MULTI-FIDUCIAL TECHNIQUES FOR TRACKING LARGE PHASE SPACE DISTRIBUTIONS IN NON-LINEAR FIELDS

S.M. Lidia*
E.O. Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA

Abstract

One of the challenges in tracking intense beams through linacs is to account for the differences in non-linear forces experienced by portions of the beam separated by large regions of phase space. In many situations, the high-order maps generated by a single fiducial trajectory fail to capture or describe the dynamics of distant particles within the beam. I describe here a technique which overcomes this difficulty by piecing together lower-order maps induced by multiple fiducial orbits. This atlas of maps can more accurately track the evolution of a beam spread over a large phase space region. I discuss applications of this technique to simulating beam dynamics in two-beam accelerators.

1 INTRODUCTION

Particle dynamics in relativistic klystrons pose several thorny problems for simulations. Possibly the most important element of the dynamics in a relativistic klystron two-beam accelerator (RK-TBA) [1] occurs in the longitudinal phase space. The beam is modulated at high frequencies (11 - 40 GHz), and each bunch carries a charge of 10's-100's of nC. Space charge effects will produce debunching forces (a capacitive impedance) which is counteracted by (inductively) detuning the rf output structures. A bunch will undergo numerous synchrotron oscillations during transport through the full-scale device. Also, the bunches are not short compared to the rf wavelength; they typically subtend 60°-120° of rf phase. Hence, they sample very non-linear fields in the rf output structures.

From this description we can identify the main problems present in a device simulation. The beams are sufficiently intense that space charge forces present more than a small perturbation. The beamline elements are necessarily spaced close together, and this requires treatment of overlapping, non-linear fringe fields. Transverse focusing is strong so that a complete betatron oscillation occurs between rf output cavities (~1 m). Transverse emittance, while low, is still sufficiently large that particles at the beam edge sample significant non-linearities present in the beamline elements. The instantaneous energy spread is large (~10%) to handle the low-frequency BBU, and to produce the bunching by rf rotation. The particle simulation, of necessity, must track many, many particles to provide adequate sampling of both the beam phase space and the fields experienced.

One of the main disadvantages of tracking by mapping can be seen immediately. The resultant map is fundamentally a power series expansion about the initial and final coordinates of the fiducial orbit. When the extent of the beam distribution in phase space is no longer 'small' in some sense, then the map generated about the given fiducial orbit loses accuracy when applied to the outlying particles. There are various solutions that may be applied to this problem. The pre-calculated fiducial may not faithfully represent the orbit of the beam centroid, but another fiducial may be found which does. The order of non-linearity carried by the calculation may be too low to adequately describe the given external field structure. Increasing the order of the calculation may be sufficient. However, these 'fixes' make for a good solution only when the beam occupies a small enough region of phase space such that a single fiducial orbit and the map it induces captures the essential dynamics.

In many applications of intense, modulated beams, however, this is not the case. These beams are most often present in single pass, linear beamlines, with constantly changing parameters. The beams may also have a relatively long pulse length with respect to any time-dependent rf fields they encounter. To sufficiently capture that interaction via a single map would require a degree of non-linearity far too high, and involve the computation of too many map coefficients, that the intrinsic efficiency of the method would be quickly lost. This problem is only compounded when self-field effects are included.

2 CONSTRUCTION OF A MULTI-FIDUCIAL MAP

The construction and evaluation of the multi-fiducial map upon the particle coordinates is straightforward. Algorithmically, it may be described by a sequence of simple steps. In any calculation, the beamline under consideration is initially divided into a set of mapping intervals. Beam particles are propagated by mapping successively through each interval. We define a 'center' fiducial as a single orbit that continuously threads through all the mapping intervals. This single fiducial is important to maintain as it provides a single reference frame, and hence a reference 'clock' and 'meter stick', for

* The author may be contacted via email at SMLidia@lbl.gov.

the problem. Each mapping interval is defined by referring to the coordinates of this center fiducial.

For each interval, the beam phase space is partitioned and a set of nominal values of coordinates are selected, one set for each region. These nominal coordinate sets provide the initial values for ‘sub-fiducials’ and their associated maps to be constructed. The particles in each region are then propagated according to these local maps. The number and method of partitioning is highly dependent upon the physics to be modeled. Partitioning should represent a balance between the order of non-linearity carried by the calculation, the degree of non-linearity present in the external fields and sampled by the beam, and the non-homogeneity of the beam density profile when self-fields are a concern. For example, in a magnetostatic transport region, only a single fiducial and map may need to be calculated. Whereas, in a region with time-dependent rf fields where the wavelength and period of the external fields is comparable to the bunch length and transit time of the beam, then perhaps 10 or more fiducials may be required. When self-field effects are included, the number of partitions may depend upon the non-uniformity of the beam density profile.

3 LONGITUDINAL BEAM DYNAMICS IN AN RF CAVITY

As an example to illustrate the method, I consider the problem of tracking a bunched beam through an rf cavity. Here, I follow only a single bunch, where the modulation carried by the beam is comparable to the rf period of the cavity, as in the case of an RK-TBA. Also, I will only consider the longitudinal phase space. The center fiducial is placed at the center of the beam distribution, with the sub-fiducials at locations that span the interval in arrival time (t) of the bunch at a given beamline position (z). The initial bunch distribution and fiducial positions is shown in Figure 1.

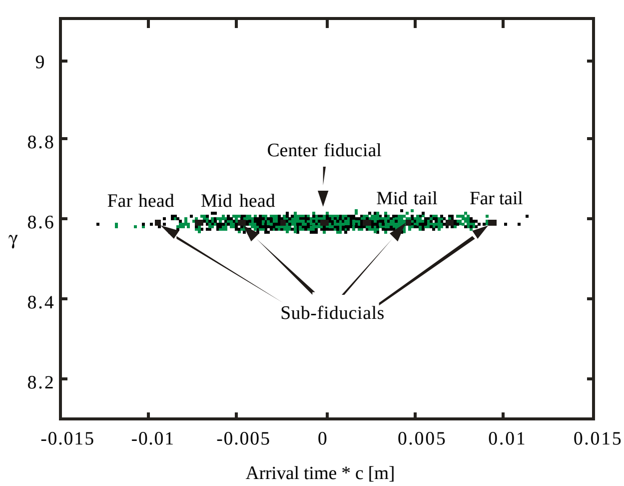

Figure 1. Initial fiducial coordinates and beam distribution in longitudinal phase space.

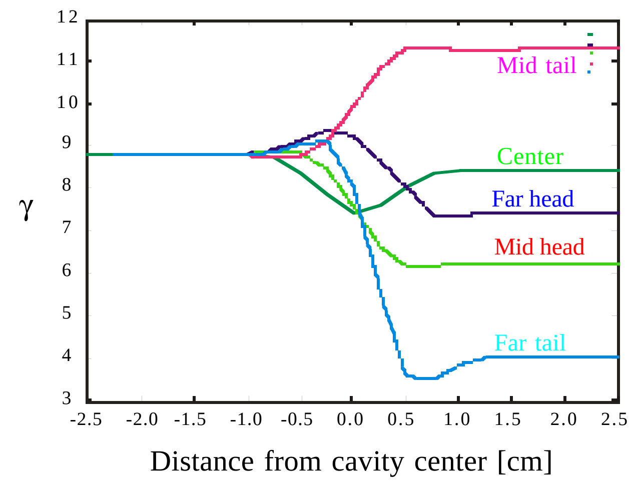

Figure 2. Fiducial trajectories in an rf cavity

3.1 Fiducial Particle Trajectories

The fiducial trajectories are calculated using the exact single-particle equations of motion. During transit through the rf cavity field, different fiducials will experience different forces due to arrival time differences. The final coordinates of the various fiducials will generally differ in a non-linear way. The fiducial trajectories in this case are shown in Figure 2.

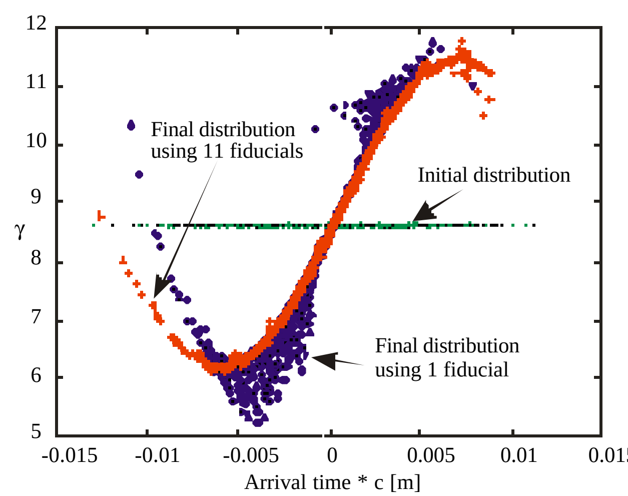

Figure 3. Final longitudinal beam distribution under a 3rd order map with multiple fiducials

3.2 Comparison of Single Vs. Multiple Fiducial Calculation

The maps induced by the fiducial orbits can, in principle, be calculated to arbitrary order. The accuracy of the mapping for outlying particles is determined by this order parameter. In Figure 3 is shown the results of the mapping through the rf cavity for two cases. The first case uses a 4^{th} order Hamiltonian, with a single fiducial (identical to the center fiducial in Figures 1 and 2). The second case uses a 3^{rd} Hamiltonian, but with 11 fiducials. In the first case, the combined effect of sampling only low-order field variations and tracking outlying particles leads to inaccuracies. The final distribution suffers additional local energy spread growth, and does not exhibit the appropriate curvature, as seen in the head of the bunch. The second case avoids these problems by sampling the fields at the location of the outlying particles. The effects of errors in the field evaluation and in large coordinate deviations between particles and their associated fiducials are kept small.

4 INCLUSION OF SELF-FIELD EFFECTS

4.1 Split Operator Algorithm

Two separate mappings are applied to the particle phase space coordinates using split-operator techniques [2]. These techniques are based on splitting the Hamiltonian into pieces that can be solved exactly (or through some desired order of accuracy), and then combining the separate maps to produce an approximate map for the full Hamiltonian. Split operator symplectic integration algorithms, including the well known 'leap-frog' algorithm of plasma physics simulations [3], are widely used in the treatment of Hamiltonian systems.

The total Hamiltonian is represented in the form

$$H_{tot} = H_{kin} + H_{ext} + H_{self}\,, \qquad (1)$$

where H_{kin} is the kinetic portion describing single-particle motion in the absence of all fields, H_{ext} is the contribution from external fields, and H_{self} is the contribution from self-fields. The maps from the first two contributions are calculated together, while the map resulting from self-forces is calculated separately. A combined map is then produced to advance particles over an interval τ. Accurate through second order in this step, the combined map is expressed as

$$M_{tot}(\tau) = M_{kin+ext}(\tau/2)\; M_{self}(\tau)\; M_{kin+ext}(\tau/2). \qquad (2)$$

The self-fields are determined by numerical solution of Poisson's equation on a 3-D Cartesian grid in the beam's rest frame. Standard techniques are used to solve for the fields at the grid nodes. For accurate representation of the 3-D fields from a bunched beam, we may use grid sizes up to 64x64x512 nodes, as well as 10^4-10^5 macroparticles.

4.2 Changing Representations

Since we are solving the Maxwell equations at a given instant in time, the particle distribution be represented at a given moment. However, for the single particle maps, we represent the particles at a given beamline location (z), with a spread in arrival time (t). To faithfully calculate the self-forces, then, we must change the distribution from a 'constant-z' to a 'constant-t' representation.

The multi-fiducial approach again provides some utility. A visual aid to the process of changing representations is provided by the 'world-line' diagram of the fiducials, Figure 4. The fiducial trajectories (z vs. t) are shown as world-lines. The constant-z beam distribution is shown as black line segments. The first step is to calculate the motion of the sub-fiducials about the center fiducial, and the associated non-linear maps. This brings all the fiducials to the same time, but at different beamline positions (green lines and arrows). The particles attached to a given sub-fiducial are still spread in time (red line segments). The last step is to apply a linear transformation to all the particles associated with a given fiducial, to bring them to the nominal time (blue line segments). The beam distribution now exhibits a spread in beamline position consistent with observation at a given moment in time.

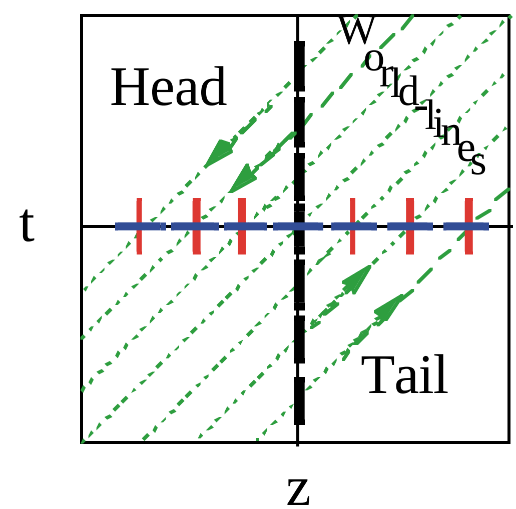

Figure 4. Changing beam distribution representation from constant-z to constant-t.

5 CONCLUSIONS

I have presented a new method for improving the accuracy of tracking algorithms, by using low-order integrators and multiple fiducials. Extensions to self-field calculations have also been made.

6 ACKNOWLEDGMENTS

I would like to thank Simon Yu and Glen Westenskow for their continued support of this work. This work was performed under the auspices of the U.S. Department of Energy under contract AC03-76SF00098.

7 REFERENCES

[1] A.M. Sessler and S.S. Yu, *Phys. Rev. Lett*. **54**, 889 (1987); G.A. Westenskow and T.L. Houck, *IEEE Trans. Plasma Sci.* **22**, 750 (1994).

[2] E. Forest and R.D. Ruth, *Physica D* **43**, 105 (1990); H. Yoshida, *Phys. Lett. A* **150**, 262 (1990); E. Forest, et. al., *Phys. Lett. A* **158**, 99 (1991).

[3] C.K. Birdsall and A.B. Langdon, *Plasma Physics Via Computer Simulation*, Adam Hilger, Bristol,1991.