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MUTUAL COMPENSATION OF WAKEFIELD AND CHROMATIC EFFECTS OF INTENSE LINAC BUNCHES*

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Abstract

Mutual compensation of transverse and chromatic effects for intense electron bunches in a high-energy linac is a recent Novosibirsk idea which provides a new control of emittance enlargement. In this paper we elaborate on the principles and constraints for this new technique which requires careful matching of internal bunch parameters with external forces. With specific values of the bunch length, bunch intensity, and klystron phasing, the transverse-wakefield-induced forces within the bunch can be cancelled by energy-dependent forces from the quadrupole lattice at all positions along the linac. Under these conditions the tolerances for quadrupole alignment, dipole stability, and injection launch errors are significantly relaxed.

1. Introduction

The primary goal of linear colliders is to maximize the integrated luminosity for the experimental program. Small transverse emittances are a necessary condition for maximum luminosity. However, a number of single-particle and current-dependent effects cause emittance enlargement.[1] In particular, the spectrum of particle energies within the bunch allow chromatic phase-space mixing. Furthermore, current-dependent wakefield effects can result in emittance dilution. A technique[2,3] emanating from the Institute of Nuclear Physics in Novosibirsk called "autophasing" can be used to mutually cancel these chromatic and wakefield effects. A description of the physics principles of this technique, the method of application, and an example for the SLC will be discussed in this note.

In Section 2 the difference between autophasing and present practice is discussed. In Section 3 we start from the equation of motion of a particle in the presence of transverse wakefields and derive the condition for autophasing. Furthermore, we attempt an intuitive discussion of autophasing. In Section 4 we describe the procedure used to arrange the linear focusing of a lattice in order for autophasing conditions to hold. Finally, in Section 5 we apply the results of this analysis to the SLC linac.

2. Present Situation

Off-axis particles traversing an accelerating structure generate transverse wakefields which deflect all trailing particles. There are many causes for a bunch to be off-axis—betatron oscillations, local bumps, head-tail transverse offsets, or collimator deflections. These wakefield deflections accumulate along the linac and cause emittance enlargement of the beam. This beam blowup by wakefields can be reduced by BNS damping. The effect of BNS damping is to reduce the effective defocusing nature of the wakefield force by providing extra focusing for the core and tail particles. This is accomplished by lowering the energy of the trailing particles relative to the head so that the quadrupole lattice focuses them more strongly. The trailing particles are differentially lowered in energy by back-phasing klystrons early in the accelerator and forward-phasing downstream klystrons to keep the energy spectrum small at the maximum energy. The overall goal is to minimize the emittance at the end of the linac using as little extra acceleration as possible. The best configuration depends on many machine parameters and must be calculated for each case. However, with this adaptation of BNS damping the effective emittance is not controlled in the middle of the linac, and sensitivity to local errors is a result.

In this paper we present a new method of controlling transverse wakefields originally suggested by Balakin et al.[2,3] This method is based on the phenomenon of autophasing: all particles within the bunch—taking into account all forces acting on them—should oscillate with the same amplitude, phase and frequency. The significance of autophasing is that the bunch will remain coherent in its motion independently of any dipole-like perturbations to its trajectory at all locations, and hence the emittance will remain unchanged.

3. Autophasing

Let us assume a relativistic bunch whose transverse dimensions are zero. Let $x(z,s)$ denote the displacement of a particle in the bunch as a function of z, the longitudinal position relative to the center of the bunch (z is positive towards the head of the bunch), and s, the distance along the accelerator.

If we assume that the beam energy increases linearly with s as a result of acceleration and that the distance to double the energy is long compared to the betatron wavelength, then the equation of motion for $x(z,s)$ can be written:

$$\frac{d^2}{ds^2}\,x(z,s)+k^2(z,s)\,x(z,s) = \frac{r_0}{\gamma(z,s)}\int_z^\infty dz'\rho(z')\,W_\perp(z'-z)\,x(z',s) \quad , \tag{1}$$

where $\gamma(z,s)$ is the beam energy at position s in units of mc^2; p is the line density of the particles in the bunch normalized to the total number of particles in the bunch N; $eW_\perp x$ is the traverse field produced by a point charge displaced from the axis by x at a distance $z'-z$ behind that point charge; $k(z,s)=2\pi/\lambda(z,s)$ where $\lambda(z,s)$ is the betatron wavelength; and $r_0=e^2/mc^2$ is the classical radius of the electron. In writing this equation we have assumed that the betatron focusing is provided by a smooth function, rather than from a series of discrete quadrupole magnets.

To find the conditions for autophasing we attempt to find an identical solution for all particles independent of longitudinal position within the bunch. Let us consider an expression of the form

$$x(z,s)=x_0\cos\left(k_0 s+\phi_0\right) \quad , \tag{2}$$

where ϕ_0 is an arbitrary initial phase, and derive the condition that needs to be satisfied in order for this expression to be a solution to Eq. (1). Inserting Eq. (2) into Eq. (1) and noting that Eq. (2) is independent of z, and hence that it can come out of the integral, we find

$$k^2(z,s)=k_0^2+\frac{r_0}{\gamma(z,s)}\int_z^\infty dz'\rho(z')\,W_\perp(z'-z) \quad . \tag{3}$$

To gain some insight into the purpose of the previous mathematical manipulations, we first discuss the meaning of Eq. (3) and then we come to the solution of the equation of motion, Eq. (2). A particle at position z within the

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bunch, subjected to wakefield forces from all the other particles ahead in the bunch, will experience a frequency shift given by the second term on the right-hand side of Eq. (3). The first term on the right-hand side is the square of the frequency with which all particles are required (by the form of the solution) to oscillate. For this condition to be possible, the external focusing forces must be such that when they act *alone* on the particle, its betatron oscillation frequency is given by $k(z,s)$. Hence, by carefully balancing the two frequencies—one coming from the chromatic effects, the other from the transverse wakefield effects—all particles in the bunch will oscillate with the same amplitude, phase, and frequency, as Eq. (2) suggests. It is important to note that the autophasing condition, Eq. (3), is independent of the transverse offset x_0 of the bunch. This condition is to be satisfied at all linac positions s.

To further realize the significance of this result, suppose that a dipole-like error perturbs the trajectory of a bunch along the linac, giving rise to a betatron oscillation. Since the autophasing condition is independent of the transverse displacement, it still holds true in the presence of betatron oscillations of arbitrary amplitude, and hence in the presence of any type of errors causing betatron oscillations in the machine! Dipole-like errors may come from injection errors in position and angle, from quadrupole displacements, RF kicks, or dipole strength changes. In the presence of any of these errors, the bunch will remain "compact," maintaining the beam emittance constant. Further, this technique is expected to work successfully for any charge density, as long as Eq. (3) is satisfied for each point in the linac, thus ensuring the emittance preservation throughout the whole machine! [It should be pointed out that chromatic effects can still occur when beam steering is done on a scale that is short when compared to $\lambda(s)$.]

Next we demonstrate how the lattice parameters can be adjusted so that the external energy-dependent forces compensate the internal wakefield forces, and the condition for autophasing is satisfied.

4. Lattice Adjustment: k_E

Let us define $k_W(z)$ as the right-hand side of the autophasing condition, Eq. (4). The subscript W serves as a reminder that this expression depends on the wakefields and on the internal parameters of the bunch. The goal now is to adjust the quadrupole lattice and klystron phasing so that the energy-dependent forces cancel the transverse wakefield forces at all points in the linac. Equivalently, we need to determine an energy-dependent function of z, $k_E(z)$, such that

$$k_W(z) = k_E(z) \quad . \tag{4}$$

In Fig. 1 we give a pictorial representation of Eq. (4). Figure 1(a) is a plot of the bunch density along the bunch. The Gaussian bunch length in this example is 1.75 mm and we assumed 3.5×10^{10} particles per bunch. Figure 1(b) displays the integrated transverse wakefield as a function of the longitudinal position z. By adding the frequency of the bunch head k_0 to the integral according to Eq. (4), we obtain Fig. 1(c) which is a plot of k_W as a function of z. Figure 1(d), on the other hand, shows the beam energy along the bunch. Longitudinal wakefields have been taken into account and the klystron phase has been chosen to be 8° in this particular example. Finally, Fig. 1(e) is a plot of the betatron frequency along the bunch as determined from lattice considerations and the bunch energy. The goal is to match the shape of Fig. 1(c) to the shape of Fig. 1(e) by varying the bunch length and the klystron phases.

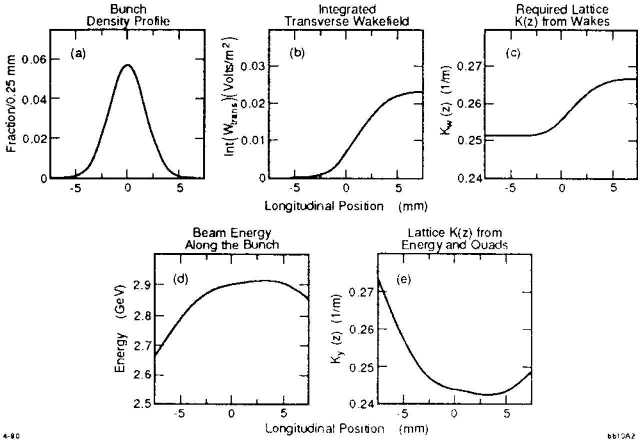

Fig. 1. Schematic representation of the mutual compensation of wakefield and chromatic effects of a bunch. This example is taken at the 100 m position in the linac.

From the definition of the chromaticity of a lattice ξ:

$$\frac{\delta\nu}{\nu} = \xi \frac{\delta E}{E} \quad , \tag{5}$$

k_E is given by

$$k_E(z) = k_0 \left[\frac{E_0 + \xi(E(z) - E_0)}{E_0} \right] \quad , \tag{6}$$

where E_0 is the energy of the head of the bunch. The energy $E(z)$ corresponding to the betatron wave number $k_E(z)$ is given by

$$\begin{aligned} E(z) \; &= E_{\text{inj}} + \sum_{i=1}^{n} \Bigg[\Delta E_i \cos\left(\phi_i + \phi(z)\right) \\ &+ \Delta s_i \int_{z}^{\infty} W_{\parallel}(z' - z)\, \rho(z') dz' \Bigg] \quad , \end{aligned} \tag{7}$$

where E_{inj} is the injection energy; ϕ_i are the klystron phases (which are free parameters); $\phi(z) = 2\pi z/\lambda_{\text{RF}}$, where λ_{RF} is the RF wavelength; ΔE_i is the maximum energy gain in the distance Δs_i; and the last term on the right-hand side of Eq. (7) is the longitudinal wakefield contribution to the particle energy.

Thus, the goal of lattice parameter adjustment is to determine the values of the bunch length and the klystron phases so that $k_E = k_W$ at every point along the bunch and the linac. In the following section we use the SLC linac as an example to demonstrate how this careful matching can be done in a realistic situation.

5. Autophasing Applied to the SLC

The applicability of autophasing to the SLC linac was studied with the goal of potentially improving the luminosity. A computer program was written to match the lattice and wakefield-determined spatial frequencies $k_E(z)$ and $k_W(z)$ over the length of a bunch at every longitudinal position along the linac. In parallel, a proposal to experimentally study this effect on the SLC has been made.[6]

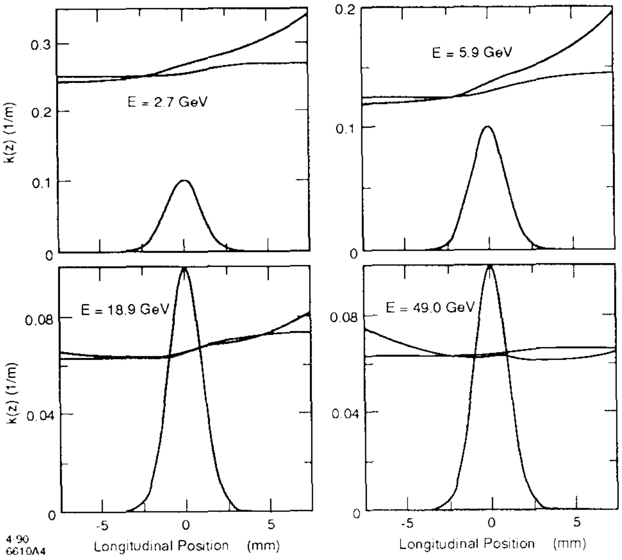

Fig. 2. Case 1. Klystron phases $\phi_i = -25°, -25°, -25°, 18°, 18°, 18°, 18°$, and $18°$; bunch length $\sigma_z =$ 1 mm; and final energy spread $\sigma_E/E = 0.30\%$.

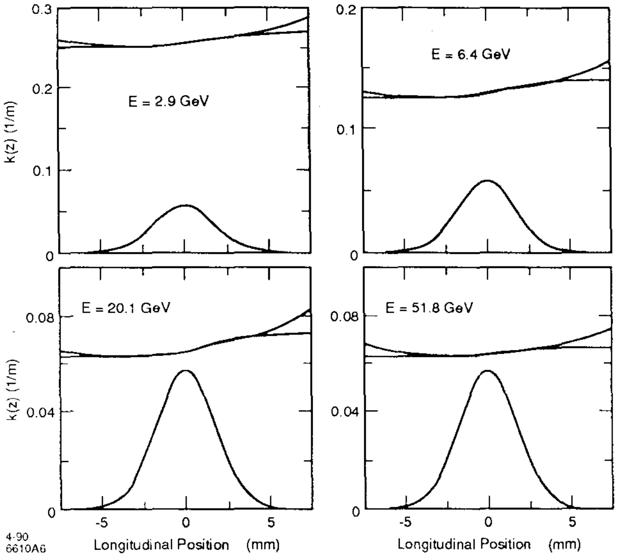

Fig. 3. Case 2. Klystron phases $\phi_i = -7°, -10°, -20°, -1°, -1°, 0°, 0°$, and $0°$; bunch length $\sigma_z =$ 1.75 mm; and final energy spread $\sigma_E/E = 1.40\%$.

Computation inputs were made as realistic as possible. The transverse and longitudinal wakefields were those calculated for the SLAC linac.[7] The betatron focusing was assumed to be provided by a smooth function. A detailed description of the computer program can be found elsewhere.[8] The autophasing match was studied for 6 SLC linac conditions, assuming 3.5×10^{10} particles per bunch. Only two of the six cases studied are presented here. The rest can be found in Ref. 8. The functions shown in the following plots resulting from the calculations are for $k_E(z)$, $k_W(z)$, and $\rho(z)$ at four locations in the linac.

Case 1

Corresponds to the presently used BNS damping conditions and is shown in Fig. 2. The autophasing match is not good, as shown by the separation of k_E and k_W in the region of the bunch core.

Case 2

The bunch length was increased to 1.75 mm and the phases then were optimized. Results are shown in Fig. 3. A very good match is obtained over the entire bunch. The internal frequencies of the curves match very well.

Conclusions

In this paper we have studied the requirements of autophasing. By carefully matching the internal bunch parameters with external forces, mutual compensation of transverse and chromatic effects for intense electron bunches can be achieved. With this compensation it is expected that the alignment and launch tolerances can be significantly relaxed.

We started from the equation of motion of a particle in the presence of transverse wakefields and derived the condition for autophasing. We then discussed the physical meaning of the result and concluded that its significance lies in the fact that the bunch will remain coherent in its motion independently of any dipole-like perturbations to its trajectory, and hence the emittance ideally will remain unchanged.

These results were then applied to the SLC linac. In our numerical computations we used the transverse and longitudinal wakefields calculated for the SLAC linac, but assumed smooth focusing. We determined the appropriate klystron phases and bunch lengths for autophasing to hold in several positions in the linac. These initial studies indicate that autophasing is possible for the SLC, and that more studies and possibly beam experiments should be pursued. Additional studies of tolerances and applicability using FODO lattices are being made. The effects of non-Gaussian bunch shapes (e.g., those produced by the SLC bunch length compression system) will also be studied, and reported elsewhere.

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