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# SIMULATION OF A STANDING-WAVE FREE-ELECTRON LASER*

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## Abstract

The standing-wave free-electron laser (FEL) differs from a conventional linear-wiggler microwave FEL in using irises along the wiggler to form a series of standing-wave cavities and in reaccelerating the beam between cavities to maintain the average energy. The device has been proposed for use in a two-beam accelerator (TBA) because microwave power can be extracted more effectively than from a traveling-wave FEL. The standing-wave FEL is modeled in the continuum limit by a set of equations describing the coupling of a one-dimensional beam to a $TE_{01}$ rectangular-waveguide mode. Analytic calculations and numerical simulations are used to determine the time variation of the reacceleration field and the prebunching required so that the final microwave energy is the same in all cavities. The microwave energy and phase are found to be insensitive to modest spreads in the beam energy and phase and to errors in the reacceleration field and the beam current, but the output phase appears sensitive to beam-energy errors and to timing jitter.

## Introduction

The next generation of linear colliders is expected to require accelerating gradients of 100 MeV/m or greater. For the high-gradient structures that have been tested, this field strength corresponds to a microwave power of about 100 MW/m, and the required frequency is typically in the range of 10-30 GHz. The microwave free-electron laser (FEL)[1] and the relativistic klystron (RK)[2] have both demonstrated the required power level in this frequency range, and they have been proposed as collider power sources in a configuration known as the "two-beam accelerator" (TBA)[3], in which a high-current "drive" beam generates microwave energy in a beamline that parallels the high-gradient structure. Both the RK/TBA and the FEL/TBA have practical problems. The RK operates best in the X-band (8.4-12 GHz) and so cannot be used with many high-gradient accelerator designs. Conventional FELs, in contrast, have no fundamental frequency limitation, but experiments have shown that microwave extraction is difficult.[4]

The cavity-coupled FEL/TBA[5] has been devised to sidestep the problems found with RKs and conventional FELs. This new device would replace the usual FEL waveguide by a series of short standing-wave cavities, each about a wiggler period in length. The cavities would be separated by irises that allow the beam to pass but reflect most of the microwave power, and between cavities there would be induction accelerating cells to maintain the beam energy. Microwave energy from these cavities would be connected to the high-gradient structure by couplers and would oscillate between the two beam lines with a period that is much longer than the beam time scale but much less than the resistive loss time. This coupling scheme was proposed by Henke[6] for a RK/TBA and is discussed elsewhere.[5]

In this paper, we present preliminary numerical simulations of the standing-wave FEL (SWFEL) used in the cavity-coupled FEL/TBA. The SWFEL has two important differences from conventional FEL amplifiers. One difference is that the standing wave phase $\phi$ develops in time only at each cavity location $z$, whereas the wave phase in conventional devices evolves in $z$ along with the "particle phase" $\theta_j = (k_s + k_w)z - \omega_s t$, where the subsript $j$ denotes the $j$th particle. This difference works against the preservation of a nearly constant average bucket phase $\langle \psi_j \rangle = \langle \theta_j + \phi \rangle$ that is needed for good bunching. As a consequence, the SWFEL requires an unusual form of prebunching, as we discuss later. A second difference is the use of frequent reacceleration to maintain a nearly constant beam energy. Reacceleration is used rather than tapering of the wiggler strength because it is more appropriate for the very long beamlines expected in linear colliders and because it in principal allows the beam energy to be adjusted in time as well as in $z$. Since the unusual phase evolution is a critical novel aspect of SWFEL physics, we choose a very simple simulation model that retains this feature but ignores other arguably important features, such as the discrete nature of the standing-wave cavities and the competition between waveguide modes. The next section describes this model briefly and is followed by a section on simulation results. We offer some tentative conclusions in a final section.

## Model

### Assumptions and Equations

Simulation particles are modeled by a pair of wiggle-averaged equations for the total energy $\gamma_j$ in units of $m_e c^2$ and the particle phase $\theta_j$. Radial motion and the effects of the transverse beam structure are neglected, and the beam is assumed to couple only with a $TE_{01}$ waveguide mode, which is usually most strongly coupled mode. The signal wavenumber for this mode in a rectangular waveguide with height $h$ and width $w$ is $k_s = (\omega_s^2/c^2 - \pi^2/h^2)^{1/2}$. For the fields, we assume an idealized linear wiggler with a vector potential

$$\underset{\sim}{A}_w = \frac{m_e c^2}{e} a_w \cos(k_w z)\, \hat{\mathbf{x}} \tag{1}$$

and an appropriate form for the signal field

$$\underset{\sim}{A}_s = -\frac{m_e c^2}{e} a_s\, \underset{\sim}{q}_\perp (x,y) \cos(k_s z - \omega_s t + \phi), \tag{2}$$

where $\underset{\sim}{q}_\perp = -\sin(\pi y/h)\, \hat{\mathbf{x}}$ is the transverse structure for a $TE_{01}$ mode. A number of other conventional assumptions are made that are suitable for most Compton-regime FELs and significantly simplify the equations. The energy is taken to be sufficiently high that $a_w/\gamma_j \ll 1$, and the energy spread is assumed small enough that all particles have effectively the same axial velocity $V_b$. We treat the signal amplitude $a_s$ as small

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compared with $a_w$, and the both $a_s$ and $\phi$ are assumed to be slowly varying compared with $k_s z$ and $\omega_w t$. This last assumption makes the equations inappropriate for modeling waveguide modes near cutoff.

The wiggle-averaged particle equations are identical to those in a conventional single-mode microwave FEL. Taking $z$ to be the independent variable, we write the equations as

$$\frac{d\theta_j}{dz} = k_w + k_s - \frac{\omega_s}{c} - \frac{\omega_s}{2c\gamma_j^2}\left[1 + \frac{a_w^2}{2} - 2D_x a_w\left(\hat{a}_r\cos\theta_j - \hat{a}_i\sin\theta_j\right)\right] \quad (3a)$$

$$\frac{d\gamma_j}{dz} = -D_x\frac{\omega_s}{c}\frac{a_w}{\gamma_j}\left(\hat{a}_r\sin\theta_j + \hat{a}_i\cos\theta_j\right) - \frac{eE_z}{m_e c^2}. \quad (3b)$$

Here, the coupling coefficient $D_x$ is given for a $\mathrm{TE}_{01}$ mode by

$$D_x = \frac{1}{2}\left[J_0(\xi) - J_1(\xi)\right], \quad (4)$$

where $\xi = \omega_s a_w^2/(8ck_w\gamma_j^2) \approx (a_w/4)(1+a_w^2/2)$. An equation for the complex signal amplitude $\hat{a} \equiv \hat{a}_r + i\hat{a}_i = a_s\exp(i\phi)$ is obtained by assuming that $\hat{a}$ evolves only in time and requiring that the wiggle-averaged equations conserve energy. Taking the distance back from the beam head $s \equiv V_b z - t$ as the "time" coordinate, this procedure gives the field equation

$$\frac{\partial\hat{a}}{\partial s} = i\eta\left\langle\frac{\exp(-i\theta_j)}{\gamma_j}\right\rangle, \quad (5)$$

where the coefficient $\eta$ in general depends on $s$ and is given by

$$\eta = \frac{4\pi}{hw}\frac{eI_b}{m_e c^3}\frac{c}{V_b}\frac{c}{\omega_s}D_x a_w. \quad (6)$$

While this equation implicitly assumes an infinitesimal cavity length and ignores field coupling through the cavity irises, it does model the novel signal evolution expected in a SWFEL.

### "Single-Particle" Solution

Some understanding of the SWFEL equations is gained by looking at a $z$-independent "single-particle" solution, in which the full beam current is assigned to a single phase-space point. Linearizing the equations for small $\delta\gamma = \gamma - \gamma_r$, where $\gamma_r = \omega_s(1+a_w^2/2)/2c(k_w + k_s - \omega_s/c)$ is the resonant energy, we obtain the approximate particle equations

$$\frac{d\theta}{dz} \approx 2\left(k_w + k_s - \frac{\omega_s}{c}\right)\frac{\delta\gamma}{\gamma_r} \quad (7a)$$

$$\frac{d\delta\gamma}{dz} \approx -D_x\frac{\omega_s}{c}\frac{a_w}{\gamma_r}\left(\hat{a}_r\sin\theta + \hat{a}_i\cos\theta\right) - \frac{eE_z}{m_e c^2}. \quad (7b)$$

Requiring $\delta\gamma$ to be $z$-independent gives the $E_z$ required for equilibrium:

$$E \equiv -\frac{eE_z}{m_e c^2} \approx D_x\frac{\omega_s}{c}\frac{a_w}{\gamma_r}\left(\hat{a}_r\sin\theta + \hat{a}_i\cos\theta\right). \quad (8)$$

If $\delta\gamma$ is initially zero, then $\theta$ is likewise independent of $z$ and equal to some arbitrary $\theta_0(s)$. The components of $\hat{a}$ in Eq. (8) are obtained by integrating the linearized field equation, which gives

$$\hat{a}(s) = \hat{a}(0) + \frac{i}{\gamma_r}\int_0^s ds'\eta(s')\exp[-i\theta_0(s')]. \quad (9)$$

As a practical special case, we consider a beam with constant $\eta$ which is prebunched at a frequency $\omega_s + \Delta\omega$, so that $\theta_0(s) = \alpha - (\Delta\omega/V_b)s \equiv \alpha + \beta s$. The components of $\hat{a}$ are then given by

$$\hat{a}_r(s) = \hat{a}_r(0) + \frac{\eta}{\beta\gamma_r}\left[\cos(\alpha) - \cos(\alpha+\beta s)\right] \quad (10a)$$

$$\hat{a}_i(s) = \hat{a}_i(0) - \frac{\eta}{\beta\gamma_r}\left[\sin(\alpha) - \sin(\alpha+\beta s)\right], \quad (10b)$$

and the corresponding reacceleration field is

$$E = D_x\frac{\omega_s}{c}\frac{a_w}{\gamma_r}\left[\hat{a}_r(0)\sin(\alpha+\beta s) + \hat{a}_i(0)\cos(\alpha+\beta s) + \frac{\eta}{\beta\gamma_r}\sin(\beta s)\right]. \quad (11)$$

The bucket size shrinks with increasing $E/|\hat{a}|$, and it is straightforward in this case to calculate the minimum extent of the bucket in $\theta$ and $\gamma$ as the signal develops in $s$. These minimum values are approximately

$$\Delta\theta \approx 3.4\left(-\frac{\beta\gamma_r|\hat{a}(0)|}{\eta}\right)^{1/2} \quad (12a)$$

$$\Delta\gamma \approx 3.3\left(\frac{\omega_s}{c}\frac{D_x a_w}{k_w + \delta k}\right)^{1/2}\left(-\frac{\beta\gamma_r}{\eta}\right)^{1/2}|\hat{a}(0)|, \quad (12b)$$

where $\delta k = k_s - \omega_s/c$. This result shows that the bucket vanishes when $\beta$ is zero or negative and that the longitudinal acceptance $\Delta\theta\ \Delta\gamma$ increases with a larger initial signal and larger $-\beta/\eta$. It is found that Eq. (12) underestimates the acceptance for distributions with spreads in $\theta$ and $\gamma$ because the required reacceleration field in such cases is somewhat lower than for the single-particle case. There is also a weak dependence of the acceptance on the initial average particle phase $\alpha$, with the largest acceptance occurring for $\alpha + \phi(0) = 0$.

## Simulation

### Parameters

The operating frequency $\omega_s$ and the final energy per unit length $W_{out}$ left in the cavities are determined in practice by the TBA requirements. With these quantities given, the specification of the waveguide dimensions $h$ and $w$, the wiggler wavelength $\lambda_w = 2\pi/k_w$, and the wiggler strength $a_w$ sets the principle beam parameters. The beam energy is determined by the resonance condition, and the total beam charge, given by $I_b L_b$ when the current is constant, is set by $W_{out}$. Since the initial spreads in $\theta$ and $\gamma$ are usually determined by the intrinsic emittance from the accelerator and the additional emittance introduced by prebunching, the values are not considered free parameters.

Two remaining beam quantities, the beam-current envelope $I_b(s)/\max\ I_b$ and the prebunching factor $\beta$ can be chosen by practical considerations. Since the acceptance is found to be proportional to $I_b^{-1}$, it is preferable for the current to be low near the beam head, where the bucket reaches its minimum size. It is also found from the single-particle equations that a current that increases like $s$ or faster leads to a monotonically increasing $E_z$ for $s \leq L_b$, which is an easier field to generate than a short pulse. For these reasons, we use a beam with a uniform current ramp as our standard case. The prebunching factor is chosen by considering the $\beta$-dependences of various beam quantities in the single-particle solution. We find that the

**Table 1** Nominal standing-wave FEL parameters

| | | |
|---|---|---|
| peak beam current | $I_b$ | 2.17 kA |
| beam length | $L_b$ | 180.0 cm |
| initial energy | $\gamma_r$ | 27.6 |
| initial $\theta$-spread | $\Delta\theta_0/2\pi$ | 0.1 |
| initial $\gamma$-spread | $\Delta\gamma_0/\gamma_r$ | 0.01 |
| wiggler strength | $a_w$ | 8.86 |
| wiggler wavelength | $\lambda_w$ | 25 cm |
| wiggler length | $L_w$ | 20 m |
| waveguide height | $h$ | 3 cm |
| waveguide width | $w$ | 10 cm |
| signal frequency | $\omega_s/2\pi$ | 17.1 GHz |
| cavity Q | $Q$ | $10^4$ |
| input power | $P_{in}$ | 50 kW / m |
| output energy | $W_{out}$ | 10 J / m |

required beam charge and the longitudinal acceptance increase with $\beta L_b$, while the maximum reacceleration field decreases. Since the beam emittance is difficult to decrease in induction accelerators, we choose $\beta L_b = \pi$, although a lower value might be selected if the limited acceptance of the SWFEL is not found to be a problem.

The nominal parameters used in the simulations here are listed in Table 1. These values are appropriate for a generic TBA, and little effort has been made to optimize the waveguide size or the wiggler strength and wavelength.

## Initialization

The simulation initialization parallels the single-particle solution. A distribution with prescribed spreads $\Delta\theta_0$ and $\Delta\gamma_0$ in $\theta_j$ and $\gamma_j$ is loaded so that $\langle\theta_j\rangle = \alpha + \beta s$ and $\langle\gamma_j\rangle = \gamma_r$. Sinulation particles are uniformly distributed within this phase-space rectangle, and different random position are chosen for each beam slice. Such a distribution is not realistic, but it allows the longitudinal acceptance to be tested systematically.

The reacceleration field required to keep $\langle\gamma_j\rangle$ constant is given by

$$E = D_x \frac{\omega_s}{c} a_w \left( \hat{a}_r \left\langle \frac{\sin\theta_j}{\gamma_j} \right\rangle + \hat{a}_i \left\langle \frac{\cos\theta_j}{\gamma_j} \right\rangle \right). \qquad (13)$$

This field could be recalculated at each $z$ and $s$ value, but this algorithm introduces a high-frequency noise component in E that increases exponently with $z$. A more practical approach is to calculate $E(s)$ at $z = 0$ and to use it at all subsequent $z$ positions. With this second technique, the calculated $E$ is noise free and reduces to Eq. (11) in the limit that $\Delta\theta_0$ and $\Delta\gamma_0$ are zero.

We set the initial signal level $|\hat{a}(0)|$ by assuming some input microwave power per unit length $P_{in}$ and balancing this with cavity-wall losses, specified by an assumed cavity Q.

## Results

The output microwave energy $W_{out}$ and phase $\phi$ for a beam with the nominal parameters and a linearly increasing $I_b$ are shown in Fig. 1. The spreads $\Delta\theta_0 = 0.1$ and $\Delta\gamma_0 = 0.01$ used here are small enough that the distribution remains trapped and the output signal is reasonably insensitive to beam and field errors. The principle $z$ dependence in this case is the initial ripple in $W_{out}$ due to synchrotron motion. This ripple would probably be much reduced for a realistic distribution.

For the standard case, the greatest sensitivity to parameter errors is found for fluctuations in the initial energy. When the reacceleration field is calculated for a beam at the resonant energy and the simulation is run with an energy that is 2% higher, $W_{out}$ is nearly unaffected, but $\phi$ has a ripple of about $\pi/2$ with a wavelength in $z$ corresponding to the synchrotron wavelength in the initial field. This phase ripple can be reduced by choosing larger $a_w$ or $\lambda_w$, which gives a higher resonant energy. In contrast to this energy sensitivity, a 2% error in $I_b$ has a negligible effect on either $W_{out}$ and $\phi$. A change of 2% in the magnitude of $E_z$ likewise has little effects on either the output energy or phase for the standard case, but introducing a 0.1 ns time lag in the reacceleration field again causes a long-wavelength ripple of about $\pi/2$ in $\phi$ due to beam-energy loss during the initial period when $E_z = 0$.

Studies with a constant-current beam show that the final wave phase is as stable as for a beam with a linear current ramp, but there is a 10% ripple in $W_{out}$ that persists in $z$. A beam with constant $I_b$ also begins to lose particles when errors in energy or current exceed about 1.5%, indicating the reduced acceptance for this current envelope.

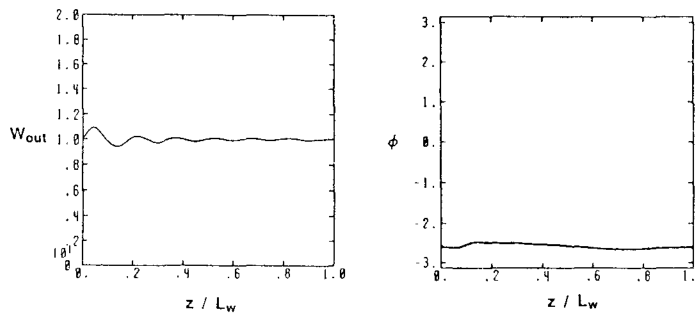


Fig. 1 Output energy per unit length $W_{out}$ and wave phase $\phi$ as functions of $z$ for the standard case.

## Conclusions

From the 1-D simulations discussed here, a standing-wave FEL appears to be a possible microwave source for a two-beam accelerator. Using a beam with modest current and energy, we find that the final microwave energy in cavities is adequate to drive a high-gradient structure, and this energy remains effectively constant in $z$ for fluctuations in energy, current, the reacceleration field, and timing of up to 2%. The final signal phase appears to be more sensitive, with 2% errors in energy or timing causing a phase variation in $z$ of up to $\pi/2$. Work is underway to determine how such a phase error would affect the performance of the high-gradient structure. The need for tight prebunching is another potential shortcoming of the standing-wave FEL.

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