THEORY AND 3-D SIMULATIONS OF AN RF ACCELERATING CAVITY WITH ASYMMETRIC POWER FEED*

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Abstract

We investigate the transverse wake potential of a side-coupled RF accelerating cavity both analytically and using a 3-D gridded code. We find that the asymmetry leads to a transverse wakefield which is not 90° out of phase with the accelerating potential. As a result, there is a net deflection, as well as a transverse tilt, of a micropulse whose center is in phase with the peak of the accelerating potential. The transverse tilt can be reduced to zero using a dummy compensating stub. The net deflection, however, is not affected by such a stub, and depends only on the average power flow into the cavity. The analytic model predicts values of the cavity Q and the amplitude and phase of the transverse wakefield which agree well with the 3-D simulations. Results for a 1 MV accelerating cavity are given.

Introduction

Single-cell RF cavities are being investigated for beam acceleration in high-power free-electron lasers.[1,2] Among the advantages of such cavities over conventional multi-cell designs are that they match well to existing power supplies, and damping of beam-induced modes is easier to apply.[1] Like multi-cell designs, the proposed single-cell cavities are fed from one side, producing a dipole asymmetry.[3,4] We have made a detailed study of the nature of this asymmetry, and its effect on beam pulses. In the following, we present the results of 3-D numerical wakefield calculations and an analytic model derived using matched eigenmode expansions.

3-D Wakefield Calculations

The gridded 3-D code SOS was used to model the single-cell RF accelerating cavity shown in Fig. 1. The geometry consists of a 25 cm radius cylindrical accelerating cavity, which is side-coupled to a rectangular (40 cm × 20 cm) waveguide through a rectangular (10 cm × 20 cm) slot. Opposite to the waveguide, we have a compensating stub of 0–10 cm in length. The stub has the same transverse dimensions as the waveguide, and is terminated with

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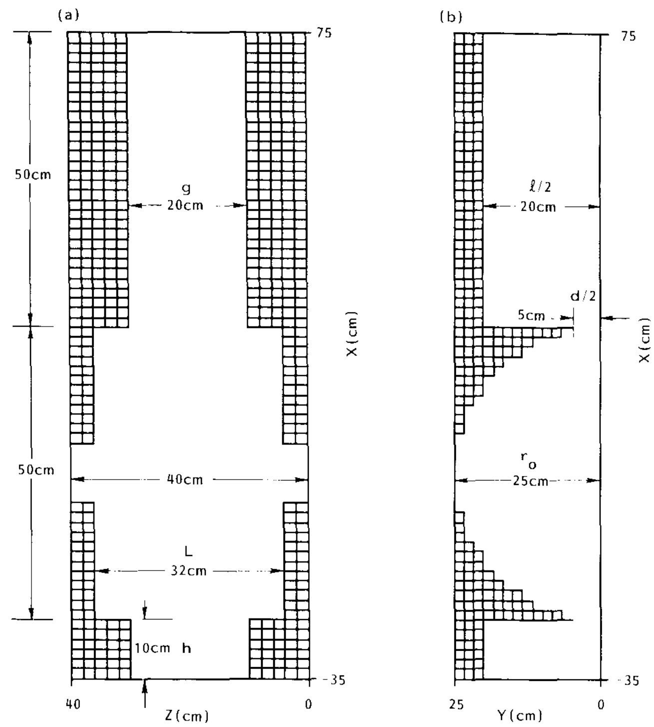

Fig. 1. Geometry of 3-D SOS cavity simulation in (a) the $y = 0$ midplane, and (b) the $z = 20$ cm midplane.

a conducting boundary. The grid sizes used are $\Delta x = \Delta y = 1.666$ cm, $\Delta z = 2$ cm. The timestep is $\Delta t = 3 \times 10^{-11}$ s, which is just less than the Courant limit. We make use of the symmetry of the cavity and the fields of interest about the $y = 0$ plane to cut the mesh size in half. Along the top boundary in Fig. 1, wave-transmitting boundary conditions are applied.

To measure the cavity Q, an ingoing TE_{10} wave with frequency equal to the fundamental TM mode of the cavity was launched from the top boundary in Fig. 1, and built up the cavity energy over many cycles. The TE wave was then turned off, and from the decay rate of the cavity energy, we measured to external Q to be $Q_e \approx 3{,}980$. This agrees well with the analytic estimate in Eq. (7).

Asymmetry of Undriven Cavity

Initial simulations were performed to measure the wakefields in an undriven cavity, since this is the case treated by our analytic model. The transverse and longitudinal wakefields are defined by

$$W_\perp(t) = \int_0^L \left[E_\perp \left(t + \frac{z-L}{v} \right) + v\hat{z} \times B_\perp \left(t + \frac{z-L}{v} \right) \right] dz \quad (1)$$

$$W_\parallel = \int_0^L E_z \left(t + \frac{z-L}{v} \right) dz \quad (2)$$

where L is the length of the cavity, and v is the particle z-velocity. The coupling slot produces only a non-zero x-component of $W_\perp$. We will denote the amplitudes of W_x, $W_\parallel$ by $\overline{W}_x$, $\overline{W}_\parallel$, respectively. Plots of W_x, $W_\parallel$ without and with a 10 cm compensating stub are shown in Fig. 2. Scaling the results to a 1 MV value for $\overline{W}_\parallel$, we find that without the stub, W_x has amplitude 6.8×10^3 V, and leads $W_\parallel$ by $\Delta\phi \approx 1.51$, which is close to $\pi/2$. When the 10 cm stub is present, the amplitude of W_x drops by a factor of 11.5 to 5.9×10^2 V, but $\Delta\phi$ is now 1.06, a large shift from $\pi/2$. We can understand this result by writing

$$W_x(t) = W_x^{(1)} \sin\omega_0 t + W_x^{(2)} \cos\omega_0 t \quad (3)$$

where $W_x^{(1)} = \overline{W}_x \cos\Delta\phi$, $W_x^{(2)} = \overline{W}_x \sin\Delta\phi$, and we assume $W_\parallel(t)$ varies as $\sin\omega_0 t$. The term $W_x^{(1)} \sin\omega_0 t$ is obtained from the component of the deflecting field B_y which is in phase with the accelerating electric field E_z. Therefore, the expression $W_x^{(1)}\overline{W}_\parallel$ is proportional to the time-averaged Poynting flux through the coupling slot, which is insensitive to the presence of the compensating stub. Thus, $W_x^{(1)} =$ const. for a given accelerating potential. The term $W_x^{(2)} \cos\omega_0 t$, on the other hand, is obtained from the component of B_y which is $\pi/2$ out of phase with E_z, i.e., the standing-wave component of B_y. From this one expects that $W_x^{(2)}$ depends on the length of the compensating stub, and can be made to vanish by terminating the stub at a position which is symmetric with the first zero of E_z in the waveguide. This implies a quarter-wavelength stub,[5] which is 27 cm for the present parameters.

The results in Fig. 2 are consistent with this picture. The 10 cm stub causes a large drop in $W_x^{(2)}$, but leaves $W_x^{(1)}$ unchanged, so that $\Delta\phi$ must depart from $\pi/2$.

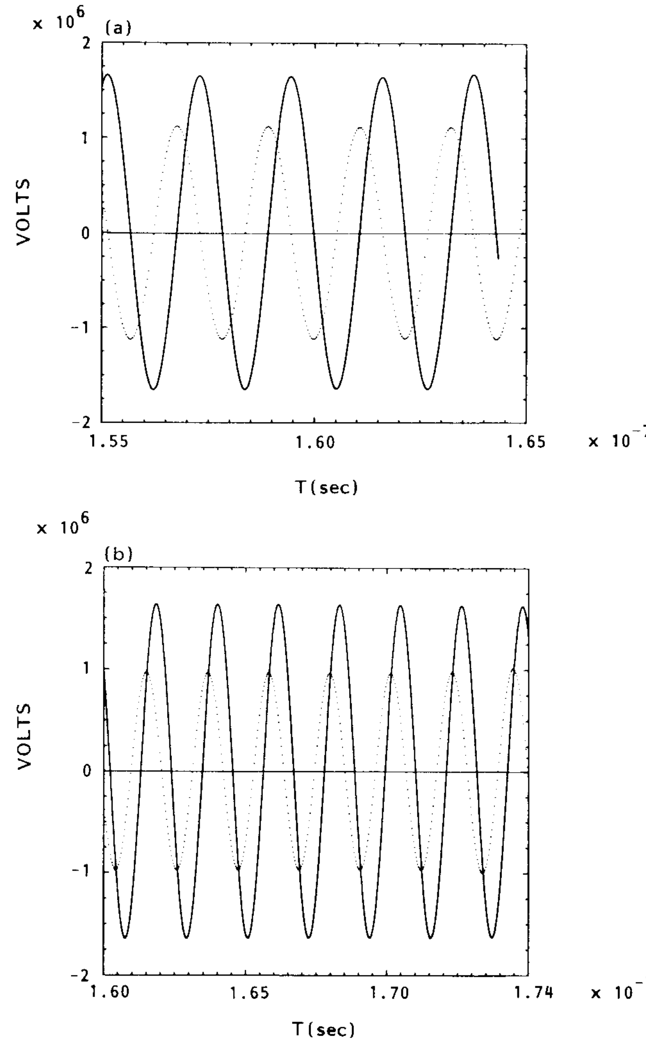

Fig. 2. Time histories of the longitudinal (solid lines) and transverse (dotted lines) wake potentials for cavities (a) without and (b) with a 10 cm compensating stub. W_x has been multiplied by 10^2 in (a) and by 10^3 in (b). Note that the time scale is different for (a) and (b).

Asymmetry of Driven Cavity

When the effect of driving fields entering from the waveguide are included, Eq. (3) becomes modified to

$$W_x(t) = \left(W_x^{(1)} - W_x^{(3)} \right) \sin\omega_0 t + W_x^{(2)} \cos\omega_0 t \quad (4)$$

where $W_x^{(3)}$ is obtained from the component of B_y produced by the incoming fields. The case $W_x^{(1)} = W_x^{(3)}$ corresponds to the case where the power entering the cavity equals the power leaving through the coupling slot. Due to beam loading, there must be a net power influx to maintain the accelerating field. If the total

Q of the cavity is Q_{tot} then, assuming a steady-state accelerating amplitude, we can write

$$\begin{aligned} W_x^{(1)} - W_x^{(3)} &= W_x^{(1)}(1 - Q_e/Q_{tot}) \\ &= -W_x^{(1)} Q_e/Q_b \end{aligned} \quad (5)$$

where Q_b is the effective Q due to beam loading. In terms of $W_x^{(1),(2)}$, the deflection and shear (defined as the difference in deflection between head and tail) of the pulse of duration τ_p whose center is in phase with the peak of the accelerating potential are given by

$$\begin{aligned} \theta_d &= eW_x^{(1)} Q_e/Q_b \gamma mc^2 \\ \theta_s &= eW_x^{(2)} \omega_0 \tau_p/\gamma mc^2 \end{aligned} \quad (6)$$

Analytic Model of Undriven Cavity

To obtain an analytic dispersion relation for the geometry in Fig. 1, we match eigenmode expansions of the electric and magnetic fields in the three regions (waveguide, cylinder and stub). An outgoing-wave boundary condition is applied in the waveguide. The curvature of the cylindrical eigenfunctions is neglected over the width of the matching regions. The fields E_z and B_y are used to do the matching (see Ref. 5 for details). The expression obtained for the cavity Q is

$$Q_e \approx \frac{(L/g)\pi^5}{8k_x\ell}\left(\frac{r_0\ell}{d^2}\right)^2 S^2 \quad (7)$$

where $k_x = \sqrt{\omega_0^2/c^2 - \pi^2/\ell^2}$ is the wavenumber of the outgoing wave in the waveguide, and $S \approx 1.2$. For the parameters in Fig. 1, this gives $Q_e \approx 4{,}000$, in good agreement with SOS. Without the compensating stub, we find

$$\overline{W}_x \approx \frac{\beta(\omega_0 r_0/c)(d/r_0)^2}{S\pi^3(L/g)}\overline{W}_\parallel \quad (8)$$

$$\Delta\phi \approx \frac{\pi}{2} - \frac{k_x\ell}{S}\left(\frac{2d}{\pi\ell}\right)^2 \quad (9)$$

where $\beta = v/c$. With a stub which is more than a few cm long, we find

$$\overline{W}_x \approx \frac{4\beta(\omega_0 r_0/c)(d^2/r_0\ell)^2(k_x\ell)}{S^2\pi^5(L/g)\sin(k_x h)}\overline{W}_\parallel \quad (10)$$

$$\Delta\phi \approx \frac{\pi}{2} - k_x h \quad (11)$$

For the parameters in Fig. 1, these expressions give $\overline{W}_x \approx 6.5 \times 10^3$ V, $\Delta\phi \approx 1.52$ for the case with no stub, and $\overline{W}_x \approx 580$ V, $\Delta\phi \approx 1.0$ for a 10 cm stub, in good agreement with the numerical results in Fig. 2. The above expressions also predict that $W_x^{(1)} \equiv \overline{W}_x\cos(\Delta\phi)$ is independent of the length of the stub, in agreement with the numerical observations.

Effect of Asymmetry on a Micropulse

For $\overline{W}_\parallel = 1$ MV, with beam loading such that $Q_b = 5{,}400$, the predicted deflection and shear for a $\gamma = 30$, 25 psec micropulse passing through the cavity in Fig. 1 are

	Deflection (μrad)		Shear (μrad)	
No Stub	13.3	(14.6)	33	(31)
10 cm stub	14.2	(14.7)	2.1	(2.3)

where the numbers in parentheses are from the analytic model.

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