A NEW TECHNIQUE FOR TRANSVERSE PHASE ELLIPSE MEASUREMENT

P.B. Vasilev, A.M. Tron
Moscow Engineering Physics Institute Kashirskoe shosse 31, 115409, Moscow,USSR

ABSTRACT

To reconstruct transverse beam's emittance as a 5 parameter ellipse using the results of its three profile measurements a new technique is presented. Compared with known before the new one gives an additional information on ellipse's position in phase space and its relative contents of particles. An emittance determination precision as a function of detector parameters and profile measurement errors has also been submitted. For beam particles' paraboloidal or Gaussian transverse phase space density distribution an error of technique has been determined. Means of improving of theese technique precision are under discussion.

INTRODUCTION

Measurement of beam's boundary in transverse or longitudinal phase space in existing accelerators is of particular interest for computor control of accelerating process.
As a rapid, practically nonperturbing a primary beam, a three cross-section method (CS-method) is intended to be used for that purpose (see [1],[2],[3]). The method deals with reconstruction of elliptical phase boundaries by measuring of three beam widths in three points or transverse planes along accelerator axis. It is assumed that transforming matrix of an accelerator's section is known and an emittance is invariant. Therefore, a measurement of transverse (or longitudinal) beam sizes is a base for determination of its phase boundaries. The sizes' measurement error depends on profilometer parameters. To simplify analysis let's consern that beam width is measured along one of coordinates (x), which is perpendicular to its axis, and a profilometer is a set of thin wires, having been stringed equidistantly and orthogonally to beam axis in one same plane.

Inspite of rather large number of papers, devoted to beam's emittance measurement, a method's error problem is studied unsatisfactory. That's why assuming beam phase boundary have been elliptical and concerning profile and beam width (for given density level of particles' distribution) measurement error negligibly small, let's see a correlation between measured and true values of emittance and beam relative content of particles within phase ellipse. As a model of particles' two dimensional distribution in transverse phase space a Gaussian and elliptic paraboloidal ones were taken. This choice is ensured by facts that the lines of equal density of both distributions are ellipses and both of them are in good correspondance (for a large number of cases) with experimentally obtained distributions.

DEFINITIONS

To ease further consideration a number of definitions and terms must be done. Let $j(x,x')$ be a density of particles' distribution in transverse phase space (x,x'), where x and x' are respectively distance from an accelerator's axis and angle between it and particle trajectory. Than we mean as a beam profile that is directly measured experimentally a function

$$j_e(x)=\int_{[x']} j(x,x')\,dx'$$

where $[x']$ -domain of a function and it is called an experimental profile. A true, real profile of $j(x,x')$ function is its orthogonal progection in (j,x) plane and it has been signed as $j_\mu(x)$ (see fig.1). A cross section on relative level h from $j_\mu(x)$ or $j_e(x)$ maximum defines beam width on h level $(0<h<1)$ and it has been signed $D_\mu(h)$ or $D_e(h)$. This way a cross section of two dimensional distribution $j(x,x')$ on h level is a square, that is in this case limited by elliptical curve G. This section having been msasured experimentally or having been defined from model of phase spase distribution is named an experimental $S_e(h)$ or true (real) $S_\mu(h)$ beam emittance or simply beam emittance on h level. Than relative content values of beam particles $m_e(h)$, $m_\mu(h)$, $n_e(h)$, $n_\mu(h)$ for corresponding sections of distributions $j_e(x)$, $j_\mu(x)$, $j_e(x,x')$ and $j_\mu(x,x')$ on h level are defined in following way

$$m_e(h)=\int_{x_{e1}}^{x_{e2}} j_e(x)\,dx \Big/ \int_{[x_e]} j_e(x)\,dx \tag{1}$$

$$m_\mu(h)=\int_{x_{\mu1}}^{x_{\mu2}} j_\mu(x)\,dx \Big/ \int_{[x_\mu]} j_\mu(x)\,dx \tag{2}$$

$$n_e(h)=m_e(h) \tag{3}$$

$$n_\mu(h)=\iint_G j(x,x')\,dx\,dx' \Big/ \int_{[x]} dx \int_{[x']} j(x,x')\,dx' = V_\mu(h)/V_\mu , \tag{4}$$

where $x_{e1}, x_{e2}, x_{\mu1}, x_{\mu2}$ are coordinates that limit beam width on h level, $[x_\mu]$, $[x_e]$ -domains of $j_\mu(x)$ or $j_e(x)$ functions, $V_\mu(h)$ -volume that is limited by (x,x') plane, by elliptical cylinder that contains G curve and by $j(x,x')$ distribution, V_μ -volume that is limited by (x,x') plane and by $j(x,x')$ function and its value is proportional to full number of beam particles or beam current I_0.

CS METHOD ERROR

A CS method error problem in determining emittance and relative beam content of particles which depends on h level is discussed here for two types of $j(x,x')$ distribution. Given errors can be written in a next way:

$$\Delta S(h)/S(h)=|S_e(h)-S_\mu(h)|/S_\mu(h)= =2|D_e(h)-D_\mu(h)|/D_\mu(h) \quad (5)$$

$$\Delta I(h)/I_\mu=|n_e(h)-n_\mu(h)|/n_\mu(h)= =|m_e(h)V_\mu-V_\mu(h)|/V_\mu(h) \quad (6)$$

First a transverse phase distribution $j(x,x')$ as an elliptical paraboloid was taken. In some point z along accelerator's axis it is

$$j(x,x')=1-x^2/a^2-x'^2/b^2 \quad (7)$$
$$-a\leqslant x\leqslant a,\ -b\leqslant x\leqslant b.$$

In point z a measured beam profile is given

$$j_e(x)=4b/(1-x^2/a^2)^{3/2}/3 \quad (8)$$

It can be shown for every $z_1\neq z$ that measured profile has more general formulation

$$j_e(x)=j_{em}(1-x^2/x_m^2)^{3/2}, \quad (9)$$

where j_{em} is profile maximum and x_m is its width. Measured profile (9) and real one $j_\mu(x)$ that were obtained from the same distribution $j(x,x')$ are displayed in fig.1b, here are also beam widths on h level

$$D_e(h)=2x_{ed}=2a\sqrt{1-h^{2/3}} \quad (10)$$
$$D_\mu(h)=2x_{\mu d}=2a\sqrt{1-h} \quad (11)$$

Relative beam content of particles $m_e(h)$ within emittance $S_e(h)$ can be calcelated with a use of next equation

$$m_e(h)=I(h)/I_o=4(3h^{1/3}/2+h)\sqrt{1-h^{2/3}}/3\pi + +2\arcsin\sqrt{1-h^{2/3}}/\pi, \quad (12)$$

where full volume $I_o=1\pi ab/2$ is limited by surface (7). Eq.(12) was obtained by integrating (9) within interval (10).

Relative beam content of particles $n_\mu(h)$ within emittance $S_\mu(h)$ is determined by common volume of cylinder $x^2/a^2+x'^2/b^2=$ $=q^2=1-h$, surface $j=1-x^2/a^2-x'^2/b^2$ and plane $j=0$, it is equal

$$n_\mu(h)=1-h^2 \quad (13)$$

Dependences $n_\mu(h)$ (12) and $m_e(h)$ (13) are displayed in fig.2, a difference between them (6), which determines an error of method, is in fig.3. A needable emittance measurement error dependence on h level value for given $j(x,x')$ may be found out of (5), (10),(11) as

$$\Delta S(h)/S(h)=2(\sqrt{(1-h^{2/3})/(1-h)}-1) \quad (14)$$

this equation is displayed in fig.3.

Let's see the same dependences for Gaussian $j(x,x')$ distribution

$$j(x,x')=\frac{1}{2\pi\sigma_x\sigma_{x'}\sqrt{1-\rho^2}}\exp\left[\frac{-1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_x^2}-2\rho\frac{xx'}{\sigma_x\sigma_{x'}}-\frac{x'^2}{\sigma_{x'}^2}\right)\right] \quad (15)$$

where $\sigma_x, \sigma_{x'}$ -dispersion, ρ -correlation ratio. A measured profile $j_e(x)$ will be

$$j_e(x)=j_{em}\exp(-x^2/2\sigma_x^2) \quad (16)$$

A true profile can be given by the same way

$$j_\mu(x)/j_{\mu m}=\exp(-x^2/2\sigma_x^2) \quad (17)$$

Hence Gaussian distribution profiles $j_\mu(x)/j_{\mu m}$ and $j_e(x)/j_{em}$ are equal and according to (5) an error of emittance measurement method is zero. But relative content of particles within real emittance differs from that in measured one. A bound of section q on $j(x,x')$ h level was fonded to be (while $\rho=0$)

$$h=j(x,x')/j_m=\exp(-q^2/2)= =\exp(-x^2/2\sigma_x^2-x'^2/2\sigma_{x'}^2) \quad (18)$$

From (18) $q=\sqrt{-2\ln(h)}$ and x or x' variates in interval from $-q\sigma_{x,x'}$ to $q\sigma_{x,x'}$. An amount of particles into a volume limited by plane $j=0$, cylinder $x^2/\sigma_x^2+x'^2/\sigma_{x'}^2=q^2$ and by surface (15) is

$$I_\mu(h)=I_o[2\Phi_o(\sqrt{-2\ln(h)})]^2 \quad (19)$$

Here I_o is a full amount of beam particles,

$$\Phi_o(w)=\frac{1}{2\pi}\int^w \exp(-t^2/2)dt$$

Eq. (19) shows a real emittance content, while experimental number of particles have to be obtained by integrating an experimental profile (16) by x from $-q\sigma_x$ up to $q\sigma_x$:

$$I(h)=2I_o\Phi(\sqrt{-2\ln(h)}) \quad (20)$$

Dependences $m_\mu=I_\mu(h)/I_o$ and $n_e=I_e(h)/I_o$ are displayed in fig.2,curves 3,4 respectively, an absolute difference between them related to real $I_\mu(h)/I_o$ is shown in fig.3 with broken line.

Adduced examples has made clear "sources" of errors in determining relative particles content within emittances, in measuring beam boundary and thus calculating the very emittances. Meamwhile it becomes clear that beam sizes measured on the same h level of profile belongs to the same section of $j(x,x')$ distribution. Next procedure of determining beam emittance with particle contents close to 100% may have been submitted: to measure profile with h>.1, then to reconstruct it for lower levels h≤.1 and finally to calculate beam sizes and emittance in terms of equations written before.

Determination of function $j(x,x')$ section's bound for h>.1 may be possible after process of iteration had been realised and it can be described in further publication.

RESTORATION OF PHASE BOUND

To reconstruct beam phase boundary as a five parameter ellipse by measuring three beam widths in three z_1,z_2,z_3 accelerator's axis points a next procedure is presented. Z axis is assumed to coinside with accelerator axis, $z_1<z_2<z_3$.General inlet ellipse equation in z_1 is

$$Ax^2+2Bxx'+Cx'^2+2Dx+2Ex'=F, \quad (21)$$

and here $AC-B^2=1$.It is known that ellipse's centre has coordinates

$$x_o=BE-CD \quad (22)$$
$$x_o'=BD-AE \quad (23)$$

Let x_{max} and x_{min} denote coordinates of most and least, respectively, distant points on ellipse's bound measured along x axis from zero point. Subject to (22),(23), from (21) this values are

$$x_{max\atop min}=x_o\pm\sqrt{x_o+FC+E^2} \quad (24)$$

where

$$x_o=(x_{max}+x_{min})/2. \quad (25)$$

The product of x_{max} and x_{min} in z_1 point is

$$(-x_{max}x_{min})=FC+E^2 \quad (26)$$

Let $\|t_{ij}\|$ be radius vector reverse transformation matrix from z_2 to z_1 ,i.e.

$$\begin{bmatrix}x_1\\x_1'\end{bmatrix}=\begin{bmatrix}t_{11} & t_{12}\\t_{21} & t_{22}\end{bmatrix}\cdot\begin{bmatrix}x_2\\x_2'\end{bmatrix} \quad (27)$$

The ellipses ratii in z_2 section (distinguished by line) may be expressed in terms of $\|t_{ij}\|$ matrix and ratii in z_1 as

$$\bar{C}=At_{22}^2+2Bt_{12}t_{22}+Ct_{22}^2 \quad (28a)$$
$$\bar{E}=Dt_{12}+Et_{22} \quad (28b)$$

By substituting (28) into right (26) a connection between product $\bar{x}_{max}\bar{x}_{min}$ having been measuring in z_2 point and ellipse's ratii in z_1 can be found out of

$$-(\overline{x}_{max}\overline{x}_{min}) = t_{12}^2(AF+D^2) + 2t_{12}t_{22}(BF+DE) + t_{22}^2(CF+E^2) \qquad (29)$$

Naturally, when beam boundary $x_{max\,i}, x_{min\,i}$, $i = 1,2,3,\ldots$ are known, one can write a set of equations

$$-(\overline{x}_{max}, \overline{x}_{min})_i = u_1 a_{i1} + u_2 a_{i2} + u_3 a_{i3} \qquad (30)$$

with unkknown u_1, u_2, u_3 and solve it for them. Here $a_{i1} = (t_{12}^2)_i$, $a_{i2} = 2(t_{12}t_{22})$, $a_{i3} = (t_{22}^2)_i$. To obtain ellipse's (21) ratii by known u_1, u_2, u_3 its centre coordinates must have bee known. But according to (25) only x coordinate can be measured in each of three points and while transformation matrixes T_1 from second to first point and T_2 from third to first are known, one can calculate

$$x_o|_1 = (x_{max}|_1 + x_{min}|_1)/2 \qquad (31a)$$

$$x_o'|_1 = [(t_{22})_1 x_o|_3 - (t_{22})_2 x_o|_2] / [(t_{12})_1 (t_{22})_2 - (t_{12})_2 (t_{21})_1], \qquad (31b)$$

where $x_o|_{1,2,3}$ - centre coordinates along x axis in 1 or 2 or 3 point z, $x_o'|_1$ - an angle coordinate in first (basic) point, $(t_{ij})_{1,2}$ a ratii of transfer matrix from z_2 to z_1 (subscript 1) and from z_3 to z_1 (subscript 2). Meanwhile, one can obtain from the set (30) with regard to (31)

$$A = (u_1 + x_o'^2)/Q, \quad B = (u_2 - x_o x_o')/Q,$$
$$C = (u_3 + x_o^2)/Q, \quad F = (u_1 u_3 - u_2^2)/Q,$$
$$Q = \sqrt{(u_3 + x_o^2)(u_1 + x_o'^2) - (u_2 - x_o x_o')^2} \qquad (32)$$
$$D = -Ax_o - Bx_o', \quad E = Bx_o - Cx_o'$$

It follows from an adduced consideration that algorythm having been described allows to reconstruct a five parameter ellipse employing results of measuring beam width in three points along accelerator's channel, i.e. by three profilometers.

CHOICE OF PROFILOMETER PARAMETERS

Even if three profilometers are identical, a beam emittance measurement error depends on beam phase portrait on measurement system inlet, on profilometers' mutual disposition and accelerating channel's optical qualities, on profile digitization number N which is equal to relation of beam width to spatial step of profilometer grid Dx, on grid's period $K = Dx/Dx_o$, where Dx_o is a single wire diameter, and on wire signal measurement error ε. Hence obtaining of general analytical dependence of this type is impossible because of large number of factors. That's why when a measuring system having been under designing a mathematical simulation of measuring procedure is suggested.

Results of simulation for 600 MeV proton beam with incoming canonic phase ellipse of 18 mm* 2.8 mrad are displayed in fig. 4. Dependences were calculated when profilometers have been situated into free of field intervals with equal distance between them and profiles were assumed to be Gaussian. As it follows from fig.4, when number of digitization N=20 includes 95% of beam current, an error not more 1% can be ahieved in a free of field length equal to 5 meters.

It should be mentioned in conclusion that x variable may be not only distanse between particle's trajectory and axis, but also its phase along accelerating wave. Hence results can be extended to measuring phase spectrum, phase length and beam longitudinal emittance.

REFERENCES

1. I.H.Chiang, J.A.Smith, A.Soukas, IEEE Trans.Nucl.Sci., NS-30(1983),2331.
2. K.Ebihara, M.Juema, T.Kawakubo, Nucl. Instrum. & Meth.,A202(1982),403.
3. V.I.Artyomov, S.A.Dobromirov, F.A.Peev, D.A.Styopin,Pribory i Tehnika Experimenta, 1989, No4,42.

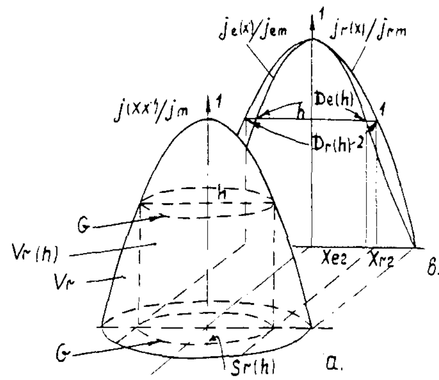
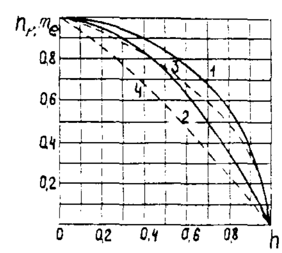


Fig.1 A model of paraboloidal j(x,x') distribution (a) and its profiles in (j,x) plane.



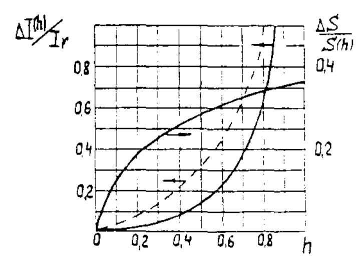


Fig.2 Relative content m_e, n_r of particles within cross-sections for two types of j(x,x') distribution. Solid line--elliptical paraboloid, broken line--Gaussian; 1,3 experimental, 2,4 real.

Fig.3 Relative error of emittance measurement $\Delta S/S$ and its content of particles $\Delta I/I$. Solid line-paraboloid, broken line -Gaussian distribution.

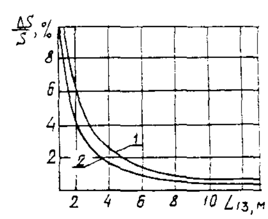


Fig.4 A CS method relative error. $L_{13} = z_3 - z_1$
curve 1: N=20, k=10, ε =.1
curve 2: N=40, k=10, ε =.1