DISPERSION AND INTERBUNCH ENERGY VARIATION FOR AN e^+e^- LINEAR COLLIDER

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Abstract

Recent studies concerning optimization parameters for e^+e^- super linear colliders use multiple particle bunches for each rf pulse to increase the luminosity and overall efficiency. Requirements for final focusing of the beams severely restrict the bunch to bunch energy variation during the rf pulse. To accurately determine the accelerating fields and energy variation, the dispersion related transient behavior of the rf drive pulse must be considered. A numerical study of dispersion effects on several different accelerating structures is presented.

Introduction

High-gradient accelerator (HGA) structures driven by relativistic klystrons (RK) are part of a collaborative experimental effort on high power microwave sources between the Lawrence Livermore National Laboratory (LLNL), Lawrence Berkeley Laboratory (LBL), and Stanford Linear Accelerator Center (SLAC).[1,2] As the duration of the rf drive pulses is about one fill time of the structure, the propagation of the rf pulse is dominated by transient effects related to dispersion. Several questions concerning the performance of these structures driven by the short RK generated pulses are posed:

1. How can transient fields be determined?
2. Where are the regions of high field stress?
3. What is the energy gain of the electrons?
4. Are there limits on the shape of the rf drive?
5. How does the energy vary between bunches?

These questions are important in the design of high energy e^+e^- linear colliders. Steady state conditions are not approached for several structure fill times. However, to minimize power requirements, high energy linacs are designed to operate with drive pulses approximately equal to one structure fill time. Methods for minimizing energy variation over a bunch train use a partially filled structure.[3]

Analysis of Dispersion

The structures used in the RK experiments are iris-loaded, cylindrical waveguides of constant geometry design. These structures were modeled as band pass filters and analyzed using Fourier analysis.[4] The axial field for the fundamental accelerating mode can be expressed as:

$$E_z(z,t) = \int_{-\infty}^{+\infty} E(\omega)\, e^{i(\omega t - kz)}\, d\omega \ , \tag{1}$$

where

$$E(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E_z(0,t)\, e^{-i\omega t}\, dt. \tag{2}$$

The wave number, $k = \beta + i\,\alpha/2$, is a function of frequency determined by the dispersion relationship of the structure. The attenuation coefficient, α, is due to wall losses and can be treated as constant for the narrow frequency pass bands typical of high-gradient structures. For constant geometry, the dispersion relationship is:[4]

$$\omega(\beta) = \omega_m - \frac{B}{2}\cos(\beta d)\ , \tag{3}$$

where B is the bandwidth, ω_m is the mid-band frequency, and d is the structure cell length. Equation (3) assumes B/ω_m and the attenuation are both <<1.

Selecting the operating frequency, group velocity, v_g, operating mode, and structure efficiency, determines the dispersion relationship. Given the rf drive pulse incident on the structure, equations (1) and (2) can be used to calculate the accelerating fields for any position along the axis of the structure at any later time.

This analysis will also determine the energy gain of an electron transiting the structure. The total energy gain is:

$$U = e\int_0^L |E_z(z,t)|\cos\{\theta(z,t)\}\, dz\ , \tag{4}$$

where L is the structure length, and $t = t_o + z/v_e$ where t_o is the time the rf pulse was applied prior to injection, and v_e is the electron's velocity . $|E_z|$ and θ represent the magnitude and phase of the accelerating field at the electron's position.

Comparison to Experiment

The accuracy of the band pass filter theory was verified by predicting the rf pulse at the exit of the HGA structure in the RK experiments. The magnitude and phase of the incident rf pulse was measured and digitally recorded. A discrete FFT routine transformed the pulse, frequency components were adjusted for dispersion, and the adjusted pulse transformed back to the time

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domain. A comparison with the measured output is shown in Fig. 1.

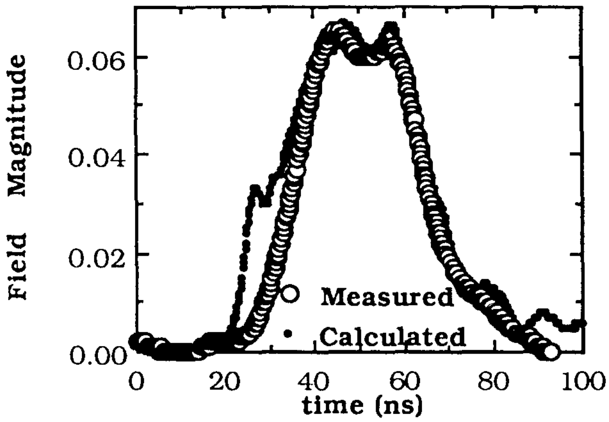

Fig. 1. Comparison of predicted to measure output pulse for the HGA in the RK experiments.

For the case shown in Fig. 1, the peak fields at the input were about 40 MV/m, and the peak fields of the output pulse were about 35 MV/m. At the moderate power level used, beam loading due to dark current was negligible.

Computer Simulations

The computer simulations used a square incident rf drive pulse. Beam loading was assumed negligible. Six different structures were studied (see Table I). The operating mode was $2\pi/3$ for all structures except number 2 which was $\pi/2$. The operating frequency was 11.424 GHz for all cases.

Regions of High Field Stress

An unanticipated finding of the simulations concerned regions of high field stress. The structures studied were constant impedance and have uniformly decreasing fields along the axis for steady state. This is not true while the rf pulse is initially filling the structure. Fig. 2 shows the accelerating field along the axis of structure 1 for two different propagation times. A vertical height of 1.0 corresponds to the incident pulse strength.

TABLE I
Simulated Acceleration Structures

Structure	Length (cm)	Fill Time (ns)	Structure Efficiency
1	26.25	27.57	0.74
2	26.25	27.57	0.62
3	52.5	55.14	0.54
4	100	55.6	0.76
5	150	68.0	0.71
6	200	111.2	0.57

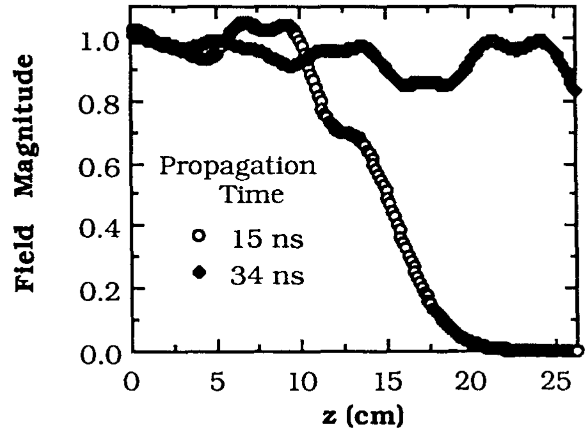

Fig. 2. Accelerating field along axis of a HGA.

The field can have peak values larger than the incident field. For times somewhat larger than one fill time, the fields at the end of a $2\pi/3$ operating mode structure are comparable to incident field strengths and are sustained for a significant portion of a fill time (~20%). For the $\pi/2$ mode, high field levels are experienced at the end of the structure, but for a shorter duration.

This is an interesting result as damage related to electrical breakdown was found at the end of the HGA used in the RK experiments. Electrical arcing at the end of an S-band, constant gradient structure has been reported.[5] As less rf conditioning is accomplished towards the end of an accelerating structure, this region would be more susceptible to breakdown.

Energy Gain Under Negligible Beam Loading

The energy gain of an electron for different injection times was calculated using equation (5) assuming an initial drive field of 100 MV/m and duration 30 ns. As can be seen in Fig. 3 for

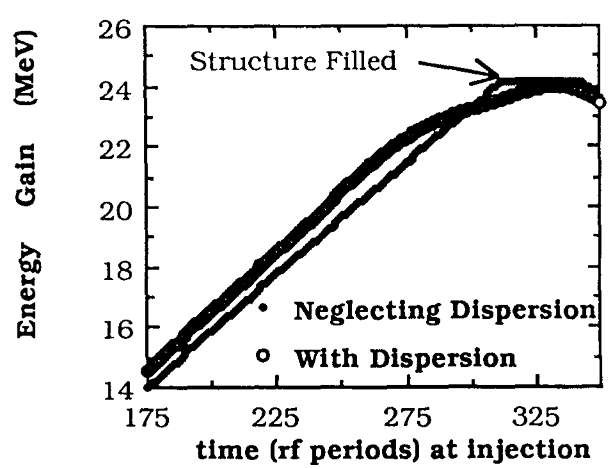

Fig. 3. Energy gain as a function of injection time.

structure 1, there is a noticeable difference between the gain when dispersion is included than when it is neglected.

The increase gain for earlier injection times with dispersion is due to the faster group velocity of the lower frequency components of the drive pulse. Although field levels are lower, the field propagates further . As the fill time is approached, the lower frequency components are exiting the structure and no longer contribute to acceleration/energy gain. Steady state is reached when the slower, high frequency components arrive at the end. Although the energy difference for injection after one fill time amounts to only a few percent of the total gain in the $2\pi/3$ operating mode (and is negligible in the $\pi/2$ mode),the rate of energy gain is different and impacts energy variation over a train of electron bunches.

Minimizing Energy Variation of a Train

For a train of bunches, the accelerating field is a superposition of the rf drive and beam loading. A numerical study of energy variation was performed by subtracting the effects of beam loading from the energy gain calculated from filter theory for a square drive pulse of 100 MV/m.

Beam Loading

For the fundamental accelerating mode, the wake field for a short bunch with charge q after propagating a distance L is given by:

$$E_{wake} = -2\,kq\,e^{-\alpha L/2}\,, \tag{5}$$

where α is the attenuation coefficient and k is the loss parameter. The loss parameter can be related to the bunch efficiency factor, η, by:

$$\eta = 1 - \frac{(E_z - E_{wake})^2}{E_z^2} \approx \frac{4\,kq}{E_z}\,. \tag{6}$$

The energy loss of the second bunch due to beam loading over the length, L, of the accelerating structure can be expressed as:

$$\Delta V_2 = \int_{\Delta s'}^{L} \left\{-\,2\,kq\,e^{-\frac{\alpha}{2}\left(\frac{v_g}{v_e}[z - \Delta s'] + \Delta s'\right)}\right\} dz\,, \tag{7}$$

where $\Delta s'$ is related to the bunch spacing, Δs, by:

$$\Delta s' = \Delta s\left(1 + \frac{v_g}{v_e - v_g}\right). \tag{8}$$

For subsequent bunches, the energy loss is a sum of integrals similar to equation (7), and for the Nth bunch can be expressed as:

$$\Delta V_N = -\frac{4kq}{\alpha}e^{-\gamma}\frac{v_e}{v_g}\left(e^{-\beta}\,\frac{1-e^{-(N-1)(\beta+\gamma)}}{1-e^{-(\beta+\gamma)}} - e^{-\delta}\,\frac{1-e^{-(N-1)\gamma}}{1-e^{-\gamma}}\right), \tag{9}$$

$$\text{with}\ \ \gamma \equiv \frac{\alpha\,\Delta s}{2}\,,\ \beta \equiv \frac{\alpha\,\Delta s'}{2}\frac{v_g}{v_e}\,,\ \text{and}\ \ \delta \equiv \frac{\alpha L}{2}\frac{v_g}{v_e}\,. \tag{10}$$

Results of the Numerical Study

For the six different structures listed earlier, equation (9) was used to calculate the energy loss for various bunch spacings and bunch efficiency factors for a train of 10 bunches. The values were subtracted from the energy gains corresponding to different injecting times into the partially filled structure. Minimum energy variation parameters for the different structures are listed in Table II.

Table II
Parameters for Minimizing Energy Variation

Struc -ture	η (%)	spacing (# λ)	$\Delta V/V_{ave}$ (%)	V_{ave}/V_m (%)
1	2.0	3	0.093	80.5
2	1.6	3	0.054	88.3
2	2.2	4	0.078	86.2
3	1.7	6	0.084	70.4
4	1.9	6	0.072	75.3
5	1.8	7	0.096	77.4
6	2.0	14	0.153	79.6

The structure number refers to the structures listed in Table I, ΔV is one half of the energy range over the train, V_{ave} is the average energy of the train, and V_m is the maximum single bunch energy gain for a filled structure. For synchroneity, spacing must be an integral number of wavelengths, λ. Small changes in η or spacing will cause a large increase in energy variation.

Conclusions

The questions asked in the Introduction are adequately addressed with filter theory. Transient fields can be accurately calculated as well as the energy gain of accelerated electrons. High field stresses are expected at both entrances and exits of structures as noted experimentally. The leading edge "squareness" of a drive pulse will be limited by the frequency band of the structure. For multi-bunches , dispersion will lower the average energy of a bunch train for a desired energy variation.

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