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A NUMERICAL STUDY OF SUPERCONDUCTING CAVITY COMPONENTS*

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Abstract

Computer programs which solve Maxwell's equations in three dimensions are becoming an invaluable tool in the design of RF structures for particle accelerators. In particular, the lack of cylindrical symmetry of superconducting cavities with waveguide couplers demands a 3-D analysis for a reasonable description of a number of important phenomena. A set of codes, collectively known as MAFIA, developed by Weiland and his collaborators, has been used at CEBAF to study its five-cell superconducting accelerating cavities. The magnitude of RF crosstalk between cavities is found to depend critically on the breaking of cylindrical symmetry by the fundamental power couplers. A model of the higher order mode coupler exhibits an unexpected mode which is in good agreement with measurement.

1. Introduction

The waveguide couplers necessary to damp higher order modes of superconducting cavities break the cylindrical symmetry of these accelerating structures in an essential way. Numerical modeling requires three-dimensional computations for both a qualitative and a quantitative description of a number of significant electromagnetic effects. In this paper studies of cavity-to-cavity cross talk and coupler modes are presented which are based on the 3-D code MAFIA of Weiland and his collaborators[1] which is available on NERSC.

2. Resonant Mode of HOM Couplers

Higher order modes (HOMs) of a superconducting cavity, which have intrinsically high Q values of order of 10^9, can be detrimental to beam quality if left undamped. In particular, multipass regenerative beam breakup studies of the CEBAF linac have shown that these HOM Qs must be reduced to the levels $< 10^7$ for stable operation at currents of hundreds of microampere. For a CEBAF cavity (Figure 1), a waveguide type HOM coupler was developed for efficient damping of higher order modes, especially those with high $\frac{R}{Q}$ values. Extensive measurements have shown that all important HOMs including quadrupole and sextupole modes are reduced successfully to $5 \times 10^2 \leq Q \leq 1.7 \times 10^5$. The CEBAF HOM coupler is sketched in Figure 2 with coordinate systems defined on a cross sectional area of the coupler. The two waveguide arms of the coupler, designed to couple effectively to both polarizations of deflecting modes, extend further to HOM loads through bent elbow segments made of niobium.[2]

It had been assumed that the HOM couplers do not support any resonant mode. However, a recent careful analysis of cavity mode measurements compared with URMEL[3] calculations and previously measured HOMs data suggested the existence of a trapped mode in the structure. Resonant modes below the lowest TE_{10} waveguide mode cutoff frequency, if they exist, are good candidates for trapped modes. All waveguide modes are evanescent in this case. Consequently, if the arms are long enough, the structure can effectively be regarded as closed for these modes. We note that the dimension of the rectangular waveguides is 7.899 cm × 3.81 cm. Therefore, the lowest TE_{10} mode is cut off at 1898 MHz.

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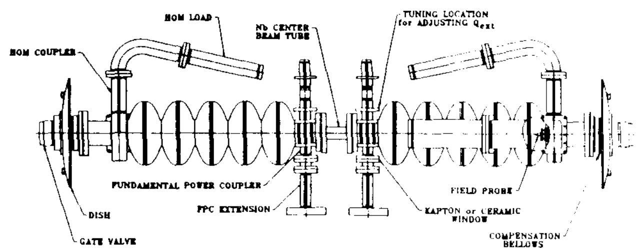

Figure 1. A CEBAF cavity pair.

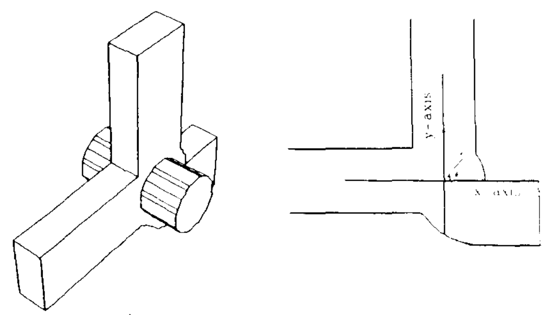

Figure 2. CEBAF HOM coupler.

The lowest five modes found by the MAFIA code for the coupler structure shown in Figure 2 are summarized in the following Table 1. Two waveguide arms are shorted at 19.95 cm from the pipe center; the beam pipe is cylindrical with $r = 3.5$ cm.

TABLE 1
Summary of Modes

Mode	Frequency (MHz)	Maxwell's Laws		
		⟨div(D)⟩	⟨curlcurl(E)⟩	⟨div(B)⟩
1	1782.193	0.1×10^{-11}	0.2×10^{-9}	-0.9×10^{-13}
2	2042.745	0.9×10^{-12}	0.1×10^{-9}	0.8×10^{-15}
3	2048.346	0.3×10^{-12}	0.5×10^{-10}	0.1×10^{-12}
4	2239.128	0.5×10^{-12}	0.8×10^{-10}	-0.1×10^{-12}
5	2430.439	0.3×10^{-12}	0.3×10^{-10}	0.3×10^{-13}

It is clear from Table 1 that only the lowest mode at 1782.193 MHz will survive as a resonant mode for the actual structure since the higher modes are above the 1898 MHz cutoff. We note that the ends of the waveguides and beam pipe are open. If a different boundary condition applied to the open end side of the structure makes a significant change in parameters which characterize a mode, the mode may not exist as a resonance. We find that the 1782 MHz mode indeed is well confined within the region near the stub and beam pipe center, and would stay resonant in the open structure as a result. The frequency agrees quite well with the cavity mode measurements. Experimentally, the Q of this mode is less than 10^5. Field plots for this mode at the midplane of the HOM coupler are shown in Figure 3. Despite a substantial change to the field patterns due to the fully three dimensional nature of the structure, the mode has a significant TM_{010} component. The mode, however, with all six components of electromagnetic fields nonvanishing, does not allow any simple characterization. We also notice that the position of maximum E_z has moved from the usual beam pipe center to the pipe edge along the waveguides.

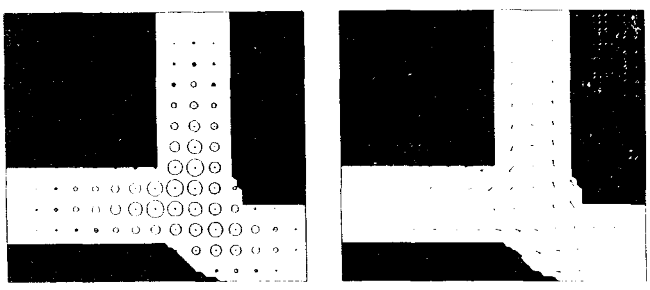

Figure 3. Electric and magnetic field plots of 1782 mode.

As a consequence of the non-vanishing transverse gradient of E_z at the beam pipe axis, the mode can deflect the beam passing the structure as well as cause an energy loss. As the mode can couple with the total charge of the bunch, the beam does not have to enter the structure off axis to excite the mode in contrast to deflecting modes commonly discussed in relation to beam breakup. Let us define the longitudinal (transverse) impedance by $R_{\|(,\perp)} = |V_{\|(,\perp)}|^2/P$, where P is the power dissipated in the structure walls. $V_{\|(,\perp)}$ is the maximum longitudinal (transverse) voltage gained by a traversing charged particle. The impedances generally depend on both coordinates, r and θ. We find that the following expressions provide a reasonable first order approximation to the impedances in the region $r \leq 1.5$ cm:

$$\frac{R_\|}{Q} = 53.0(1 - \frac{0.066r}{1\ \mathrm{cm}}\cos(\theta + \frac{\pi}{6}))^2\Omega\ ,$$

$$\frac{R_\perp}{Q} = 1.5\,\Omega\ .$$

This is consistent with the description of the mode as a superposition of the TM_{010}-like mode and the deflecting TM_{110}-like modes. The magnitude of the coupling, defined as the ratio of the maximum longitudinal voltage gain due to the dipole component to that of the TM_{010} mode assuming the relation to be valid in the whole beam pipe region, is about 0.23.

The main concern with the transverse impedance is its ability to cause beam breakup above a certain threshold current. At CEBAF extensive computer studies have been carried out to predict beam breakup threshold currents caused by various deflecting modes of superconducting cavity.[4] Compared to most dangerous dipole modes of the cavity, the transverse impedance, $\frac{R_\perp}{Q}$, of 1.5 Ω appears about a factor of 30 to 40 smaller. Therefore, since its Q is comparable to the strongest modes, this 1782 MHz mode is not expected to be a significant factor in limiting the current handling capability of the linac.

3. Coupling Between Cavities in a Cryounit

The linac RF system at CEBAF is required to have the ability to control the accelerating field gradient and phase of each cavity individually. Individual cavity controls will provide maximum flexibility in the operation of the RF system, which will allow maximum utilization of the individual cavity gradient performance. A CEBAF cryounit consists of two five cell superconducting cavities each equipped with its own fundamental power coupler (FPC) and HOM couplers connected through a 9.393 cm-long niobium adapter. The cavity beam pipe radius is 3.5 cm. The distance between the two nearest end-cells of the two cavities is one and a quarter λ, the wavelength of accelerating mode at 1497.0 MHz. (See Figure 4, where a CEBAF cavity pair with FPCs is illustrated. Only a single cell of the superconducting cavity is shown.) To ease the task of building individually controlled RF system which meet CEBAF design requirements, it is imperative to avoid significant cross talk between the two superconducting cavities at the design operating frequency of 1497.0 MHz.

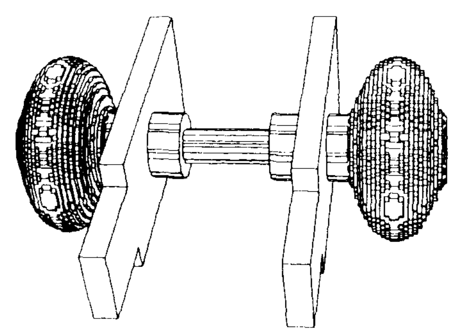

Figure 4. A CEBAF cavity pair with fundamental power couplers.

A few simplifications have been made to keep computing time and memory space under control. The cryounit with the full two elliptical 5-cell superconducting cavities along with couplers demands many mesh points, which is presently not practical to model, even with a Cray-2. For the study of cross coupling, we should be able to obtain a fairly accurate estimate by substituting a properly adjusted pill-box cavity for an elliptical 5-cell cavity. We note that the 1497.0 MHz mode is the π mode. The pill-box cavity chosen has a radius of 7.88 cm and is 10.0 cm long and turns out to be a quite adequate substitute for a single-cell elliptical CEBAF cavity given in Figure 4. As we will see later, the TM_{010} mode frequency, which is our only interest in this section, is in excellent agreement with the operating frequency. We also neglect the presence of higher order mode couplers for this study. It is assumed that their effect on this coupling is minor compared to that of fundamental power couplers as the HOM couplers are located outside – not in between – the cavities. On the other hand, a realistic modeling of the FPCs has been provided with precisely dimensioned short stubs. The open end of the FPC, which consists of a 13.44 cm × 2.50 cm rectangular wave guide, has to be shortened to a convenient length.

The following Table 2 summarizes the results of MAFIA runs, which are to determine the eigenfrequencies of the coupled system and the degree of a cross talk.

TABLE 2
Eigenmodes of Coupled Cavities

Without FPC

r_{inner} = 3.5 cm	1498.659534
	1498.659542
	8

With FPC

	Boundary conditions on FPC	
Case	E	M
r_{inner} = 3.5 cm	1498.878301	1498.884335
	1498.878170	1498.884260
	131	75
r_{inner} = 2.5 cm	1498.879664	1498.885098
	1498.879655	1498.885093
	9	5
r_{inner} = 1.5 cm	1498.880697	1498.885660
	1498.880697	1498.885660
	0	0

The parameter r_{inner} is the radius of the niobium inner adapter placed between the FPCs to reduce crosstalk. In Table 2, the top line in each case lists frequencies for the symmetric mode and the second line frequencies for the antisymmetric mode in MHz. The difference Δf is given in the third line in hertz.

When there are no FPCs, the coupling of the cavity fields through the TE_{11} waveguide mode is not possible at 1497.0 MHz due to the axial symmetry. However, once the symmetry is broken by the introduction of FPCs, this is no longer the case. Indeed, one can easily check that there can be two orders of magnitude difference in field strengths transferred from one cavity to another depending on whether the coupling to the TE_{11} waveguide mode is allowed or not. Even though this fact alone does not translate directly to the magnitude of Δf, the MAFIA result is certainly consistent with this picture.

We may discuss the problem of a cross coupling by considering the motion of two identical harmonic oscillators of angular frequency ω, linearly coupled symmetrically and with a damping. As is well known, in the case of a free oscillation, the system has two eigenmodes: a symmetric and an antisymmetric modes. The difference in their frequencies is proportional to the coupling constant.

We need to look at the case of a stationary state of a forced oscillation when the driving frequency is one of the normal modes of the coupled system. Let us assume a force in the form $f(t) = \frac{\omega^2}{Q}\cos(\omega t + \phi)$ is applied only on the left cavity. One then obtains a steady state solution after transients die out:

$$V_{left}(t) = \frac{\sqrt{1+K^2}}{\sqrt{1+4K^2}}\sin(\omega t + \phi - \phi_\ell),$$

and

$$V_{right}(t) = -\frac{K}{\sqrt{1+4K^2}}\sin(\omega t + \phi + \phi_r),$$

where $\phi_\ell = \arctan\frac{K}{1+2K^2}$, $\phi_r = \arctan\frac{1}{2K}$, and $K = Q\frac{\Delta\omega}{\omega}$.

Now the ratio of the amplitude of the fields in the right cavity to that of the left one is given by:

$$\frac{K}{\sqrt{1+K^2}}\ .$$

Let us now apply the above results to the CEBAF cryounit with a 3.5 cm adapter. We have f = 1497 MHz, Δf = 150 Hz, and $Q = 6.6 \times 10^6$. f and Q are design values. Δf is an estimate based on results from MAFIA runs summarized in the above Table 2. We have K = 0.66 and 55% in the right cavity due to the coupling.

In steady state, one expects about 3% in the right cavity with the design Q and Δf = 7 Hz. To reduce the beating to a less than 1% level in the stationary case, $\Delta f \leq 2.3$ Hz is required. From Table 2, we may conclude that this seems clearly possible with $r_{inner} \leq 2.0$ cm. An adapter radius of 1.74 cm has been chosen and was found experimentally to meet the 1% criterion.

4. Conclusions

The studies presented demonstrate the importance of three-dimensional considerations in understanding the operation of superconducting cavities with waveguide couplers. Previously, the design of such accelerating structures has required painstaking bench measurements of hardware and considerable developmental time. The good agreement of the MAFIA modeling presented here with laboratory measurements clearly indicates that 3-D computations can provide considerable support for the design of the next generation of superconducting cavities.

References

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2. For more details, see CEBAF Design Report, May 1986.
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