# SELF-CONSISTENT 3D SIMULATIONS OF LONGITUDINAL HALO IN RF-LINACS\*

John J. Barnard<sup>1</sup>, Steven M. Lund<sup>1</sup>, and Robert D. Ryne<sup>2</sup> 1. Lawrence Livermore National Laboratory, L-645, Livermore, CA 94550 2. Los Alamos National Laboratory, H817, Los Alamos, NM 87545

#### Abstract

In order to prevent activation of the beam pipe walls and components of a high power ion accelerator, beam loss must be minimized. Here we present self-consistent, 3D particle-in-cell simulations of longitudinally mismatched beams, including the effects of rf non-linearities, using parameters based on the Accelerator Production of Tritium linac design. In particular, we explore the evolution of the longitudinal halo distribution, i.e. the distribution of particles in longitudinal phase space with oscillation amplitudes significantly larger than amplitudes of particles in the main body or "core" of the beam. When a particle reaches a sufficiently large amplitude longitudinally, it can be lost from the rf bucket, and consequently loses synchronism with the rf wave. Such particles will lose energy and so be poorly matched to the transverse focusing field and consequently can be lost transversely. We compare the present simulations, in which all particles contribute self-consistently to the self-field, to predictions of a core/test particle model, in which the core distribution has uniformly distributed charge and does not evolve self-consistently. Effects of self-consistent, non-linear space-charge forces, non-linear rf focusing on envelope mismatch induced beam halo are explored through comparisons of both models.

# **1 INTRODUCTION**

Requirements on accelerator activation in high power proton linacs, have led to stringent limits on particle loss from the beam. Transverse mismatches have been shown to lead to large transverse particle oscillation amplitudes, leading to large apertures (up to a factor of 20 rms in conservative designs) to avoid intercepting the transverse beam halo (cf. refs.[1]-[4]). In the longitudinal case, the width of the stable rf bucket replaces the physical aperture as the dimension which determines when a particle can be lost from the beam. Since the half-width of the bucket can be as small as 2.3 longitudinal beam radii, a careful understanding of longitudinal halo is also warranted. Recently, the development of halos in the longitudinal direction has been explored using core test-particle models (cf. ref. [5],[6]), and using 3D PIC simulations, (ref. [7], and [8]) using rf fields which varied linearly with distance from the bunch center. In this paper we focus on the effects of the non-linear rffocusing field, and compare numerical results of a 3D PIC

code, known as Langevin3d, to the core-test particle code (CTP) reported on in references [5] and [6].

#### 2 PARAMETERS USED FOR EXAMPLES

For concreteness, we will refer to a design of the Accelerator Production of Tritium (cf. ref. [9]) for numerical examples (see table). The undepressed synchrotron wavenumber,  $k_{s0}$  represents the zero-current oscillation wavenumber of a particle at infinitesimal amplitude about the synchronous phase,  $k_s/k_{s0}$  is the tune depression due to space charge. Also,  $k_{\beta 0}$  is the zero current transverse betatron frequency, and  $k_{\beta}/k_{\beta 0}$  is the transverse tune depression. The quantities  $r_{\perp 0}$  and  $r_{z0}$  are  $\sqrt{5}$  times the rms radius in the transverse and longitudinal directions, respectively in the lab frame. The quantity  $\alpha$  represents the ratio of transverse to longitudinal radius in the comoving frame, and is the quantity of physical relevance when evaluating the fields (cf.ref[5]). Note that although the beam is nearly spherical in the lab frame at the 500 MeV point, in the comoving frame  $\alpha = 0.68$  and at other energies it is even more elongated.  $\Delta z_{max} \cong \beta_s c(-\phi_s)/2\pi\nu$  and  $\Delta z_{min} \cong \beta_s c(2\phi_s)/2\pi\nu$  are the approximate rf-bucket half-widths longitudinally, where  $\phi_s$  is the synchronous phase,  $\beta_s c$  is the velocity of the synchronous particle, and  $\nu$  is the rf frequency.

Energy (GeV)	0.1	0.5	1.0	1.8
$k_{s0}$ (rad/m)	0.285	0.107	0.055	0.030
$k_s/k_{s0}$	0.31	0.31	0.23	0.20
$k_{\beta 0}$ (rad/m)	0.474	0.160	0.151	0.147
$k_eta/k_{eta 0}$	0.35	0.32	0.48	0.65
$r_{z0}$ (mm)	6.53	4.61	4.90	4.62
$r_{\perp 0}/r_{z0}$	0.53	1.04	0.62	0.46
$\alpha \equiv r_{\perp 0} / \gamma_s r_{z0}$	0.48	0.68	0.30	0.16
$\Delta z_{max}/r_{z0}$	2.34	5.87	6.37	7.26
$\Delta z_{min}/r_{z0}$	4.68	11.7	12.7	14.5

**Table** APT parameters in superconducting coupled cavity linac, at four representative points along the linac.

# **3** CORE-TEST PARTICLE CODE, CTP

In the code CTP (refs. [5,6]), test particles are allowed to evolve from both external and space charge fields, in all three spatial directions (although the fields themselves are cylindrically symmetric). The external longitudinal field is a sinusoidally varying rf electric focusing field with the intrinsic non-linearity of a sine wave, and the external transverse field is assumed to be uniform focusing, representing the effects of the quadrupoles, in an average sense. The

<sup>\*</sup> Work performed under the auspices of the U.S. D.O.E. at LLNL under contract W-7405-ENG-48 and at LANL under support of the Division of Mathematics, Information, and Computational Sciences, the Division of High Energy Physics, and Office of Defense Programs.

space charge field is that of a uniformly charged spheroid, calculated for arbitrary  $\alpha$  (prolate, oblate or spherical). The spheroid is set to oscillate in linearized normal modes of the coupled transverse and longitudinal envelope equations, at the linearized mode frequencies, each with assumed sinusoidal time dependence. Arbitrary amplitudes and phases of the two envelope modes are allowed. Particles are typically loaded with radial coordinate r = 0, and distributed in longitude either over a bunch length or a bucket length, although more complicated loads are also possible. Stroboscobic plots of particle energy and phase relative to the synchronous particle are made at the same phase of each envelope period. CTP is well suited for studying particle resonances and the overall structure of the phase space.

#### **4 3D PIC CODE, LANGEVIN3D**

The code Langevin3d (see ref. [10,7] for details) is a Particle-In-Cell (PIC) code that allows 3D space charge field calculations as well as 3D particle orbits. The boundaries are placed at infinity by using the method of ref. [11], which is a good approximation in accelerators with large beam pipe radii. One option in the code uses a modified symplectic integrator, to allow inclusion of artificial damping and diffusion. Use of this feature drives beams to a Boltzmann distribution, to allow thermal equilibrium initial conditions. The parallel processing architecture of Langevin3d permits the use of large particle numbers, which minimizes statistical fluctuations and maximizes accuracy. Typical results displayed here were run with 0.5 million particles, but runs with several million particles are not extraordinary. Phase space plots of a randomly chosen fraction of all the particles can be viewed at a fixed time or test particles can be viewed stroboscopically. Plots with a "density cut" can be viewed which display particles up to a specified maximum density. This feature allows viewing the low density halo regions without saturating the higher density core.

#### **5 COMPARISON OF RESULTS**

Figure 1, shows a comparison between the longitudinal phase space  $(d\Delta z/ds$  vs.  $\Delta z$  generated by Langevin3d (top two plots) and CTP (lower plot). Here  $\Delta z$  is the longitudinal particle position relative to the synchronous particle, and s is distance along the accelerator. In the top plot, a density cut was made allowing the details of the halo to be observed. The relative intensity of the halo is better characterized by the middle plot, in which the density of particles displayed is proportional to the actual distribution. In the lower plot the solid line represents the position of the equilibrium bunch, with dotted lines indicating the extent of the excursions from the beam mismatch. It is apparent that the main two-lobe structure (due to the resonance between the particle oscillation frequency and half the envelope frequency) in phase space is clearly visible in the results of both codes. The extent of the halo in both  $\Delta z$  and  $d\Delta z/ds$  is also quite similar for both CTP and Langevin3d. Higher order resonances appearing within the beam in the CTP plots do not appear to be as significant in the self-consistent simulations.



**Figure 1** Comparison of Langevin3d (top, middle) and CTP results (bottom) of  $d\Delta z/ds$  versus  $\Delta z(m)$  for a 30% mismatched beam at the 500 MeV point in the APT linac. Top: Density cut; Middle: Full distribution; Bottom: Stroboscopic test particle plot.



**Figure 2** Comparison of the maximum longitudinal excursion in CTP and Langevin3d simulations as a function of mismatch parameter  $\delta r_{z0}/r_{z0}$  using parameters from the 500 MeV point. Dashed lines indicate CTP results; solid are from Langevin3d. The upper curve of each pair is the absolute value of the negativemost excursion in  $\Delta z$ , the lower is the maximum of the positive particle excursion.

Figure 2 indicates that CTP tracks the Langevin results well as a function of envelope mismatch amplitude, although the maximum excursion is somewhat less than that found in Langevin3d. Note that the particles all remain within the stable bucket, and so no particles were lost at these amplitudes.



**Figure 3** Langevin3d stroboscopic test particle plots for an average current per bunch of 5 mA (rather than 200 mA) but otherwise parameters for the 100 MeV APT normal conducting linac tabulated in ref.[5], loaded over the width of the bucket, with  $d\Delta z/ds = 0$ . Left: with non-linear rf-focusing; Right: with linear focusing.

# 6 NON-LINEAR RF HALO SUPPRESSION

In ref. [6] it was shown that for fixed focusing strengths and emittances, at very low or high currents the resonance disappears. Because the rf focusing field varies sinusoidally, the particle oscillation frequencies do not tend to an asymptotic frequency as they would in a strictly linear external focusing field. Rather, a maximum oscillation frequency is reached at some radius generally outside the bunch, and the frequency decreases for larger amplitudes. If that maximum frequency lies below half the longitudinal envelope frequency no resonance is possible. Parametrically it was found that at very low and high currents (when focusing and emittance is held fixed) the maximum frequency is below half the envelope frequency and simulations with CTP found an absence of resonance. Simulations using Langevin3d verified that for a low current case the resonance indeed vanished. Figure 3 illustrates this effect, by showing the phase space with linear focusing in which the resonance persists and sinusoidal rf focusing which suppresses the resonance.

# 7 CONCLUSIONS

We have performed 3D PIC simulations of beams with longitudinal mismatches, and have found general agreement between PIC simulations and the Core/Test Particle model, particular with respect to the size of the resonant region. The suppression of the envelope-particle resonance by the sinusoidal rf field under certain conditions predicted using CTP has been confirmed using Langevin3d. Using both codes, no particle loss through the rf bucket has been observed for mismatch amplitudes below 0.5.

#### 8 ACKNOWLEDGMENTS

The authors wish to thank Bob Gluckstern, Ingo Hofmann, Alexei Fedotov, and Tom Wangler for valuable discussions on beam halo. This research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Energy Research of the U.S. D.O.E. under contract DE-AC-03-76SF00098.

## **9 REFERENCES**

[1] J.S. O'Connell, T.P. Wangler, K.R. Krandall, Proc. 1993 Part. Accel. Conf., Washington DC, p. 3657.

[2] R.A. Jameson, 1993 Part. Accel. Conf., Washington DC, p. 3926.

[3] R.L. Gluckstern, Phys. Rev. Letters, **73**, 1247, (1994).

[4] R.D. Ryne, T.P. Wangler, Proc. Accel. Driven Transmutation Tech. Conf., Las Vegas, NV, July 1994.

[5] J.J. Barnard and S.M. Lund, "Theory of Longitudinal Beam Halo in RF Linacs: I. Core/Test-Particle Formulation," Proc. of the 1997 Particle Accelerator Conf.

[6] S.M. Lund and J.J. Barnard "Theory of Longitudinal Beam Halo in RF Linacs: II. Envelope/Particle Resonances," Proceedings of the 1997 Particle Accelerator Conference.

[7] R.L. Gluckstern, A.V. Fedotov, S.S. Kurenoy and R. D. Ryne, "Halo Formation in three-dimensional

bunches," Phys. Rev. E., in press, (October 1998).

[8] M. Pabst, K. Bongardt "Progress on Intense Proton Beam Dynamics and Halo Formation" Proceedings of the European Particle Accelerator Conference, held Stockholm, Sweden, 1998.

[9] T. Wangler, private communication (1997). [10] R. Ryne et al, "The DOE Grand Challenge in Computational Accelerator Physics," 1998 International Linear Acc. Conf. (these proceedings).

[11] R.W Hockney and J.W. Eastwood, "Computer Simulation Using Particles," [Adam Hilger, Bristol], 1988.