# **BEAM CONTROL IN THE ETA-II LINEAR INDUCTION ACCELERATOR\***

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### Abstract

Corkscrew beam motion is caused by chromatic aberration and misalignment of a focusing system. We have taken some measures to control the corkscrew motion on the ETA-II induction accelerator. To minimize chromatic aberration, we have developed an energy compensation scheme which reduces energy sweep and differential phase advance within a beam pulse. To minimize the misalignment errors, we have developed a time-independent steering algorithm which minimizes the observed corkscrew amplitude averaged over the beam pulse. The steering algorithm can be used even if the monitor spacing is much greater than the system's cyclotron wavelength and the corkscrew motion caused by a given misaligned magnet is fully developed, i.e., the relative phase advance is greater than  $2\pi$ .

#### Introduction

Recent interest in induction accelerators has focused on their applications as drivers for high power radiation sources or heavy ion fusion. Performance of these systems is generally limited by beam transport, such as beam brightness, transverse sweep and energy variation within the beam pulse. For example, the ETA-II induction accelerator has been used to drive a high-average-power microwave FEL for electron cyclotron resonance heating the MTX tokamak plasma<sup>1,2</sup>. In order to obtain 140 GHz microwave pulses with power greater than 1 GW, the ETA-II has to deliver a beam with the nominal beam parameters as 7.5 MeV  $\pm$  1 - 2% beam energy, 2-3 kA beam current, 70 ns pulse width, a moderate brightness 1-2 x 10<sup>8</sup> and a transverse sweep less than  $\pm 1$  mm at the entrance of the wiggler<sup>3</sup>. Initial performance in 1989 experiments was limited to (5-10 ns) 0.2 GW pulses due to large time varying corkscrew motion<sup>4</sup> (~1 cm) of the beam centroid within a pulse entering the wiggler<sup>1</sup>. The corkscrew motion is a differential rotation of the beam centroid between the leading and trailing portions of the beam pulse driven by chromatic aberration and misalignments of the focusing system. We have developed an energy compensation scheme<sup>4</sup> to remove energy variation and to reduce the differential phase advance within the beam pulse on the ETA-II. The ETA-II accelerator consists of six ten-cell blocks. Each cell's solenoid focusing magnet is wrapped with a sine/cosine steering coil pair to correct dipole errors in the

focusing magnetic field. The beam position monitors are located between cell-blocks. Based on this configuration, we have developed an optimal timeindependent steering algorithm<sup>5</sup> to minimize the time varying corkscrew motion with the beam position monitors spaced widely apart compared to the length of individual focusing elements and the cyclotron wavelength of the system. This steering algorithm can work effectively regardless of whether the corkscrew motion due to a given misalignment is fully developed. By implementing the energy compensation scheme and the steering algorithm, we have achieved energy sweep  $< \pm 1\%$ , transverse beam motion  $< \pm 0.6$ mm for a 40 ns flattop with 1.5 kA of beam current and 2.7 MeV energy on a 20-cell experiment<sup>6,7</sup>, and energy sweep ~  $\pm 2\%$ , transverse beam motion <  $\pm 1$  mm for a 40 ns flattop with 2.4 kA of beam current and ~6 MeV energy at the entrance of the wiggler on the whole 60-cell ETA-II system<sup>8,9</sup>. In this paper we will discuss corkscrew motion and briefly review the energy compensation scheme used on the ETA-II. We will also discuss the optimal dynamic alignment procedure used on the ETA-II, its corkscrew reduction factor and the effects of beam position monitor spacing.

#### **Corkscrew Motion**

Let us consider a misaligned solenoidal focusing system with an error field  $\Sigma \delta B_j = \Sigma \delta B_{xj} + i \Sigma \delta B_{yj}$ where j is index of the solenoids. Note that the error field  $\delta B$ , the transverse displacement and the corkscrew amplitude  $d\eta$  in this paper are complex and include both the x and y components. The magnetic flux line is displaced by  $\int \delta B(z')/B_z dz'$ . The beam centroid at location z and beam pulse time  $\tau$  will rotate around the displaced flux line with a gyroradius  $|\rho(z,\tau)|$  given by

$$\rho(z,\tau) = \int_0^z \frac{\delta B(z')}{B_z} \exp \left[ik_c(\tau)z'\right] dz'$$
$$\cong \sum_{j=1}^n \frac{dB_j(\tau)}{B_z} \exp \left[-ik_c(\tau)z_j\right] , \qquad (1)$$

and its relative transverse displacement with respect to the offset flux line is given as  $\rho(z,\tau) \; exp[-ik_c(\tau)z]$ . Here, n is the index of the last magnet within distance z, and

$$dB_{j}(\tau) = \int_{-\infty}^{\infty} \delta B_{j}(z') \exp\left[ik_{c}(\tau) (z'-z_{j})\right] dz' \qquad (2)$$

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is the Fourier transform of the j-th error field relative to its local origin  $z_j$ . The phase of gyration caused by a single misaligned solenoid is accumulated from the magnet's origin. If a beam has an energy variation over its length, different parts of the beam will rotate at different cyclotron frequencies with different gyro-radii as the beam propagates in a solenoidal focusing system (Fig. 1). The differential gyration



Fig. 1 Beam centroid gyrates around an offset magnetic flux line. The beam slices with different energy have different gyro-radii and phases.

within the beam pulse is called corkscrew motion, and the corkscrew amplitude, i.e., the beam centroid's transverse displacement at  $\tau$  and z relative respect to the time averaged displacement, is given as

$$d\eta(z,\tau) = \left\langle \rho(z,\tau) \exp\left[-ik_c(\tau)z\right] \right\rangle - \rho(z,\tau) \exp\left[-ik_c(\tau)z\right], \quad (3)$$

where " $\langle \rangle$ " denotes time averaging over the beam pulse. When the accumulated relative phase advance  $\delta \psi = \delta k_c z$  within the pulse is much less than  $\pi$ , the corkscrew amplitude is a linear function of the energy variation, i.e.,

$$d\eta(z,\tau) \cong \delta \psi \left[ i\rho(z,\tau) - \frac{1}{z} \frac{\partial \rho(z,\tau)}{\partial k_c} \right] \exp \left[ -ik_c(\tau)z \right] .$$
(4)

After the beam has traveled some distance, the relative phase advance is greater than  $\pi$ . The beam will resemble a corkscrew. The corkscrew motion is then "fully developed" with an amplitude given by

$$d\eta(z,\tau) \cong -\rho(z,\tau) \exp\left[-ik_c(\tau)z\right] . \tag{5}$$

The amplitude of the corkscrew motion caused by randomly tilted magnets  $(\delta \theta_{rms})$  as a function of energy variation and the accelerator length (n =  $z/\lambda$ ) is shown in Fig. 2, where  $\lambda$  is the solenoid length.

### **Energy Compensation**

A beam generated by a space charge limited injector has an intrinsic energy variation near the head and the tail of the pulse. Inevitably, the beam will develop corkscrew oscillations in the transverse direction as it propagates down a misaligned accelerator. Since corkscrew motion is simply the differential gyration of a beam, limiting accumulation of the relative phase advance by means of energy compensation schemes can control growth of corkscrew oscillations. For example, it is possible to unwind the beam corkscrew completely at a given location by using some cleverly designed voltage pulses to reverse

the sign of energy variation within the beam so that the relative phase advance vanishes. However, this



Fig. 2 The corkscrew oscillations caused by randomly tilted solenoids as a function of energy variation and the accelerator length (n).

approach can not prevent corkscrew motion from growing downstream. Obviously, the best energy compensation scheme for limiting corkscrew oscillations is to remove the energy variation at the beginning of the system. We have applied this energy compensation scheme to the 60-cell ETA-II. The first 20 cells use a waveform with an "ear" at each end to compensate for the normally falling energy near the head and tail of the injector pulse. The remaining 40 cells then use a flatter accelerating pulse. We have obtained an energy variation less than  $\pm 1\%$  for 40 ns for both 20-cell and 60-cell operation. Simulation results presented in Ref. 4 show that the energy compensation scheme can reduce the corkscrew amplitude on the 60-cell ETA-II by a factor of four.

### **Beam Steering**

The energy compensation scheme described above does not completely remove corkscrew beam motion. We can reduce corkscrew motion further by using a steering method with local transverse dipole correction coils that compensate the error field of each misaligned solenoid. Since the Fourier component of each error field at the cyclotron wavenumber determines the gyro-radius, convention wisdom indicates that the beam should be steered back to the axis at the locations of beam position monitors at intervals usually much shorter than the cyclotron wavelength in order to remove the corkscrew motion. However, this is generally impractical from the viewpoint of diagnostic hardware and data acquisition channels required. Furthermore, this method of steering can not be used in the strong focusing field case where the solenoid length is comparable to the cyclotron wavelength. Consequently, we have developed a time independent steering algorithm that can reduce the time dependent transverse beam motion by minimizing the time-averaged corkscrew amplitude with beam position monitors spaced widely apart compared to the distance between focusing elements and cyclotron wavelength of the system. This steering algorithm

can work effectively even if the corkscrew motion due to a given misalignment is fully developed.

The time averaged corkscrew amplitude A(z) at the beam position monitor at location z is defined as

$$A(z) = \left\langle \left| d\eta(z,\tau) \right|^2 \right\rangle^{1/2} . \tag{6}$$

When the corkscrew motion is fully developed, Eq. (5) indicates that this steering algorithm will minimize the beam gyration. However, minimizing the time averaged corkscrew amplitude can result in reduction of the relative gyration phase and the differential gyro-radius of the beam simultaneously. It is less apparent that implementing the steering algorithm will allow us to minimize the beam gyration effectively. We will estimate the corkscrew amplitude reduction in the following.

For simplicity, we assume the differential gyroradius within the beam is small so that

$$\rho(z,\tau) \approx \rho(z,\tau_o) + \frac{\partial \rho(z,\tau)}{\partial k_c} \bigg|_{\tau_o} \delta k_c(\tau) \quad , \qquad (7)$$

where "o" denotes the reference point in the pulse. The net gyro-radius of a steered beam is

$$\rho_{\rm s} = \rho_{\rm B} + I_{\rm c} \rho_{\rm c}$$

where  $\rho_B$  and  $I_c.\rho_c$  are the gyro-radii of the beam gyration caused by the error field and the correction field with an assumption that the correction field is linearly proportional to its current excitation  $I_c = I_x +$  $i I_y$ . Let us assume that both the error field and the correction field are localized upstream from the monitor position  $z_m$ . We can then drop the z dependence in  $\rho$ . According to Eqs. (4) and (6), the optimal current for zeroing the time averaged corkscrew amplitude at monitor position  $z_m$  is given by

$$I_{m}(z_{m}) = -\frac{\rho_{B}(\tau_{o})}{\rho_{c}(\tau_{o})} \left[ 1 + i \frac{\partial \rho_{B}(\tau)/\partial k_{c}}{z_{m} \rho_{B}(\tau)} - i \frac{\partial \rho_{c}(\tau)/\partial k_{c}}{z_{m} \rho_{c}(\tau)} \right]_{\tau_{o}}$$
$$\cong -\frac{\rho_{B}(\tau_{o})}{\rho_{c}(\tau_{o})} + O\left(k_{c}\lambda \frac{\lambda}{z_{m}}\right) , \qquad (8)$$

where the last two terms are usually very small, and  $\lambda$  is the effective magnet length. The residual corkscrew amplitude is then much smaller than the uncorrected amplitude, i.e.,

$$\frac{\rho_{s}(\tau)}{\rho_{B}(\tau)} \cong i \left[ \frac{\partial \rho_{c}(\tau) / \partial k_{c}}{z_{m} \rho_{c}(\tau)} - \frac{\partial \rho_{B}(\tau) / \partial k_{c}}{z_{m} \rho_{B}(\tau)} \right]$$
$$\approx O \left[ \left( k_{c} \lambda \right) \left( \frac{\lambda}{z_{m}} \right) \right] \qquad . \tag{9}$$

If the corkscrew motion is fully developed at the monitor location, the optimal current setting is

$$I_{m}(z_{m}) = -\frac{\rho_{B}(\tau_{o})}{\rho_{c}(\tau_{o})} \qquad , \qquad (10)$$

and the residual corkscrew amplitude is

$$\frac{\rho_{\rm s}(\tau)}{\rho_{\rm B}(\tau)} \approx \left[ \frac{\partial \rho_{\rm B}(\tau)/\partial k_{\rm c}}{\rho_{\rm B}(\tau)} - \frac{\partial \rho_{\rm c}(\tau)/\partial k_{\rm c}}{\rho_{\rm c}(\tau)} \right] \delta k_{\rm c}$$
$$\approx O\left[ \left( k_{\rm c} \lambda \right) \left( \delta k_{\rm c} \lambda \right) \right] \quad . \tag{11}$$

According to Eqs. (8)-(11), the steering algorithm is most effective when the monitor spacing is very large.

## Time Independent Steering Algorithm

To begin the beam steering during accelerator operation, the focusing magnet should be set to produce a chosen magnetic profile for the target beam quality. The steering procedure can be incorparated into a computerized data acquisition and control system, such as the MAESTRO<sup>10</sup> program used on the ETA-II. The control system acquires and processes signals from the beam position monitors. The beam displacements  $x(z,\tau)$  and  $y(z,\tau)$  are recorded as functions of time  $\tau$  at the beam position monitors. The corkscrew amplitude is calculated as

$$d\eta(z,\tau) = (x - (x)) + i(y - (y)) .$$
(12)

Since the corkscrew amplitude is the differential beam displacement from the averaged centroid position, the offset of the beam position monitor will not be included in the calculated corkscrew amplitude. The time averaged corkscrew amplitude calculated by the control system is determined by the net error field which includes both the alignment error field and the steering field. Varying the excitation current on a steering magnet can change the magnitude of the time averaged corkscrew amplitude A(z). We will obtain a well defined minimum A(z) while tuning the steering coil's current to its optimal setting. Operationally the accelerator is steered iteratively, starting at the injector and sequentially adjusting the current in each steering coil for a minimum in the time averaged corkscrew amplitude observed by a downstream beam position monitor until the end of the accelerator is reached. If the relative cyclotron phase advance from the misaligned magnet to the monitor is smaller than  $\pi$ , we will fine tune the steering coils by repeating the whole procedure again and using the monitors further downstream. When the alignment errors are large, repeating the steering process for the whole accelerator may be needed to reach convergent settings on the steering coils.

Since this steering procedure is included in the control system, the complete beam steering can be done systematically and quickly. MAESTRO can tune the steering coils on the 60-cell ETA-II within 1-2 hours. As predicted by Eq. (9), we had observed one order of magnitude reduction on the 20-cell ETA-II's corkscrew amplitude by implementing the steering algorithm (see Fig. 3). When the Stretched Wire Alignment Technique (SWAT)<sup>11</sup> was used to minimize the tilts of focusing magnets, the corkscrew amplitude was only reduced slightly from 8 mm to 6 mm. However, when

the corkscrew tuning algorithm was used, the corkscrew amplitude was reduced to 0.6 mm. For the 60-cell experiments, we had observed a factor of two in corkscrew reduction by using the more distance beam position monitor spacing. The measured corkscrew amplitude at the wiggler's entrance was 1-1.5 mm for 40 ns sufficient for the FEL requirements.



Fig. 3 Measured beam position for 40 ns with (a) no steering, (b) correction of tilts by using SWAT values, and (c) the corkscrew tuning curve steering algorithm (taken from Ref. 6).

### **Tuning Curves**

A tuning curve is a curve of the time averaged corkscrew amplitude versus a given steering coil's current excitation. The ability of beam steering by using the steering algorithm presented here depends heavily on whether the tuning curve of each steering coil has a pronounced unique minimum. The net error field in Eq. (1) is a superposition of the misalignment error fields and the steering field of a given correction coil. Assume that the steering field is linearly proportional to the current excitation. The time averaged corkscrew amplitude A(z) is then a function of the steering currents,  $I_x$  and  $I_y$ , of a given pair of steering coils. The surface described by the function  $A(z; I_x, I_y)$  encloses a vertical asymmetric cone with the rounded tip pointing down as shown in Fig. 4. The corkscrew tuning curve for tuning one steering coil in an experiment lies on the surface of the corkscrew tuning



Fig. 4 The corkscrew tuning cone for tuning a steering coil pair.

cone. Observed corkscrew tuning curves on the 20-cell ETA-II presented in Ref. 6 are shown in Fig. 5. The optimal steering current for a given steering coil in one



Fig. 5 Observed ETA-II corkscrew tuning curves for the horizontal and the vertical steering coils at the last injector cell (taken from Ref. 6).

transverse plane is unique regardless of what the current excitation is for the steering coil in another transverse plane. Hence, we generally need to obtain a corkscrew tuning curve only once for each steering coil.

When we have more than one steering coil pair between two beam position monitors, we minimize the observed time averaged corkscrew amplitude to  $A_1$  at a downstream beam position monitor by obtaining a corkscrew tuning curve for the first steering coil and setting the excitation to its optimal current. Then, we further reduce the time averaged corkscrew amplitude from  $A_1$  to  $A_2$  by tuning the excitation on the second steering coil to its optimal current. According to Eqs. (9) and (11), by repeating this procedure on the subsequent steering magnets, we can eventually reduce the corkscrew amplitude by a large factor and satisfy the performance requirements of the accelerator if enough steering coils are used.

## Monitor Spacing

In order to use the corkscrew tuning algorithm successfully, reduction in the corkscrew amplitude  $\Delta A(z)$  observed at a monitor has to be larger than the greater of the fluctuations  $\delta_A$  in the corkscrew amplitude due to shot-to-shot variations and the monitor resolution limit  $\delta_m$ . This requirement sets a restriction on the beam position monitors' spacing. Assume that there are n (n >>1) randomly misaligned solenoids within one monitor spacing. The rms error field of the system is  $\delta B_{rms}$ . The time averaged corkscrew amplitude of a fully developed corkscrew motion is approximately given as

$$A(z) \cong \sqrt{2n\pi} \rho_{\rm rms} \quad , \tag{13}$$

where  $\rho_{rms} = \delta B_{rms}/B_z$  is the rms value of the gyroradius. By assuming that we can eliminate the corresponding corkscrew motion caused by one of the misaligned magnets completely by steering, we obtain the corkscrew amplitude reduction  $\Delta A(z)$  as

$$\Delta A(z) \cong \partial A(z) / \partial n$$
$$\cong \sqrt{\pi/2n} \rho_{\rm rms} \qquad (14)$$

The error bars  $\delta_A$  in the corkscrew amplitude due to shot-to-shot noise is then given by

$$\delta_{\rm A}(z) \cong \sqrt{2n\pi} \,\, \Delta \rho_{\rm rms} \quad , \qquad (15)$$

where  $\Delta \rho_{\rm rms}$  is the variation in gyro-radius due to fluctuations in beam energy and focusing field. In general, the shot-to-shot noise  $\delta_A$  in the observed corkscrew amplitude is bigger than the monitor resolution limit  $\delta_m$ . Hence, when the relative phase advance is much greater than  $\pi$ , the number of misaligned magnets per monitor spacing has to satisfy the following inequality:

$$n \le \frac{\rho_{rms}}{2\Delta\rho_{rms}} \qquad (16)$$

When the relative phase advance is much less than  $\pi$ , the time averaged corkscrew amplitude is

$$A(z) \cong \frac{n^{3/2}}{2} \rho_{\rm rms} \,\delta\psi \qquad , \qquad (17)$$

and the number of misaligned magnets within a [3] R. A. Jong et al., "IMP, a Free Electron Laser Amplifier monitor spacing is limited by

$$n \leq \frac{3}{2} \left( \frac{\Delta \rho_{rms}}{\rho_{rms}} + \frac{\Delta \delta \psi}{\delta \psi} \right)^{-1} .$$
 (18)

In the case that beam position monitors' resolution is poor and  $\delta_{\rm m} > \delta_{\rm A}$ , the monitor spacing is determined by

$$\frac{16}{9} \left( \frac{\sigma_{\rm m}}{\rho_{\rm rms} \, \delta \psi} \right)^2 \le n \le \left( \frac{\pi}{2} \frac{\rho_{\rm rms}}{\sigma_{\rm m}} \right)^2 \,. \tag{19}$$

The lower limit in n indicates that the monitor spacing should be large enough such that the corkscrew amplitude caused by a single misaligned magnet can grow to a detectable size at the monitor location. The upper limit in n given by Eqs. (16), (18) and (19) indicates that the monitor spacing for implementing the corkscrew tuning curve steering algorithm can be very large generally.

### Conclusions

We have developed a simple energy compensation scheme to reduce chromatic aberration of an accelerator system and a dynamic steering algorithm to control corkscrew motion. The steering algorithm is most effective when the beam position monitor spacing is much larger than the focusing element length. It is clear that we can minimize both the corkscrew motion and the time independent beam displacement by implementing the corkscrew tuning curve algorithm if the error fields are caused only by tilted magnets. If the error fields are due to offset magnets, the magnetic flux line at a downstream beam position monitor remains on the machine axis. We

will tilt the magnetic flux line by using the steering algorithm. Then, the beam with the minimized corkscrew amplitude will not be on axis. For a system consisting of randomly tilted and offset magnets, it is not clear whether the corkscrew tuning curve steering algorithm can also remove the time independent transverse displacement. The ETA-II experimental results indicate that the averaged beam offset is also reduced by the steering algorithm without using additional bending magnets to remove the time averaged centroid displacement. However, two additional bending magnet pairs for each transverse degree of freedom placed anywhere in the system are generally needed to steer the time independent centroid back to axis before the beam breakup instability becomes noticeable.

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