# **RF CONTROL STUDIES FOR MODERATE BEAMLINE COUPLING BETWEEN SRF CAVITIES\***

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#### Abstract

When an SRF accelerator is designed, there is motivation to move the cavities close together on the beamline. Assuming the beamline apertures are not shrunk as well, this compaction (which will increase the overall accelerating gradient and/or lower the dynamic cryogenic heat load) increases the inter-cavity coupling. Within certain limits, the control system can compensate for this coupling by retuning each of the cavities. This paper describes constraints on the RF system, tuners, couplers, and control systems that are required to provide stable operation of cavities in the presence of inter-cavity coupling that exceeds the loaded bandwidth of an individual cavity.

## **1 INTRODUCTION**

Many cost optimizations have discussed the tradeoff between accelerator capital and operating costs, acceleration gradient, cryogenic capacity, and RF Power. One contributing term in those equations is  $\eta$ , the ratio of the active accelerating cavity length to the overall accelerator length, sometimes called the "filling factor" or "packing fraction." While the effect of packing fraction on a new machine is debatable (since the relative cost of cavity meters and other meters can be hard to identify), when upgrading an existing machine with fixed total length,  $\eta$  has a clear effect, particularly on the cryogenic load. A cavity cell and the manufacturing technology will set the familiar cavity parameters  $\omega$ ,  $Q_0$ , and the shunt impedance per unit length (r/Q). For a given acceleration voltage V and active accelerator cavity length  $\eta L$ , the cryogenic power dissipation is

Power = 
$$\frac{V^2}{\eta LQ_0(r/Q)}$$
.

Thus,  $\eta$  is the only parameter under the designers' control that can change the relationship between voltage gain and dynamic cryogenic load.

Historic values of  $\eta$  are in the 0.3 to 0.6 range. The existing CEBAF accelerator linac has  $\eta = 0.42$ , and the baseline design of our Energy Upgrade studies calls for raising  $\eta$  to 0.58, by increasing the number of cells per cavity from 5 to 7[1].

Between cavities is a beam pipe of length b, whose radius a is normally set by beam impedance or beam aperture needs. The coupling  $d\omega$  between two cavities is given by  $\Delta \omega \cdot \exp(-kb)$ , where  $k^2 = k_0^2 - (2.405/a)^2$ , and the proportionality constant  $\Delta \omega$  represents the coupling between the resonant fields in the end cell and the evanescent TM<sub>01</sub> fields in the beam pipe.

Increasing  $\eta$  will normally involve decreasing b, at which point the coupling  $d\omega$  has the potential to rise enormously. Accelerators to date have kept  $d\omega$  much smaller than the cavity bandwidth, to avoid potential problems that could arise from stronger coupling. The rest of this paper will discuss those problems in detail, and how they could be worked around.

# **2 DEFINING EQUATIONS**

In the most general form, ignoring wall losses in the cavity  $(\beta \gg 1)$ , in the rotated coordinate (phasor) sense where the instantaneous gradient E(t) in a cavity is given by  $\operatorname{Re}\{Ee^{j\omega_0 t}\},\$ 

$$\frac{dE}{dt} = -(\omega_f - j\omega_d) \cdot E - jd\omega_- \cdot E_- - jd\omega_+ \cdot E_+ + 2\omega_f \sqrt{R_c} \cdot K - \omega_f R_c \cdot I_b$$

For cavities at the end of a string, this formula involves phantom cavities, which should be treated as if they had zero gradient.

Table 1: notation	
$\omega_f$	bandwidth, $\omega_0/2Q_L$
$\omega_d$	frequency offset from $\omega_0$
$E_{-}$	gradient of upstream cavity
$E_{+}$	gradient of downstream cavity
$d\omega_{-}$	coupling to upstream cavity
$d\omega_+$	coupling to downstream cavity
K	specific drive amplitude
$R_c$	coupling impedance per length, $Q_L(r/Q)$
$I_b$	beam current

Typical self-consistent units of K and  $R_c$  are  $\sqrt{\text{Watts/m}}$  and  $\Omega/\text{m}$ . The quantities E, K, and  $I_b$  are complex numbers; the rest of the parameters are real. The definitions of E, K,  $R_c$ , and I make physical and numerical sense on a macroscopic scale. Over a length l, a beam of current I gains El energy using  $KK^*l$  power, when the matching condition  $E = IR_c$  holds.

The per-cavity  $\omega_0$  component of the beam current depends on the string beam current  $I_{bs}$  according to  $I_b$  =

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 $I_{bs}e^{j\phi}$ , where  $\phi$  is the phase-of-flight along the beamline to cavity of interest.

If we restrict ourselves to the steady-state where dE/dt = 0, and additionally set  $E_{-} = v_{-}Ee^{-j\theta_{-}}$ ,  $E_{+} = v_{+}Ee^{j\theta_{+}}$ , and  $I_{b} = i_{b}Ee^{j\phi}$ ,

$$\frac{K}{E} = \frac{1}{2}\sqrt{R_c}i_be^{j\phi} + \frac{1}{2\sqrt{R_c}\omega_f}\left(jd\omega_-v_-e^{-j\theta_-} - \omega_f - j\omega_d + jd\omega_+v_+e^{j\theta_+}\right)$$

When the right hand side is separated into its real and imaginary components, the quantity  $\omega_d$  (controlled by the tuners) only appears in the imaginary part. Moving the tuners to minimize the power therefore results in the zeroing of that imaginary part, which happens when

$$\frac{\omega_d}{\omega_f} = i_b R_c \sin \phi + v_- \frac{d\omega_-}{\omega_f} \cos \theta_- + v_+ \frac{d\omega_+}{\omega_f} \cos \theta_+$$

Indeed, the tuner operation is equally sensitive to tuning with no inter-cavity coupling, when all cavity voltages are held fixed. The remaining real part gives the power at optimum cavity tune,

$$Power = \frac{1}{4} R_c E^2 \left( i_b \cos \phi + \frac{1}{\omega_f R_c} (v_- d\omega_- \sin \theta_- + \omega_f - v_+ d\omega_+ \sin \theta_+) \right)^2$$

If  $\sin \theta_{-} \neq 0$  and  $\sin \theta_{+} \neq 0$ , only small amounts of coupling can be tolerated without requiring excessive Klystron power. The bad cases are one end of the string (which end depends on the sign of  $\sin \theta$ ), and any case where  $v_{-}d\omega_{-}\sin\theta_{-}$  does not exactly cancel  $v_{+}d\omega_{+}\sin\theta_{+}$ . Ordinarily one wants the ability to adjust each cavity gradient independently based on its performance capabilities. This independence can be recovered by setting  $\sin \theta_{+} = \sin \theta_{-} = 0$ , at which point the coupling terms all but disappear from the Power equation.

### **3 DISCUSSION**

The coupling between the cavities can provide an opportunity to measure the relative phases of the cavities. Without this measurement, the cavities' phase relative to the beam must be individually measured. Of course, with strong coupling, the relative phases must be set properly or the limited klystron power will not allow operation at full gradient. Figure 1 shows this phenomenon—curves show the input drive required as a function of phase for 10 Hz and 1 kHz coupling, under no-beam conditions. The flat reference line shows the power needed for 400  $\mu$ A beam current. Other parameter values assumed for this example are  $\omega_f = 235 \text{s}^{-1}$ ,  $R_c = 1.34 \times 10^{10} \Omega/\text{m}$ , E = 12 MV/m, l = 0.7 m, and  $v_- = v_+ = 1$ .

One would certainly hope to operate a cavity string by attaching single-cavity control systems to each individual



Figure 1: Drive required *vs.* neighboring cavity phase for 10 Hz coupling (solid) and 1 kHz coupling (dash-dot). A reference line (dashed) is given for the power required with beam.

cavity, so that  $2\omega_f \sqrt{R_c}K = A(s) \cdot (E_s - E)$ , where A(s) is the (diagonal matrix) gain and  $E_s$  represents the setpoints. The undesirable alternative is some hopelessly complex centralized control that understands the  $n \times n$  inverse of the coupling matrix. The equation of motion of the simpler system is

$$M \cdot E = A(s)E_s$$

where if we make the simplifying assumption that all cavities and coupling terms are identical,  $M_{i,i} = s + \omega_f + j\omega_d + A(s) = p$  and  $M_{i,i+1} = M_{i+1,i} = jd\omega$ . The stability criteria are based on the zeros of det(*M*); to first order in  $d\omega$ ,

$$\det(M) = p^{n-2}(p_i^2 + (n-1)d\omega^2),$$

where *n* is the number of cavities in the string. One can clearly see that there are n-2 unperturbed stability criteria based on zeros of  $s + \omega_f + j\omega_d + A(s)$ . There are also two more cases involving global string modes, based on zeros of  $s + \omega_f + j\omega_d + A(s) \pm j\sqrt{n-1}d\omega$ .

Strong inter-cavity coupling makes a string more difficult to turn on. One sensible approach is to turn them all on slowly and proportionally, so that  $v_-$  and  $v_+$  are constant. The tuners can center the system for minimum RF power under low gradient conditions, when there is plenty of available RF power. The only phenomenon the tuners have to compensate for during the ramp to full gradient is the ponderomotive frequency shift with gradient.

If a cavity trips, it's probably best to immediately turn RF off to the whole string and let it coast to a stop. Even if the adjacent cavities' RF systems could maintain their fields (a job made easier since the beam is probably turned off by the first cavity trip), some of their power would flow to the tripped cavity, which is not desirable.

Longitudinal alignment sensitivity can be significant. Historical assembly patterns at CEBAF, where that axis of alignment was not relevant, showed fluctuations of 6 mm. Between thermal contraction, manufacturing and assembly errors, and coarse tuning, 2-4 mm is a reasonable target for our 0.5 to 0.7 m long,  $\lambda$ =20 cm, cavities. Still, that implies uncontrolled phase differences between cavities of  $\phi$ =0.1 radians. This error can be accommodated in two ways: If  $\sin \theta = 0$ , power does not flow between cavities. RF characteristics without beam are ideal, but individual cavities are miscrested with respect to the beam. The total crest for the module can still be zeroed, but the overall voltage gain has slipped some. If  $\sin \phi = 0$ , individual cavities are crested properly, but large amounts of RF power move from cavity to cavity. For a given (measured) set of cavity gradient and Klystron power capabilities, inter-cavity and external coupling bandwidths, and longitudinal cavity positions, numerical optimization can construct a set of phases that optimizes the total voltage delivered to the beam.

As discussed above, the sensitivity of a coupled cavity string to tuner motion is unchanged from the uncoupled case. That is, the curvature of the Power vs. tuner position relationship is not affected by the coupling. On the other hand, the position of that curve's minimum becomes sensitive to the voltage ratio between adjacent cavities  $(v_{-} \text{ and } v_{+})$ . This sensitivity is one more reason to demand a tuner subsystem capable of short response times and no backlash. If a coarse tuner/fine tuner pair is used, the required range of the fine tuner is the sum of all effects that must be corrected. To the traditional list of pressure and ponderomotive compensation, inter-cavity coupling adds another term: the estimated maximum value of  $v_{-}d\omega_{-}\cos\theta_{-} + v_{+}d\omega_{+}\cos\theta_{+}$ . A reasonable choice for the maximum voltage ratio is in the 2 to 3 range, when cavities are more mismatched than that one would probably

turn off the weak cavity.

When a cavity is turned off, it must also be detuned. Normally this is done to keep the cavity fields low in the presence of exitation from the beam. In a coupled-cavity system, excitation will also come from neighboring cavities. The neighbors of a detuned cavity must have their tuning corrected to reflect the changed pattern of voltage setpoints.

## 4 CONCLUSIONS

With the right phase relationship between cavities, coupling equal to or greater than the bandwidth of the cavity appears technically feasible. Tuning individual cavities can cancel all the reactive elements of the system. For any particular gradient pattern, proper tuning will result in drive requirements unchanged from the no-coupling case. The dynamic stability of the string is perturbed, and some reduction in feedback performance should be expected. The control system needs to be agile enough to measure the coupling and implement turn-on scenarios that involve coordination between the cavities.

#### **5 REFERENCES**

[1] J. R. Delayen, "Development of an Upgrade of the CEBAF Acceleration System," these proceedings.