# DESIGN STUDIES OF LOW ENERGY H- BEAM TRANSPORT WITH ELECTROSTATIC LENSES* 

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#### Abstract

We report the results of design studies for low-energy transport of a $35 \mathrm{kV}, 30 \mathrm{~mA} \mathrm{H}{ }^{-}$beam using electrostatic quadrupole (ESQ) and einzel lenses. Emittance growth is obtained from a modified PARMILA code for the ESQ configuration and from SNOW for the einzel-lens system. A comparison of important design considerations and parameters for the two electrostatic systems will be presented.


## Introduction

In this paper we report the results of conceptual design studies for a $35 \mathrm{kV}, 30 \mathrm{~mA} \mathrm{H} \mathrm{H}^{-}$LEBT system using either electrostatic quadrupole (ESQ) or einzel lenses for matching the beam into a Radio Frequency Quadrupole (RFQ) accelerator. The $\mathrm{H}^{-}$beam is assumed to have an initial normalized rms emittance of $0.069 \pi \mathrm{~mm}$-mrad and an initial radius of approximately 1 mm . These numbers were chosen to achieve an emittance in the range of $0.1 \pi$ to $0.2 \pi \mathrm{~mm}-\mathrm{mrad}$ at the entrance of the RFQ, which corresponds to the parameters of the BEAR project ${ }^{1}$ and the $S S C^{2}$. The spacing between source and RFQ was taken to be 30 cm for the ESQ design and 20 cm for the einzel lens system. To minimize nonlinear effects we limited the maximum beam radius, $R_{\text {max }}$, to about 10 mm .

There are several important differences between electrostatic quadrupole lenses and einzel lenses: (1) The ESQs provide strong, first-order focusing while the einzel lenses are weak, second-order focusing devices. Consequently, ESQs in doublet or triplet configurations - operate with lower voltages than einzel lenses to achieve the same focusing strength per meter. (2) For matching purposes, one needs a minimum of only two einzel lenses, but at least four quads (two doublets) in the ESQ case. (3) Spherical aberrations are generally more severe in axisymmetric than in quadrupole field configurations. Consequently, the fraction of the lens aperture ("linear aperture") available for the beam is larger in ESQs than in einzel lenses. For our design studies we chose $R_{\text {max }}$ to be about $75 \%$ of the aperture radius of the ESQs and $50 \%$ in the einzel lens system. (4) Since the electric field lines are predominantly in the transverse direction in ESQs and longitudinal in einzel lenses, chromatic aberrations tend to be more severe in ESQs than in einzel lenses. This effect limits the ESQ voltages to less than about $20 \%$ of the beam voltage while the einzel lens voltages can be almost as high as the beam voltage.

In our design studies, the minimum spacing $d$ between any two electrodes with potential difference $V$ must satisfy

[^0]the voltage breakdown criterion ${ }^{3}$
\[

$$
\begin{equation*}
d_{[\mathrm{cm}]} \geq 1.4 \times 10^{-3} V_{[\mathrm{kv}]}^{3 / 2} \tag{1}
\end{equation*}
$$

\]

## Electrostatic Quadrupole (ESQ) Design

The basic design studies for the ESQ system were performed with a linear beam optics code developed in house ${ }^{1}$ that solves the coupled K-V envelope equations for $X(z)$ and $Y(z)$ given by

$$
\begin{align*}
& X^{\prime \prime}+\kappa_{x} X-\frac{2 K}{X+Y}-\frac{\epsilon^{2}}{X^{3}}=0  \tag{2a}\\
& Y^{\prime \prime}+\kappa_{y} Y-\frac{2 K}{X+Y}-\frac{\epsilon^{2}}{Y^{3}}=0 \tag{2b}
\end{align*}
$$

$\kappa_{x}$ and $\kappa_{y}$ are assumed to be hard-edge focusing functions of length $\ell$ and separation $L$ with values $\left|\kappa_{x}\right|=\left|\kappa_{y}\right|=$ $\left(V_{q} / V_{b}\right)\left(1 / R_{q}^{2}\right)$, where $V_{b}$ is the beam voltage, $V_{q}$ and $R_{q}$ the quad voltage and radius, respectively. $K=\left(I / I_{0}\right)\left(2 / \beta^{3}\right)$ is the generalized perveance, with $I_{0}=3.1 \times 10^{7} \mathrm{~A}, \beta=v / c=$ $\left(2 q V_{b} / m c^{2}\right)^{1 / 2}$, and has the value of $K=3 \times 10^{-3}$ for our 35 $\mathrm{kV}, 30 \mathrm{~mA} \mathrm{H} \mathrm{H}^{-}$beam. $\epsilon$ is the effective emittance given by $\epsilon=4 \tilde{\epsilon}=0.276 \mathrm{~mm}-\mathrm{mrad}$, where $\tilde{\epsilon}=\epsilon_{\mathrm{rns}}$.

In order to minimize spherical and chromatic aberrations we tried to satisfy the constraints ${ }^{4} R_{\max } / \ell \leq 0.10, R_{\max } / R_{q} \leq$ $0.75, V_{\max } / V_{b} \leq 0.05$, where $V_{\max }=V\left(R_{\max }\right)$.

After many runs with the K-V envelope code we chose the six-lens design shown in Fig. 1, with the parameter values listed in Table 1. Note that the lenses 1 and 6,2 and 5 , and 3 and 4 are identical so that we have essentially two mirror-image triplets. This is a mere convenience for the computation, and in an experiment one may wish to vary all six voltages rather than pairing the lenses. The relative variation of the hard-edge focusing function $\kappa_{x} / \kappa_{0}$ is shown at the top and the two beam envelopes $X(z), Y(z)$ at the bottom of Fig. 1. We note that the two lenses (3 and 4) at the center act like a cell of a FODO transport channel. The first and last lens doublets accomplish the actual matching of the diverging beam into a converging one. Thus, for matching alone one needs only about 20 cm . The $10-\mathrm{cm}$ FODO cell at the center is only necessary to cover the total distance of 30 cm . We note from Table 1 that we were not quite successful in meeting the aberration constraints. However, our design is not yet optimized.

The second step of our study was to design an actual lens system equivalent to the hard-edge approximation and to calculate the realistic focusing function $\kappa(z)$ using a Laplace solver. This lens configuration is shown in Fig. 2 (top) and


Figure 1: Hard-edge variation of the focusing function $\kappa(z)$ at top and beam envelopes $X(z), Y(z)$ at bottom.

Table 1: Parameter values for the ESQ Lens System

| Quad No. | $1 / 6$ | $2 / 5$ | $3 / 4$ |
| :--- | ---: | ---: | ---: |
| $\ell[\mathrm{~mm}]$ | 25.00 | 60.00 | 50.00 |
| $L[\mathrm{~mm}]$ | 2.50 | 2.50 | 5.00 |
| $R_{\max }[\mathrm{mm}]$ | 4.00 | 9.83 | 8.86 |
| $R_{q}[\mathrm{~mm}]$ | 6.00 | 15.00 | 15.00 |
| $V_{q}[\mathrm{kV}]$ | 3.53 | 6.47 | 6.00 |
| $V_{\max } / V_{b}[\mathrm{in} \%]$ | 4.5 | 7.9 | 6.0 |

the $\kappa(z)$ function at the bottom of the figure. A special computer code calculates the nonlinear force components up to third order with the aid of integral functions of $\kappa(z)$. The results, including chromatic aberrations, are then used as input - in the form of fringe-field matrices - for our modified PARMILA code, as described in Ref. 4. The results of our best PARMILA runs so far for the six-lens ESQ system are shown in Fig. 3. The two phase space plots at the beginning ( $z=0$ ) and after the last lens ( $z=29 \mathrm{~cm}$ ) illustrate the evolution of the particle distribution and the effects of nonlinear forces. We used lens voltages that differ somewhat from the values of Table 1 for this run to obtain better agreement with the K-V results. These voltages are shown in Table 2. The normalized rms emittances obtained from this PARMILA run are listed in Table 3. The average final rms emittance (normalized) between the $x$ and $y$ direction is $0.111 \mathrm{~mm}-\mathrm{mrad}$ and the corresponding emittance growth is 1.60 , or $60 \%$, which is not excessive.

## Einzel Lens Design

The einzel lens system is being studied as an option for the SSC, as reported elsewhere. ${ }^{2}$ The design differs from that


Figure 2: Geometry with potential distribution of ESQ system (top) and variation of $\kappa(z)$ along $z$-axis (bottom).

Table 2: Modified Quad Voltages for PARMILA Run.

| Quad No. | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{q}[\mathrm{kV}]$ | 3.460 | 6.395 | 5.700 | 5.750 | 6.010 | 3.360 |

Table 3: PARMILA Results for Normalized RMS Emittance

|  | $\tilde{\epsilon}_{n}^{(i)}$ | $\tilde{\epsilon}_{n, x}^{(f)}$ | $\tilde{\epsilon}_{n, y}^{(f)}$ | $\bar{\epsilon}_{n}^{(f)}$ | $\Delta \overline{\tilde{\epsilon}}_{n}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| mm-mrad | 0.069 | 0.120 | 0.101 | 0.111 | 0.042 |

for the ESQ configuration in several respects. First, the total effective length of the matching section between ion source and RFQ entrance is only about 20 cm . This was dictated by the desire to use only two einzel lenses. (It is equivalent to leaving out quads 3 and 4 in the ESQ design.) Second, all of the design studies were done with the SNOW code. Third, the initial radius and slope of the beam emerging from the source were $R_{i}=1.2 \mathrm{~mm}, R_{i}^{\prime}=0$, as compared to $R_{i}=1.0$ $\mathrm{mm}, R_{i}^{\prime}=0.020$ in the ESQ design. The larger radius and zero slope make it easier to match the beam in the einzel lens design.

The SNOW code in its present form can only handle a laminar beam. The corresponding distribution in phase space $\left(r, r^{\prime}\right)$ is a line. The nonlinear forces due to the spherical aberrations of the lens fields deform the line into an S-shaped curve. From this curve an effective emittance can be calculated.


Figure 3: Phase-space plots of the particle distribution from the PARMILA code at the beginning and end of the six-quad ESQ system.

The results of the einzel-lens design with the SNOW code are shown in Figs. 4 and 5, with relevant parameters listed in Table 4.

The electrode configuration and the potential distribution are shown at the top, the ion trajectories at the bottom of Fig. 4. Note from Table 4 that very high negative potentials of -34.5 kV for the first lens and -33.7 kV for the second lens were required to achieve the desired matching conditions. The S-shaped curve representing the effects of aberrations on the laminar beam at $z=22.2 \mathrm{~cm}$, is shown in Fig. 5. The corresponding normalized rms emittance projected in the $x-x$ ' plane is calculated to be $\Delta \tilde{\epsilon}_{n}^{(f)}=0.102$ $\mathrm{mm}-\mathrm{mrad}$. Adding the thermal emittance in uncorrelated form yields

$$
\tilde{\epsilon}_{n}^{(f)}=\left[0.069^{2}+0.102^{2}\right]^{1 / 2}=0.123 \mathrm{~mm}-\mathrm{mrad},
$$

with a corresponding emittance growth of 1.78 . Thus, to the

EQUAL POTENTIAL PLOTS


ION TRAJECTORIES: R VS. Z


Figure 4: Electrode configuration of the einzel lens system with potential distribution (top) and beam profile (bottom) from the SNOW code.


Figure 5: Phase space distribution of the laminar beam from the SNOW code at $z=22.2 \mathrm{~cm}$.

Table 4: Parameter Values for the Einzel Lens System

| Lens <br> No. | Voltage | Aperture <br> Radius | Electrode <br> Thickness | Gap <br> Width |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -34.5 kV | 20 mm | 40 mm | 10 mm |
| 2 | -33.7 kV | 21 mm | 40 mm | 10 mm |

extent that the two codes are accurate and can be compared, we find that the einzel lens system produces a slightly larger emittance growth than the ESQ system.

## Comparison of the Two Systems

Our LEBT design studies for a $30 \mathrm{~mA}, 35 \mathrm{kV} \mathrm{H}^{-}$beam with initial emittance of $\tilde{\epsilon}_{n}=0.069 \mathrm{~mm}-\mathrm{mrad}$ show that either electrostatic quadrupoles or einzel lenses can accomplish the desired matching task. The ESQ system requires relatively low voltages and a minimum of four quads (two doublets) to match the beam in both directions. However, we used six quads to cover the total LEBT length of 30 cm . The einzel lens system requires high potentials - comparable to the beam voltage - and two lenses for matching within the 20 cm distance. The emittance growth was found to be slightly lower in the ESQ design than in the einzel lens system ( $60 \%$ versus $78 \%$ ). However, it is not clear whether the two codes (PARMILA and SNOW) are adequate tools to accurately evaluate the emittance growth. Further work and experiments are planned to fully assess the relative merits of the two schemes. In particular, one needs a better assessment of the capability and accuracy of the codes.

## References

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[^0]:    *Research supported by ONR/SDIO and DOE.
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