SUPPRESSION OF SINGLE BUNCH BEAM BREAKUP BY BNS DAMPING*

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Introduction

The intense narrow beams now used or planned in linear colliders lead to strong wake fields which are capable of increasing the transverse emittance to unacceptable values. A brief analysis of this phenomenon, known as single bunch beam breakup[1], was outlined by Neri and Gluckstern[2] for a coasting beam, using the cumulative beam breakup formalism of Gluckstern, Cooper and Channell[3].

Balakin, Novokhatsky and Smirnov[4] suggested that, by decreasing the energy of the tail of a bunch relative to its head, the resulting increased focusing force on the tail of the bunch could offset the effect of single bunch beam breakup. This method (a form of Landau damping now known as BNS damping) has been implemented at SLAC[1] and is being incorporated into the CLIC design[5]. Recently Balakin[6] suggested a variant of BNS damping in which all particles in the bunch oscillate with the same transverse frequency, amplitude and phase. A possible implementation of this suggestion has been analyzed by Seeman and Merminga[7] for the SLC.

The recent work[1,5,7] is primarily an application to specific accelerators (SLC and CLIC) and depends on the details of bunch shape, structure, and acceleration history. We here instead analyze single bunch beam breakup with BNS damping for a uniform coasting beam in order to explore the dependence on parameters. We use an earlier formulation for cumulative beam breakup[3] and finally express the transverse beam growth in terms of two universal parameters.

Single Bunch Beam Breakup

In order to use the formalism for cumulative beam breakup[3], we divide the single bunch into M_0 equally charged macroparticles. The equations governing ξ_M^N, the displacement of the M'th macroparticle in a coasting beam as it enters the N'th cavity are

$$\xi_M^{N+1} - 2\cos\mu\, \xi_M^N + \xi_M^{N-1} = z_M^N , \quad (1)$$

*Work supported by the Department of Energy.
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$$z_M^N = r \sum_{l=1}^{M-1} e^{-\frac{\omega\tau}{2Q}(M-l)} \sin(\omega\tau(M-l))\xi_l^N , \quad (2)$$

where z_M^N is proportional to the excitation of the N'th cavity as the M'th particle enters. Here μ is the phase advance of the transverse oscillation between cavities, ω and Q are the frequency and quality factor of the transverse deflecting mode, and τ is the time interval between macroparticles. The parameter

$$r = \frac{N_p e^2}{2M_0 W} \frac{Z_\perp T^2}{Q} L \quad (3)$$

is a measure of the charge of each macroparticle and its influence on the transverse motion. The bunch has energy W and total charge $N_p e$, and the cavities, separated from each other by a distance L, have a shunt impedance parameter $\frac{Z_\perp T^2}{Q}$, where T is the transit time factor.

In our analysis for a single bunch, we shall assume that $M_0\omega\tau$ is sufficiently small so that $\sin((M-l)\omega\tau)$ can be replaced by $(M-l)\omega\tau$ in Eq. (2). In addition, we assume a weak focusing approximation and treat the large parameters N and M as continuous variables. This permits us to rewrite Eqs. (1) and (2) as

$$\frac{\partial^2 \xi_M^N}{\partial N^2} + \mu^2 \xi_M^N = z_M^N , \quad (4)$$

$$z_M^N = r\omega\tau \int_0^M dl (M-l)\xi_l^N . \quad (5)$$

Two derivatives of Eq. (5) with respect to M lead to

$$\frac{\partial^2 z}{\partial M^2} = r\omega\tau\xi , \quad (6)$$

where $r\omega$ should really be taken to be the sum $\sum_j r_j\omega_j$, over all transverse modes which are capable of deflecting the beam.

The dominant dependence on N suggested by Eq. (1) is $e^{i\mu N}$. By redefining ξ_M^N and z_M^N so as to remove this factor and assuming that the remaining factors are slowly varying with respect to N, we obtain

$$\frac{\partial \xi}{\partial N} = \frac{z}{2i\mu} . \quad (7)$$

Equations (6) and (7) govern the asymptotic behavior of the displacement and cavity excitation for large N and M.

It is a simple matter to demonstrate that

$$\begin{aligned} \xi, z &\sim e^{ue^{-i\pi/6}} \\ u &= \tfrac{3}{2}\left(\tfrac{r\omega\tau}{\mu}\right)^{1/3} M^{2/3} N^{1/3} \end{aligned} \quad (8)$$

contains the dominant dependence for large N and M. In fact, an alternate calculation starting with the integral representation for the solution of Eqs. (1), (2), which uses a saddle point calculation for large N and M, leads to

$$\frac{\xi}{\xi_0} = \frac{1}{\sqrt{4\pi}} \Re \left(\frac{e^{i\mu N + ue^{-i\pi/6}}}{\sqrt{ue^{-i\pi/6}}} \right) . \quad (9)$$

Assuming that the dominant dependence on N is contained in the factor $e^{i\mu N}$ we can check the validity of Eq. (9) for large u by forming

$$\Lambda = \ln \frac{|\xi - \frac{i}{\mu}\frac{\partial \xi}{\partial N}|}{\xi_0} \quad (10)$$

from direct numerical simulations of Eqs (1) and (2). A plot of

$$\lambda_1 = \Lambda - \frac{\sqrt{3}u}{2} + \frac{1}{2}\ln 4\pi u \quad (11)$$

vs. u, is shown in Fig. 1 for $M_0 = 100$, $\omega\tau = 0.01$, $rM_0 = 0.06$, $\mu = \frac{\pi}{50}$. Since the value of λ_1 corresponding to Eq. (9) is zero, the figure shows that Eq. (9) is valid to an accuracy of about 10% for $u \geq 2.5$.

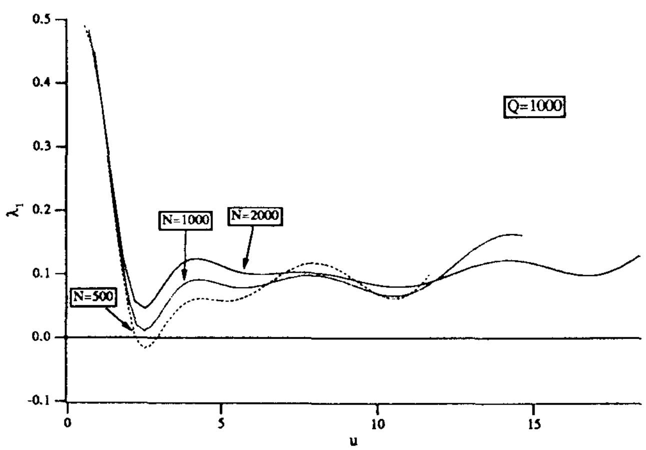

Figure 1: Check of Eq. 11 with simulation for $N = 500$, 1000, and 2000 .

BNS Damping

We shall now assume that the energy decreases linearly from the head to the tail of the bunch. Specifically, we replace μ^2 in Eq. (4) by

$$\mu^2 \to (\mu + \alpha M)^2 \cong \mu^2 + 2\alpha M \mu, \quad (12)$$

where $\frac{\alpha M_0}{\mu} \ll 1$ is the fractional increase in the phase advance per cavity during the pulse, and where μ is now independent of M. Removal of the factor $e^{i\mu N}$ from ξ and z in Eq. (4) then leads to an additional term in Eq. (7):

$$\frac{\partial \xi}{\partial N} + \frac{\alpha M \xi}{i} = \frac{z}{2i\mu} . \quad (13)$$

It is now convenient to change variables from N and M to u and v where u is given in Eq. (8) and where

$$v = \alpha\left(\frac{\mu}{r\omega\tau}\right)^{1/3} M^{1/3} N^{2/3} . \quad (14)$$

After considerable algebra, we find that, for large u, the solution of Eqs. (6) and (13) for $\frac{\xi}{\xi_0}$ can be written in terms of the universal parameters u and v, as

$$\frac{\xi}{\xi_0} = \frac{1}{\sqrt{4\pi u}} \Re \left(e^{uf(v) + g(v) + i\frac{\pi}{12} + i\mu N} \right) , \quad (15)$$

where $f(v)$ and $g(v) = \int_0^v \frac{N}{D} dv$ satisfy the following equations:

$$i + f^3 + 3vf^2 f' + \frac{9}{4}v^2 f f'^2 + \frac{1}{2}v^3 f'^3 = iv\left(2f^2 + 2vff' + \frac{1}{2}v^2 f'^2\right) , \quad (16)$$

$$\begin{aligned} N \;=\; & 24i(f + vf') - 9f'(4f + 3vf') \\ & - 2v(-2iv + 9f + 6vf') f''(v) , \end{aligned} \quad (17)$$

$$D = 4(2f + vf')(-2iv + 3f + 3vf') . \quad (18)$$

The solution of Eq. (16) corresponding to $f(0) = e^{-i\pi/6}$ has been obtained by numerical integration and is shown in Fig. 2. Fig. 3 shows the result for $g(v)$, obtained by a subsequent numerical integration using $\frac{N}{D}$.

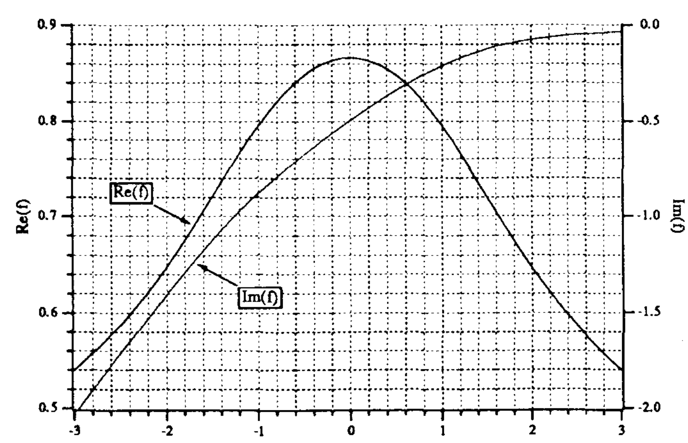

Figure 2: $f(v)$ vs. v.

The validity of Eq. (15) can be checked by forming Λ defined in Eq. (10), and

$$\lambda_2 = \Lambda + \frac{1}{2}\ln 4\pi u - u\Re f(v) . \quad (19)$$

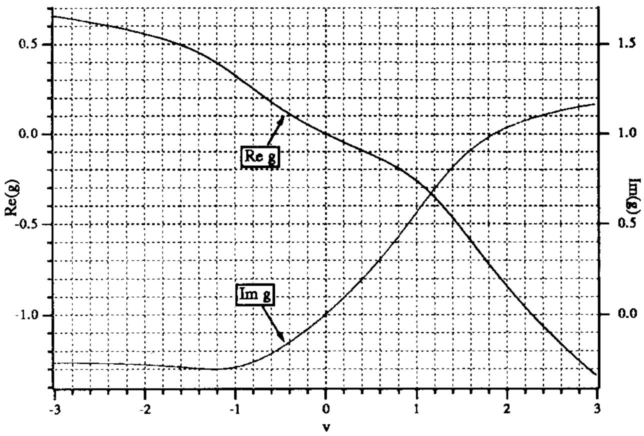

Figure 3: $g(v)$ vs. v.

The plot of λ_2 vs v in Fig. 4 for u = 14.57, 17.65, 20.35, adjusted s.t. $\lambda_2 = 0$ for $v = 0$, shows that the results are independent of u in this range, as predicted by Eq. (15). Furthermore, the agreement between λ_2 and $\Re g(v)$, shown as the dashed line in Fig. 4, clearly confirms the parametrization predicted in Eq. (15).

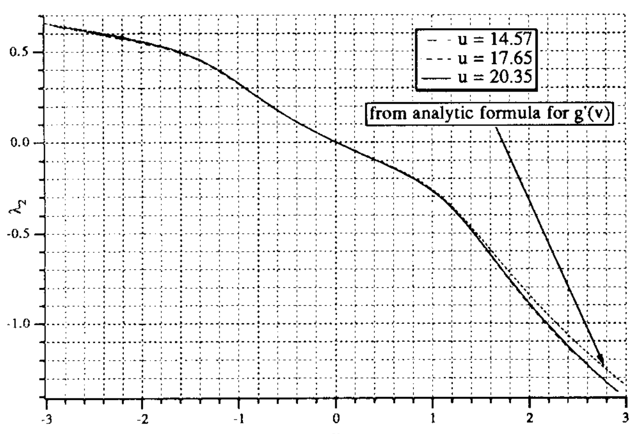

Figure 4: λ_2 and $g(v)$ vs. v for 3 values of u.

Discussion and Summary

The main result of this paper is the parametrization in terms of u and v in Eq. (15). In suppressing single bunch beam breakup one therefore determines the growth of the tail displacement in the absence of BNS damping ($v = 0$) and the value of v necessary to reduce this growth to an acceptable value.

An interesting result is obtained if one assumes that v is large and that $uf(v)$ is the dominant part of the exponent. It can be shown for large v that $f(v) \rightarrow \sqrt{\frac{8}{9v}}$, in which case

$$uf(v) \cong \sqrt{\frac{2Mr\omega\tau}{\alpha\mu}} \qquad (20)$$

independent of N. If one now looks for a solution to Eq. (6) which is independent of M, this can be achieved by requiring in Eq. (4) that

$$\xi^N \Delta\mu^2 \cong \xi^N 2\alpha M\mu = z_M^N = \frac{M^2}{2} r\omega\tau\xi^N, \qquad (21)$$

or $Mr\omega\tau = 4\alpha\mu$. The condition that the increase in focusing completely compensates for beam breakup is therefore similar to placing a numerical limit on the exponent in Eq. (15).

Finally, although our analysis is for a uniformly charged beam, the parameter $M_0 r\omega\tau/\alpha\mu$ can be written in a form independent of the bunch microstructure:

$$\frac{M_0 r\omega\tau}{\alpha\mu} = \frac{(M_0 r)(M_0 c\tau)}{(\alpha M_0/\mu)(\mu^2 c/\omega)}, \qquad (22)$$

where $M_0 r$ is proportional to the total bunch charge, $M_0 c\tau$ is the equivalent bunch length, and $\alpha M_0/\mu$ is the fractional increase in the transverse phase advance. In order to suppress single bunch beam breakup this parameter should be of the order of 1 or less.

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