## CHARACTERIZATION OF FIELD ERRORS OF LAYER WOUND SHORT SOLENOIDS\*

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# Abstract

We report on a study of the error fields of a set of laverwound short solenoids of the type used in high-current electron linacs. These error fields are mainly caused by the azimuthal variation of the winding pitch angle. The conventional winding pattern design tends to concentrate the error fields in a dipole component perpendicular to the solenoid axis. Compounding the problem are smaller contributions due to the asymmetric coil feeds and conductor crossovers from one layer to the next. We have constructed and tested a prototype quadrufilar solenoidal magnet which utilizes symmetry to cancel the effects of current leads and crossovers and, in the absence of winding errors, should have an intrinsically straight magnetic axis. This design tends to concentrate the error field in the much less harmful octupole component. Measurements of the dipole error field yield a value of the order of one milliradian for the quadrufilar magnet as compared with about six milliradians for the conventional ETA-II magnets and twelve milliradians for the ATA magnets. The improved cooling of the quadrufilar design allows DC operation at fields almost three times higher than the conventional magnets.

# Introduction

High-current linacs require magnetic focusing to avoid space-charge-induced beam loss. For achromatic beam transport and high average acceleration gradient, the magnetic field is usually generated by an array of closely spaced short solenoids, most of which are built into the induction cells. For satisfactory beam injection into extraction devices such as wigglers, the magnetic axis of the individual magnets must be aligned to a small fraction of a milliradian with respect to the mechanical axis. Problems with magnetic misalignment have been encountered in two linacs at LLNL, the 50-MeV ATA and the 6-MeV ETA-II; as a consequence, a program was undertaken to determine the source of these errors.

#### Solenoid design considerations

# Fabrication

Maximum fields of the order of several kilogauss may be required for beam transport, making necessary the forced cooling of the magnet windings. The ETA-II magnets, for example, use a 0.25-in.<sup>2</sup> hollow copper conductor covered by a ~6 mil glassdacron insulation layer heat-bonded to its surface. The magnet has four layers of about 33 turns each; a single conductor length (or filar) is used to wind two layers, so two conductor lengths, electrically in series but with parallel water cooling paths, form each magnet. The magnets are vacuum impregnated with an epoxy formulation that provides mechanical strength and dimensional stability.

Perpendicular dipole field windings (sin/cos coils) were incorporated in the newer ETA-II design to compensate for misalignments and error fields. The winding pattern is photoetched on circuit board which is then wrapped around the magnet prior to impregnation. These trim windings are limited in the field they can produce and are not effective in correcting the higher-order multipoles that may cause emittance growth. Thus, even when trim coils are used, there is a premium in producing as error-free a magnet as possible.

## Sources of error

A perfect axially symmetric short solenoid will show no azimuthal variation of its field components around its mechanical axis. I define "error fields" as deviations of the measured field from that expected from a "perfect" short solenoid. Such deviations may be due either to the choice of winding pattern or to errors in attempting to reproduce that winding pattern. The conventional single filar winding pattern described above even if accurately reproduced does not have a straight magnetic axis. Asymmetries at the ends caused by the current leads and crossovers (the region at the end of a winding layer where the conductor makes a transition to a larger winding radius in preparation for the return winding) cause all of the field lines to bend in these regions. These effects are small, but they are not random; consequently, in an accelerator the total error introduced is linear with the total number of magnets in series and can lead to loss of the beam in the absence of correction fields.

These end effects can be eliminated by the use of a multifilar winding pattern. If the symmetry number s is the number of indistinguishable positions to which the winding pattern can be turned in one complete rigid rotation around the winding axis, then s = 1 for the conventional single filar winding pattern. Winding patterns with s > 1 have no perpendicular field component along their winding axis and the magnetic axis coincides with the winding axis. Any deviation of the axis from straightness in an actual magnet with a s > 1 winding pattern is only due to symmetry-breaking winding errors which make s = 1.

Winding errors can arise in two ways, as variations either of the winding radius or of the winding pitch angle. Of these two the latter is the much more important for layer wound solenoids. The winding pattern for an error-free conductor layer is given by

$$x = R \cos \theta, y = R \sin \theta,$$
  
$$z_{i0} = [(n\theta/2\pi) \cdot i + 1] w, \qquad (1)$$

for  $i-1 < (n\theta/2\pi) < (L/w)+i-1$ . Here n is the total number of filars, i = 1...n, w is the separation center to center between adjacent conductors, L is the axial length of the winding, and R is the mean radius of the winding layer. Axial winding errors modify the pattern to

$$z_i = z_{i0} + \delta z_i(\theta) \tag{2}$$

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Commonly, layer-wound-magnet conductors are packed as closely as possible to maximize the number of turns. Close packing forces the conductors to be nested with the axial coherence length of perturbations long compared with the length of the magnet. Thus the perturbation is cyclical to lowest order and can be represented by a Fourier series,

$$\delta z(\theta) = \Sigma_{m} (\delta z)_{m} \cos(m\theta - \Phi_{m})$$
(3)

For small mean pitch angle the axial surface current density is given by

 $k_z = (I/w)[(nw/2\pi R) - (1/R) \Sigma_m (\delta z)_m m sin(m\theta - \Phi_m)]$  (4) where I is the conductor current. The azimuthal variation of the axial surface current is a source of multipole radial error fields. The dipole contribution (m = 1) produces the only asymmetric field component with a non zero value on the mechanical axis. Along this axis the normalized error field is

$$B_1/B_z = (\delta z)_1/2R.$$
 (5)

which is independent of z. To illustrate the sensitivity to this type of error, a single layer winding of radius 100 mm will need a  $(\delta z)_1 < 200 \ \mu m$  for a dipole error field less than a milliradian.

Eqn. 5 should be compared with the expression for the variation of perpendicular field along an axis tilted a small angle  $\alpha$  with respect to the magnetic axis of a perfect solenoid

$$B_1/B_z = \alpha (1 + (z/2B_z)dB_z/dz)$$
(6)

Measurement of the axial variation of  $B_1/B_z$  should aid in identifying the source of error fields.

# Measurement of error fields

Solenoid error fields were measured with a system similar to that described in Ref. 1. A F.W. Bell Model #811AB meter with an HTJ8-0608 transverse Hall probe, capable on its highest gain setting of covering the range of 0.01 to 100 gauss, was used. The sensor area is of the order of  $1 \text{ mm}^2$ . The instrument uses a precision-machined 2-in.-diam., 24-in.-long rotating horizontal brass shaft with a coaxial 60° tapered hole at each end. Axial thrust centers the shaft between 610 fixed brass dead centers, which are supported by precision x,y adjustment stages. The shaft is rotated by a stepping motor through a drive belt coupling. The Hall probe, oriented to measure the radial field component, is inset in a milled slot in the shaft and can be set at a number of axial positions and two radial positions in the slot. The instrument axis of rotation is aligned either to the mechanical axis defined by alignment rings on the magnet or to the coil i.d. (if there are no alignment rings on the magnet under test). The accuracy with which this can be done sets the experimental resolution, which we estimate to be of the order of a few tenths of a milliradian.

For each set of magnet measurements, the value of the coil current and the Hall probe signal were read out after every  $5^{\circ}$  step of azimuthal rotation for one revolution, after which the axial position was shifted. Separate measurements of the axial field profile were used to normalize the data

At each axial position the data of the radial field measurements were Fourier analyzed to obtain the amplitude and phase angle of the first five terms,  $B_0...B_4$ , of the multipole fit to the data. The azimuthally independent term  $B_0$  is not an error field but is present if the plane of the sensor is not exactly parallel to the axial field and/or is displaced with respect to the axis of rotation. The dipole and higher-order

multipole terms are the error field components. The sum of the five term Fourier series matched our measured data to a small fraction of 1%.

## **Experimental** results

Data from eighteen ATA-type and three ETA-II type magnets have been reduced. In addition we have built and tested a quadrufilar magnet (s = 4). For the set of ATA type magnets, the average value of the dipole field, normalized to the local value of the axial field and the average phase angle is shown in Fig. 1. The error bars show the standard deviation of the data.



Fig.1 Average dipole field normalized to local axial field and average phase angle of set of eighteen ATA magnets. (a/R=0.39, R=91mm, L=211mm) Solid curve is from Eqn. 6 for a tilted magnet.

The set of three ETA-II magnets all have dipole strengths in the same range with a mean value of six mrad, about one half the mean ATA value.

The quadrufilar magnet was designed to replace a conventional ETA-II induction cell solenoid with only minor modifications to the cell; it uses the same size conductor and has the same dimensions and number of turns(Fig. 2).



Fig.2 Prototype quadrufilar magnet.

A number of separate runs were made on this magnet to check the reproducibility of results. Figure 3 shows the average values and standard deviation of the multipole strengths obtained from the reduction of four runs with the sensor at a normalized radius a/R=0.44. The measurements of the normalized multipole strength are reproducible within a few tenths of a milliradian, in agreement with our previous estimates. This data shows that the quadrufilar has much smaller error field than the others tested. In Fig. 4 we compare a



Fig. 3. Strengths of first four poles of the error field of the prototype quadrufilar magnet normalized to the local value of the axial field. (a/R=0.44, R=81mm, L=237mm)

field line calculated from the ETA-II magnet data with one calculated from the quadrufilar data. For these data we have found the set of Eulerian angles to transform from the initial x,y,z coordinate system to the tilted x',y',z' system, in which the z' axis is parallel to the magnetic axis and the field line lies



Fig.4 a) Field line calculated from ETA-II magnet data. b) Field line calculated from quadrufilar data

in the x',z' plane near the origin which is located at the field maxima. The ETA-II magnet line has large displacements from the x',z' plane and changes of slope near the ends, which we

ascribe as due to the asymmetries of the feed and crossovers. The quadrufilar magnet line is much better behaved and lies in the x',z' plane over almost its entire length. In Fig 4 b) we compare these data to the paraxial expression for a field line

$$x' = -(x'_0/2)\ln(B_z/B_{zmax})$$
 (7)

Here  $x'_{0}$ , the displacement of the origin from the magnetic axis is a fit parameter equal in this case to 146 µm. The fact that the data points lie within a maximum distance of a few microns from the x',z' plane and within a maximum deviation of 6 µm from the paraxial field line fit indicates that the symmetry of the quadrufilar winding has eliminated the problems connected with the feeds and crossovers. The fit also implies the existence of a magnetic axis displaced from the origin by about 150 µm and straight to less than 10 µm.

#### Discussion

We expect the same magnitude error fields in both conventional and multifilar windings for a given pitch variation. In the conventional magnets which we have measured, the dipole field is large and shows a degree of repeatability of magnitude and phase that suggests a systematic winding error introduced by the design. The conventional magnet winding pattern with a single wedge establishing the winding pitch is prone to emphasize dipole errors if the wedge angle is not exactly right. Because these wedges have a fixed relationship to the leads and crossovers, their effect would be repeatable in magnets of the same design. The four wedges used in establishing the quadrufilar pattern, even if they are not right but are alike, should emphasize the much less harmful, octupole component. This hypothesis is supported by the data of Fig. 3 where we see that the dipole error has been greatly reduced from that of the conventional magnets while the octupole field is only exceeded by the dipole. Note that the octupole component drops rapidly away from the ends; this may be evidence that the coherence length for the shorter wavelength winding errors is short compared with the coil length.

The alignment of quadrufilar magnets with errors of the order measured in the prototype could be improved by indexing the alignment rings to the measured magnetic axis. For the existing magnet we would need to shift the ring position only by about 100  $\mu$ m at each end to accomplish this.

A dividend of the use of a multifilar winding pattern is the improved cooling due to multiple, shorter cooling channels. The field measurements of the quadrufilar were made at 800 A magnet current, the limit of the available power supply. (Data from the ETA-II magnets were taken at 100 A and from the ATA magnets at 450 A). With a cooling-water pressure drop across the magnet of 90 psi, the temperature rise was ~20 $^{\circ}$ C. From these values, we estimate that for this same pressure drop a limit of 60 $^{\circ}$ C rise would allow operation at a maximum magnet current of over 1.4 kA producing a maximum axial field of ~8.5 kG, almost three times that obtainable from the ETA-II design for the same pressure drop and temperature rise.

# Reference

1. H. Nishihara and M. Terada, J. Appl. Phys. 39, 4573(1968).