TRANSVERSE COUPLING IMPEDANCE MEASUREMENT STUDIES OF LOW-Q CAVITIES*

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Abstract

We report studies using wire and beadpull measurement techniques to determine the accuracy of transverse coupling impedance measurements, particularly for the case of very low-Q cavities. The studies were performed on simple pillbox cavities in preparation for the measurement of Dual-Axis Radiographic Hydrotest Facility (DARHT) linac cavities, which require extremely low coupling impedances for the deflecting modes of the cavity.

Introduction

This report describes two methods of measuring the transverse impedance of TM_{1n0} modes of pillbox cavities. We measured pillbox cavities to determine the accuracy of the measurement techniques before attempting to measure the actual DARHT cavities, which are low-Q ($Q \leq 5$) ferrite-loaded cavities that accelerate beam by applying a dc-voltage across the cavity gap. In order to avoid beam instabilities the transverse impedance, Z_t, where $Z_t$represents the LLNL definition[1] of transverse impedance, may need to be measurably small ($\omega Z_t/c \leq$ 400 ohms/m).

Previous coupling impedance measurements[2] were for beam pipe structures and cavities having Q's on the order of 100 or greater; however, we had not considered whether there might be difficulties inherent in measuring cavities with very low Q's, nor did we have much knowledge of the accuracies of the measurements. The present work explored both wire and bead-pull measurement techniques on high-Q pillbox cavities (a geometry in which the coupling impedances are readily calculated by the 2-D code URMEL-T and the analytical model described in Ref. 1, hereafter called the Briggs model.)

The TM_{1n0} (or dipole) modes are deflecting modes of cavities, and the transverse coupling impedance is a measure of how efficiently the beam interacts with these modes. This paper will use two different definitions of coupling impedance: the LLNL definition,[1] which has been converted to the MKS system of units

$$\frac{Z_t}{Q} = \frac{c^2[\int B_y\, dz]^2}{2\omega_0 U}\,, \qquad (1)$$

and the SLAC-DESY definition[3]

$$Z_\perp = \frac{j}{\beta I \Delta}\int_{-l/2}^{l/2}(E + v \times B)_T dz\,, \qquad (2)$$

where $\beta = v/c$, I is the beam current, v is the beam velocity, l is the length of the cavity, and Δ is the transverse distance of the beam from the cavity axis. The two definitions of have the following relationship:[4,5]

$$Z_t = \frac{c}{\omega} Re\, Z_\perp\ . \qquad (3)$$

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Beadpull and wire measurements were performed on the following configurations: an Al pillbox cavity, the Al cavity with a mu-metal insert to lower the Q, the Al cavity with large magnetically-coupled, terminated loops to lower the Q, and a steel cavity with and without the terminated loops. These couplers were conducting rods inserted at about mid-radius in the plane containing the maximum electric field. The couplers were shorted to one end of the cavity and were terminated by a resistor to the other end of the cavity. Figure 1 shows the couplers and the electric field patterns for the first three dipole modes of the cavity. Each of the cavities has a 12.7-cm radius. The Al cavity is 38 mm long; the steel cavity is 12.7 mm long. The beam pipe radius is 17.5 mm.

URMEL-T, a computer code that computes TM and TE modes in cylindrically symmetric structures, was used to calculate the transverse impedance:

$$\frac{Z_t}{Q} = \frac{1}{2\omega_0 r_0^2}\frac{|\int_{r_0} E_z\, dz|^2}{U}\,, \qquad (4)$$

where r_0 is the radial distance from the axis. URMEL-T calculates the integral of E_z at the beam tube radius; then it calculates an equivalent value for any specified r_0 by using the known radial distribution (linear for TM_{1mn}, quadratic for TM_{2mn}, and so forth).

Measurement Methods

Figure 2 shows a diagram of the cavity and test environment for the wire measurements. The cavity is transformed into a shielded, balanced coaxial TEM line by adding a section of beam pipe on each end and placing two center conductors inside the pipe. Reference 2 gives complete detail on the wire method.

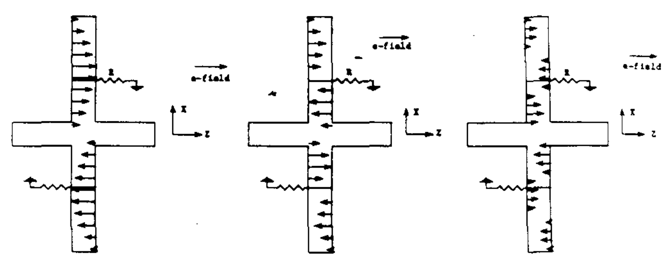

Fig. 1. Position of coupling rods with respect to electric field patterns of first three dipole modes of pillbox cavity.

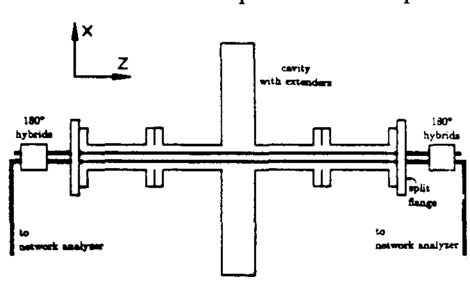

Fig. 2. Wire measurement of transverse coupling impedance of pillbox cavity.

A TSD calibration removes the effects of all mismatches and lengths of line from inside the network analyzer up to the the test cavity with its beam pipe extensions is inserted. Two 180° hybrids are used to excite the beam pipe and cavity with transverse currents. Because the cavities measured are short compared to a wavelength, the cavity is modeled as a lumped parallel resonant circuit imbedded between two sections of transmission line. The two equations that result permit the impedance to be calculated from the magnitude and phase of the measured transmission (after the transmission phase has been corrected to remove the effect of the length of the cavity):

$$re[T^2 Z_t] = \frac{2Z_0}{\Delta^2}\left(\frac{\cos\theta}{|S|} - 1\right) \qquad (5)$$

$$im[T^2 Z_t] = \frac{2Z_0}{\Delta^2}\frac{\sin\theta}{|S|}\,, \qquad (6)$$

where T is the transit time factor. In this paper, all of the following references to values for $Z_\perp$ or Z_t are referring to the real part only. The distance between the two wires is Δ, $|S|$ is the magnitude, and θ is the corrected phase of S_{21}, the transmission through the cavity. Typically, the calibration procedure can be performed with an accuracy of better than $\pm 1\%$ at 100% transmission, and involves measuring a through connection, applying the calibration, and then finding the magnitude of the transmission to be 1 ± 0.01 (calibration precision will be discussed more fully in the discussion of results). Therefore, if the pipe radius is equal to 17.5 mm, the wire radius equal to 1.6 mm, with the wires spaced 24 mm apart ($Z_0 = 200\,\Omega$), and the frequency of the TM_{110} mode approximately 1400 MHz, the minimum measurable impedance is $T^2 Z_\perp \geq 230\ \Omega/m$, or $Z_t \geq 8\ \Omega$. Errors due to field distortions caused by the wires can be minimized by the use of thin wires (calculations by Gluckstern[6] indicate that these field distortions would result in errors of 20% or less). Although we had not previously done extensive testing to determine the accuracy of these measurements, measurements on several devices had yielded results that were very close to calculated impedances.

Measurement of the coupling impedance using the beadpull method is based on the Slater perturbation theory[8], which states that the frequency shift in a cavity due to perturbation by a small, conducting sphere is

$$\frac{\delta f}{f} = \frac{(\frac{3}{2}\mu_0 H^2 - 3\epsilon_0 E^2)}{4U}\frac{4\pi a^3}{3}\,. \qquad (7)$$

Alternatively, for a small, dielectric sphere,[9]

$$\frac{\delta f}{f} = -\frac{\pi a^3}{U}\left(\frac{\epsilon - 1}{\epsilon + 2}\right)\epsilon_0 E^2\,. \qquad (8)$$

We calibrated our dielectric beads by pulling them and a metallic bead of known diameter on axis for the TM_{010} mode, noting the maximum frequency shift at the center of the cavity, and calculating a constant for the dielectric bead that represented the effects of its size and permittivity.

The transverse impedance of the dipole modes can be measured by beadpulls in two different ways. The electric field of these modes is nearly zero on axis; for a perfect pillbox, it is exactly zero, however the presence of the beam pipes causes fringe fields. The magnetic field is so strong on axis, however, that the frequency shift caused by a metal bead pull is almost entirely due to the displacement of magnetic field. Thus, for a dipole mode, one may pull a metal bead along the axis of the cavity, calculating the integral of the magnetic field. Errors in this measurement would be due to fringe electric and longitudinal magnetic fields; however, these errors should be small. Combining Eqs. 1 and 7,

$$\frac{Z_t}{Q} \simeq \frac{c^2\mu_0}{\omega_0\pi a^3}\left|\int\sqrt{\frac{\delta\omega}{\omega_0}}\,dz\right|^2\,. \qquad (9)$$

Alternatively, the measurement could be performed by pulling a dielectric bead off-axis and measuring the electric field. Because

$$B_y = \frac{1}{j\omega}\frac{\partial E_z}{\partial r} \qquad (10)$$

and because, for the dipole modes,

$$E_z = r_0\frac{\partial E_z}{\partial r}\,, \qquad (11)$$

combining Eqs. 1, 10, and 11,

$$\frac{Z_t}{Q} = \frac{c^2}{2\omega_0^3 r_0^2 U}\left|\int_{r_0} E_z\,dz\right|^2\,. \qquad (12)$$

Results

Table I shows, for the TM_{110} mode, the measurement results for the Al pillbox cavity. The bead size was 1.59 mm both for the magnetic field measurement (metallic bead, on axis) and for the electric field measurement (dielectric bead, off-axis). The degenerate dipole mode (orthogonal to the one being measured) was moved by using shorting rods in the plane x=0. Also, because significant field extended into the beam pipes on both sides of the cavity, all measurements were performed with attached beam pipes long enough (the first three modes were below cutoff for the beam pipe) to provide attenuation at the ends of the pipes.

TABLE I. RESULTS OF Al CAVITY MEASUREMENTS
R = 12.7 cm, g = 3.8 cm, b = 1.75 cm and Δ = 2.4 cm

Q	Z_t/Q Briggs	Z_t/Q URMEL-T	Z_t/Q Beakpull	Z_t/Q Wires
7040	30.5	37	36.6[a] 30.9[b]	
2450				48
360[c]			34.8[b]	
490			26.5[b]	
300[d]				25
275			20.2[b]	
22[e]				22

[a]Off-axis electric field measurements. [b]On-axis magnetic field measurements. [c]Mu-metal-lined cavity. [d]One large coupler. [e]Two large couplers.

The discrepancies between URMEL-T and the wire measurement results are fairly large ($\simeq$ 30%) for the higher-Q measurement. This may be because as the cavity impedance gets very large, the magnitude of the transmission $|S|$ becomes very small. Calibrations on network analyzers tend to yield large errors in $|S|$ as $|S|$ becomes very small. The errors in impedance vary as $\Delta Z_\perp/Z_\perp = \Delta|S|/|S|$. Figure 3 shows data taken from specifications for three Hewlett Packard network analyzers. All data are for a full 12-term 2-port calibration using precision 7-mm calibration hardware. This particular measurement with Q = 2450 corresponds to $|S| \simeq -42$ dB, which is possibly beyond the small error region for this experiment. The wire measurement technique must be used with care when measuring medium-to-high-Q cavities (beadpull methods require a high Q in order to have measureable frequency shifts). Another source of error is that the cavity gap represents 18% of a wavelength at 1420 MHz, so the lumped approximation is not very good. The measured values for Z_t (both wire and beadpull) with the couplers in place are low because the coupling rods present a large perturbation to the fields (see Fig. 1) for this mode and thus actually lowers the impedance.

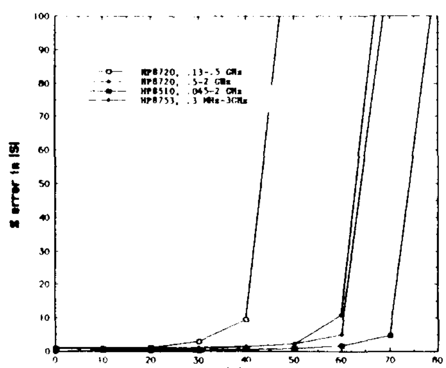

Fig. 3. Accuracy of transmission measurements.

Table II shows the results of the measurements of the steel cavity. We would expect that the beadpull measurements of this cavity may be less accurate than the Al cavity because the gap, being only 1.27 cm long, is not a lot greater than the size of a bead needed to produce measurable signal with low Q's. The bead is no longer a small perturbation to the field, and the Slater perturbation formulas do not describe the measurement as well as before. On the other hand, we would expect greater precision from the wire measurement because the shorter cavity gap will fit the lumped impedance model better than a longer one. The higher-Q measurements (ones with no couplers) were performed both with the wires close together (7.6 mm, Z_0 = 170 Ω) and with the wires placed far apart for greater sensitivity (24 mm, Z_0 = 200 Ω). The bead pull was performed using a metallic bead on axis and only for the lowest mode because we do not have a synthesizer above 2 GHz that can be phase-locked.

As expected, the wire measurements on the steel cavity agree more closely with calculations, probably because the cavity now has a lower Q and better represents a lumped impedance. With the couplers in place, the impedances again are lowered, as with the Al cavity.

TABLE II. RESULTS OF STEEL CAVITY MEASUREMENTS
(R = 12.7 cm, g = 1.27 cm, r = 1.75 cm)

Mode	f	Q	Z_t/Q Briggs	Z_t/Q URMEL-T	Z_t/Q Beadpull	Z_t/Q Wires
TM110	1410	180	16.3	16	11	21[a]
						16[b]
		12[c]				13[b]
TM120	2560	280	12.4	14		17[a]
						13[b]
		65[c]				16[b]
TM130	3730	430	7.6	10		9[a]
						7[b]
		170[c]				5[b]

[a]Close-spaced wires. [b]Wide-spaced wires. [c]Two large couplers.

Conclusions

The pillbox measurements indicate that the wire measurements can be performed with a sufficiently high degree of sensitivity and accuracy to measure the DARHT cavity models. The DARHT cavities have a gap that is short compared to the wavelength of the first two dipole modes, the wires will be small compared to the dimensions of the cavity, and the impedances will be low but well above the noise floor of the measurement. These measurements have also shown that impedance measurements of high-Q cavities may require beadpull methods and that low-Q cavities (especially ones with small gaps) require wire measurements.

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