THE USE OF A SPREADSHEET IN THE DESIGN OF ACCELERATOR COMPONENTS*

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Abstract

This work was undertaken to explore the capabilities of user-friendly technology in building an easy-to-use tool that could be applied to formulate quickly a workable design for a radio-frequency quadrupole (RFQ). The tool will be extended to include additional components. This paper outlines the derivation of scaling laws from which we obtained a set of self-consistent equations that describe the behavior of an RFQ. These equations are relations between accelerator parameters (electric field, rf frequency, zero-current transverse and longitudinal phase advances per period) and beam parameters (current, energy, and emittance) that act as guides for designing RFQs. These equations show the various tradeoffs involved in choosing RFQ designs and help to choose starting points in parameter space for optimizing an RFQ for a particular requirement. By entering values in a simple spreadsheet, the design parameters of an RFQ can be calculated. The spreadsheet is fully described.

Introduction

The design of complex devices, such as an RFQ accelerator module, has depended on the use of large design programs that are dependent for input on files of data that are treated like card files. This input method is the result of programming in the era of batch oriented computer processing. These tools continue to serve their purpose well for detailed design. However, setting-up these "decks" of input can be tedious and prone to error because the user is not given help as to what information should be supplied on what "card." Also, the information that is supplied is usually position sensitive. The initial impetus for this work grew out of discussions concerning the need for a design tool that addressed the problem of the difficulty of data entry as well as a tool that could be used by someone, physically removed from the person who normally "runs" the design tool. This led to the consideration of creating a tool that could be based on a PC-class machine, possibly portable, that could provide some ball-park estimates of the design and cost for an accelerator, and that would be user-friendly enough that its use would be intuitive. To examine these requirements an RFQ structure was chosen as a model because a new formulation of a set a scaling laws had been derived[1][2] and a spreadsheet format was chosen because of the availability of the spreadsheet software and its inherent user-friendly features. The particular spreadsheet software used is Lotus 1-2-3** based just on its availability on an IBM PC/AT.***

This paper presents the formulation of the scaling laws from the electrical field properties of the RFQ. Space-charge is included in the formulation. The formulation leads to a set of equations, four, which describe the RFQ. There is freedom to chose which of the many variables in these equations are fixed and which four are left free to vary as the four equations are simultaneously solved. There are some variables that would have little meaning to fix and some, such as beam current or emittance that a designer might be forced or at least want to fix. Finally the spreadsheet itself is described.

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Overview of Scaling Law Derivation

The RFQ[3][4] is a device that provides transverse focusing, longitudinal bunching, and acceleration of beam particles. The RFQ's electrical properties are determined by using an electrostatic potential function which is used to calculate the electric field for beam-dynamics modeling and gives the shape of the RFQ vanes. The transverse particle motion in the RFQ is modeled by the Mathieu equation and the longitudinal motion by a harmonic oscillator equation. The Mathieu equation is (X represents both transverse coordinates)

$$\frac{d^2X}{ds^2} + [\Delta_T + B\sin(2\pi s)]X = 0 \ ,$$

where s is a normalized length along the structure ($s = 1$ = one period) and Δ_T and B are constants. The parameter B is calculated from the equation for the external alternating gradient force of the RFQ and Δ_T is calculated from the transverse external and space-charge (Coulomb) defocusing forces. These parameters depend on the RFQ vane potential (V), the vane radius (a), the vane modulation (m), the average beam bunch velocity (β_s), and the synchronous phase (ϕ_s) (relative phase between the beam bunch centroid and the cavity rf phase). When $\phi_s = 0$, there is maximum acceleration with no bunching and when $\phi_s = -90^\circ$, there is maximum bunching with no acceleration. An approximate solution of the Mathieu equation is[5]

$$X(s) = X_0\sin(\sigma_T s)[1 + \frac{B}{4\pi^2}\sin(2\pi s)] \text{ where}$$

$$\sigma_T = [\Delta_T + \frac{B^2}{(8\pi^2)}]^{\frac{1}{2}}. \qquad (1)$$

σ_T is the phase advance per period (represents the average focusing force) and X_0 is a constant set by initial conditions.

The longitudinal motion is modeled by the harmonic oscillator equation

$$\frac{d^2Z}{ds^2} + \Delta_L Z = 0$$

which has the solution $Z = Z_0\sin(\sigma_L Z)$ where

$$\sigma_L = \sqrt{\Delta_L} \ . \qquad (2)$$

Linear space-charge defocusing terms are calculated from the electric-field components for a uniformly charged ellipsoid[6][7] to give Δ_{L-sc} and Δ_{T-sc} which add to the constant terms, Δ_L and Δ_T in the Mathieu equation and the harmonic oscillator equation. These space-charge terms depend on the dimensions of the ellipsoid (X_{ave}, Z_{Max}), the beam current (I), the charge-to-mass ratio [$Qe/(m_0c^2)$], and the rf wavelength (λ) that acts as a normalization factor. The Mathieu equation contains a periodic oscillating force term that is not present in the harmonic oscillator equation and that gives "flutter" to the beam. This is why we use X_{ave} instead of X_{Max}. The ratio of the transverse to longitudinal space-charge forces depends on the ratio of the transverse to longitudinal beam sizes (X/Z) through a form factor f[7]. Although $f(Z/X) = X/3Z$ is a good approximation, we use the exact equation.

Relationships between the beam parameters I, ϵ_T, ϵ_L (beam current, and transverse and longitudinal emittances defined below), and the external-force-determined parameters, σ_{T0} and σ_{L0}, which are zero-current phase advances generated by the RFQ, c.f., eqs. (1) and (2), are obtained by defining the space-charge parameters μ_L and μ_T (which are the negative of the ratio of the space-charge forces to the external focusing forces). These space-charge parameters relate the longitudinal and transverse phase advances, σ_T and σ_L, for a finite current beam with space-charge (Coulomb repulsion) forces to the transverse and longitudinal phase advances, σ_{T0} and σ_{L0}, for a zero current beam (no Coulomb repulsion). The phase advances σ_T and σ_L can be written as

$$\sigma_L^2 = \sigma_{L0}^2(1-\mu_L) = \sigma_{L0}^2 - \Delta_{L-sc} \quad \text{and}$$
$$\sigma_T^2 = \sigma_{T0}^2(1-\mu_T) = \sigma_{T0}^2 - \Delta_{T-sc}. \qquad (3)$$

If we want stable focusing, the values for μ_L and μ_T must be less than 1, otherwise the space-charge forces will overpower the external forces. Simulations have shown that the μ values should be kept less than 0.8 to minimize emittance growth.

The normalized emittances, ϵ_T and ϵ_L, are the areas/π of the beam, enclosed by phase-space ellipses having semimajor axes (X_{ave}, X'_{ave}) or (Z_{Max}, Z'_{Max}) [where $X' = (1/\lambda)dX/ds$ etc.] and are $\epsilon_T = X_{ave}X'_{ave}$, $\epsilon_L = Z_{Max}Z'_{Max}$. The significance of the emittance is that it describes the volume of phase space (position and divergence) occupied by the beam particles and is a conserved quantity for linear systems (where X, Y, and Z are decoupled) which we are modeling.

The equipartitioning theorem[8][9], which states that the thermal free energy is the same for each dimension (transverse and longitudinal), and the virial theorem, which states that the average kinetic energy equals the average potential energy for linear systems, are used to obtain $\epsilon_T\sigma_T = \epsilon_L\sigma_L$, which relates the transverse emittance and effective focusing force to the corresponding longitudinal quantities. All of the relationships defined above are used to calculate the scaling-law equations.

RFQ Functions

Using the above parameters and relationships, four equations must be satisfied by any RFQ that will transmit a beam in equilibrium with the linear forces.

$$f_1 : 0 = 2f\frac{\mu_T}{\mu_L} - (1-f)\frac{\sigma_{L0}^2}{\sigma_{T0}^2}$$

$$f_2 : 0 = \lambda - \epsilon_T \cdot \left[\frac{4\pi\epsilon_0 c}{3}\cdot\frac{\sigma_{L0}}{f\sigma_{T0}^{\frac{1}{2}}}\cdot\frac{m_0c^2}{e}\cdot\frac{\mu_L}{(1-\mu_L)^{\frac{1}{2}}}\cdot\frac{1}{QI(1-\mu_T)^{\frac{1}{4}}}\right]^{\frac{2}{3}}$$

$$f_3 : 0 = \frac{\pi(2\sigma_{T0}^2 + \sigma_{L0}^2)^{\frac{1}{2}}}{\sigma_{T0}^2}\cdot \sin\phi_s AR_r^2\phi_s^2\psi + 2\chi\left(\frac{1-\mu_T}{1-\mu_L}\right)$$

$$f_4 : 0 = \lambda - \left[\frac{4\pi^2(2\sigma_{T0}^2 + \sigma_{L0}^2)\psi}{\chi\sigma_{T0}(1-\mu_T)^{\frac{1}{2}}}\cdot\epsilon\left(\frac{R_r\frac{m_0c^2}{e}}{QE_0}\right)^2\right]^{\frac{1}{3}}$$

where χ and A depend on a, β_s, and λ through Bessel functions, ψ is the flutter factor that depends on the B term in the Mathieu equation and R_r is the ratio of the minimum RFQ vane radius divided by the maximum beam radius.

With four equations, four unknowns can be determined. It is up to the designer to pick just which four unknowns he wants to be determined. Another way of saying this is that the job of the designer at this point is to pick which parameters he wants to hold at a particular value. He must have a value for all but four of the parameters. There are three parameters for which one typically has no intuitive feel and can take whatever value is dictated by the rest of the model. The vane modulation factor, m, is one of these. The other two are the two sigmas, σ_{T0} and σ_{L0}. Allowing these three to be free to vary, one needs to pick yet another to be free to vary. This can be done by eliminating those variables that must be fixed. For example, one would not allow the particle mass or charge to vary. One would want to fix the ratio of the electric field to the Kilpatrick factor to get as much acceleration as possible. The synchronous phase is fixed because the designer knows how much acceleration he wants out of the structure. Continuing in this manner, we eliminate all variables except beam current, rf frequency and emittance. Picking one of these and allowing it to vary means the other two can be fixed to whatever value the designer wants. Thus, by deciding on values for beam current and emittance, for example, the equations will determine the frequency that the RFQ must have.

Reasons for Using Spreadsheet

Aside from the perception that spreadsheets are easy to use, there are several more subtle reasons that this particular technology was chosen and why it turned out to be a good choice. In programming languages such as Fortran, execution takes place in a linear fashion, that is the program steps are executed one at a time, working from first to last. One of the difficulties with this, especially when considering maintenance of the program, is finding dependencies. A variable, in the program, may depend on another variable which obtains a value several pages away from the point of interest. With a spreadsheet each step of the calculation is a simulated parallel computation of the whole sheet. Because of this, it doesn't matter where computation is placed on the sheet and thus the calculation of a particular variable can be localized to where the variable appears. By adopting some simple rules, such as all dependencies appear in the row corresponding to the variable in question, the organization of the computation can overcome the non-localization problem.

Spreadsheets were designed as a high-level tool to be applied to many problems that address themselves to the relatively computer-illiterate. Because of this, tools are provided for the spreadsheet designer that allow the building of menu-driven, fill-in forms that people find intuitive and easy to use. Also provided are tools that allow the designer, with relative ease, to put together graphs. Using a traditional programming language, most of a program is built around the user-interface and the display of the final answers. The spreadsheet has already taken care of these two problems. Of course, the draw-back here is that if you don't like the style of menu or what a graph looks like, that's too bad. You get what is provided. This has seemed to us a trivial point because the user-interface looks good and the graphics are flexible enough to do what we need. Also, in a traditional approach, one finds that for an iterative solution, the user is usually required to input all data on each iteration, although most of it remains the same. A programmer frequently corrects for this annoying input requirement by saving this "previously-input" data on the computer system's file system. This requires the addition of a number of program steps that are not directly related to solving the problem at hand. The menu that the spreadsheet provides keeps track of and displays the last values changed on the form, even between sessions.

The spreadsheet approach is not completely satisfactory, however. The PC/AT, the machine that the spreadsheet is implemented on, is fairly slow and because of this, the calculation is not as fast as it might be. Using a spreadsheet for a lengthy calculation has its own set of problems of which an implementor must be aware. In a straight-forward implementation, the whole spreadsheet is recalculated each time a value in the spreadsheet is changed. This can cause huge delays, a real problem especially when maybe one doesn't care, at a particular point, whether there are up-to-date values in the spreadsheet. The capability has been provided to disallow recomputation or to cause recomputation over only limited portions of the spreadsheet. Judicious use of this facility can greatly enhance the speed of calculation. The use of this facility requires some sophistication in determining just how it works and implementing it properly. Another problem is the lack of true indirect addressing (as used in a Fortran subroutine). This makes reuse of code somewhat cumbersome.

Another rather specialized problem was inherent to this particular study. As was noted above, there are four complicated functions that require an iterative method to extract the four unknowns. The Newton-Raphson method was used to solve for the four unknowns and was implemented as a spreadsheet "macro." This worked relatively well and addressed the problem of preventing recalculation of the whole spreadsheet at each step since recalculation is disabled during macro execution. The Newton-Raphson method has a few peculiarities of its own. In his book, *The Mathematical Tourist*[10], Ivars Peterson describes an area of instability in the solution space of the Newton-Raphson method that will not produce a solution at all. We found it necessary to take a small fraction of the full correction step to guarantee stability in the method.

The maximum electric field that can be achieved on the RFQ vanes without sparking is determined by a fixed multiple of the Kilpatrick relationship[11] (with present vacuum and surface preparation techniques, we can generally design for electric fields that are twice this criterion). This relationship is such that a straight-forward solution is not possible. A specialized macro was built to compute the value of the Kilpatrick field given the frequency. The relationship between the two variables, frequency and Kilpatrick field (E_{KP}) is as follows:

$$freq = 1.643 E_{KP}^2 e^{\frac{-8.5}{E_{KP}}}$$

Knowing the largest field and the smallest field that is possible and the frequencies that correspond to these fields, it is a rather straight-forward use of the method of bisection to find the field, given the frequency. A macro was implemented to use the bisection method to solve this problem.

Figure 1. shows the user interface to the spreadsheet. Each of the items at the top is a command to the spreadsheet. The first item, *DATA*, is highlighted as indicated by the heavy box around it. On the spreadsheet, this means that this item is selected. When a command is selected an explanitory line appears just below. In this case it says, "Enter New Data" indicating that we are in a data entry mode. This form of instruction is also part of the standard spreadsheet capability. The other commands indicate what the spreadsheet will do. For example, *I/S/S/M* indicates that a calculation allowing beam current (I), the two sigmas (S/S), and the vane modulation factor (M) to be the freely varying parameters is selected. *Look* allows the user to examine the spreadsheet at any location to which he wants to move, *Print* provides for printing the third column of the spreadsheet that contains all the useful numbers, and *Quit* terminates the spreadsheet. Selection is made by use of the arrow keys.

Conclusions

The use of a spreadsheet for the kind of calculation that is described above seems to have more advantages than

DATA I/S/S/M F/S/S/M E/S/S/M Look Print Quit

Enter New Data

DATA ENTRY FORM

Mass(ev)	9.38E+08
Charge No. (=1 for P)	1
Current(amps)	0.125
Emit(tot norm pi-m-rad)	1.00E-06
Emax/K field ratio	2
F Enhance fac (1.3-1.4)	1.3
Sync Phase(deg)	-85
Frequency(Hz)	5.05e+08
mu-t	0.8
mu-l	0.8
min vane/max beam	1.2

Figure 1. Spreadsheet Data Entry Screen

disadvantages. We have demonstrated that rather complex calculations can be performed in this environment. The spreadsheet provides facilities that allow the implementor to put together quickly user-friendly screens that are intuitive to use. Facilities to gather and plot data are also provided and easy to use. This allows one to do studies of parameter interactions. Thus, design calculations can be done on inexpensive hardware that is, at least in principle, portable. The calculations are accurate enough also to provide real cost figures. One can come very close to the true price of an RFQ that will perform to given specifications using an easy-to-use, inexpensive platform. The down-side of this is, of course, speed. One has to be patient while the calculation is being performed.

References

[1] E.A. Wadlinger, "Scaling Laws for RFQ Design Procedures," Proc. 1985 Particle Accelerator Conf., IEEE Trans., Nucl. Sci. 32 (5), (1985), 2596.

[2] E.A. Wadlinger, "RFQ Scaling-Law Implications and Examples," 1986 Linear Accelerator Conference Proceedings, SLAC-303, CONF-860629 , UC-28, (1986), 346.

[3] I.M.Kapchinskiii and V.A.Teplyakov, Prib. Tekl. Eksp. No. 2, (1970), 19 and No. 4, (1970), 17.

[4] K.R. Crandall, R.H. Stokes, and T.P. Wangler, "RF Quadrupole Beam Dynamics Design Studies," Proc. 1979 Linear Accelerator Conf., Brookhaven National Laboratory report BNL-51134, (1980), 205.

[5] Henri Bruck, *Circular Particle Accelerators*, Los Alamos National Laboratory translation LA-TR-72-10 Rev.

[6] R.L. Gluckstern, *Linear Accelerators*, R.M. Lapostolle and A.L. Septier, Eds., (North-Holland Press, Amsterdam, 1970), 827.

[7] Many of the equations that are stated in this paper can be found in the report by T.P. Wangler, "Space-Charge Limits in Linear Accelerators," Los Alamos National Laboratory report LA-8388.

[8] P.J. Channell, "Equipartitioning in Charged Particle Beams," Los Alamos National Laboratory report LA-9927-MS, November 1983.

[9] R.A. Jameson, "Equipartitioning in Linear Accelerators," Proc. 1981 Linear, Accelerator Conf., Los Alamos National Laboratory report, LA-9234-C, (1982), 125.

[10] Ivars Peterson, *The Mathematical Tourist*, W.H. Freeeman and Company, New York.

[11] W.D. Kilpatrick, Rev. Sci Instrum., 28 (10) (1957), 824.