pair of modulated electrodes. MAFIA calculations show that our resonator's fundamental mode is quadrupolar. Next higer frequency mode (dipole mode) is about double the fundamental frequency. The location of the peak surface magnetic field B_s is marked by arrow 1 in Fig 3. At this point the field is enhanced by a superposition of field from two conductors. The arrow 2 in Fig 3 shows the location of the peak surface electric field E_s.

The electrical unbalance on electrodes was checked by MAFIA and experiment for the unmodulated model. The difference between the center and end of the electrodes is due to transmission line effects. The measured values of the frequency tuning sensitivity for translation of the bottom cover in a modulated structure model was 3.3–0.7 KHz/mm for cover to electrode distance of 10–25 cm. The tuning sensitivity for the beam ports was ~60–5 KHz/mm for electrode to beam port gap of 2 to 4 cm.

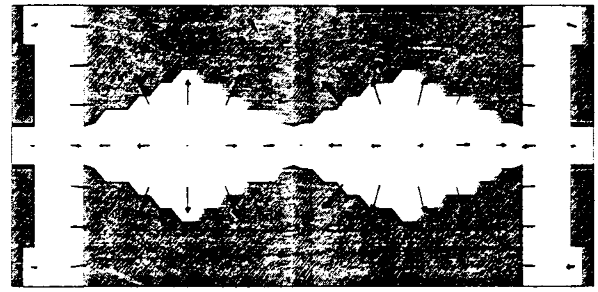

Figure 4: *Electric field distribution obtained by using MAFIA codes*

Table 1. MAFIA Results vs Measurements and Approximate Expressions

Character.	Unmodulated Model MAFIA	Unmodulated Model Meas.	Unmodulated Model Approx.(4)	Modulated Prototype MAFIA	Modulated Prototype Approx.[3]
f (MHz)	51.9	53.5	54.8(5)	54.7	60.8(5)
Q (Cu)	14300	——	13260	9952	8938
C_{total}(pF)	48.2(1)	42.2	34.2(6)	45.4(1)	24.7(6)
$E_s/V(m^{-1})$	53.6(2)	——	65.3(7)	62.0(9)	38.5(7)
B_s/V(G/MV)	1218(3)	1187	1322(8)	1365	1271(10)
$(E_a/E_s)_{mean}$	——	——	——	0.114	0.161
$\Delta V/V$(%)					
Ends	3.9	2.9	7.0	2.2	15
Centre	3.7	2.1	7.0	2.9	15

Notes:
(1) Obtained from the total stored energy.
(2) At the straight corner of rod end.
(3) At the inner top corner of rectangular support.
(4) The supports and arms were assumed to be pairs of parallel strip conductor. The inductance and resistance per unit length are[5] $\frac{L}{l} = \frac{\mu}{2\pi}[ln(b/a) + 3/2] \quad (H/m)$, $R/l = \frac{1}{\sigma\delta(a+c)} \quad (\Omega/m)$, where a and c are width and thickness of conductors. b is the conductors' seperation. σ = conductivity, δ = skin depth.
(5) Using equation (2), and C_{total} from MAFIA calculations
(6) Using equation (1). The contribution of the supports was not included.
(7) Using $E_s/V = 1.45/r_0$ as given by reference 6.
(8) It is estimated as $B_s = \frac{\mu\omega C_{total}V}{2\sqrt{2}(a+c)}$ at sharp corner of conductor and contributed by the support and arm. C_{total} is taken from MAFIA results.
(9) This value is higher than the approximation because the simulated electrodes have sharp edges.
(10) Using C_{total} from MAFIA calculations.

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References

1. A. Schempp, M. Fersh and H. Klein, *Proc. 1987 Particle Accelerator Conference* in *IEEE Trans. Nucl. Sci* **87CH2387-9**, 267(1987).
2. R.M. Hutcheon, File No. 9022/ZEBRA-TM-67, AEC, Chalk River.
3. I. Ben-Zvi, A. Lombardi and P. Paul, submitted to *Particle Accelerators*.
4. *MAFIA User Guide*, The Mafia Collaboration, DESY, LANL and KFA, 1988.
5. G.W. Carter, *The Electromagnetic Field in its Engineering Aspects*, Elsevier, New York, 1967, p.232.
6. K.R. Crandall, R.S. Mills and T.P. Wangler, *Proc. 1983 Particle Accelerator Conference, Santa Fe NM, LA-UR 83-743*.