INTRA BUNCH TRAIN TRANSVERSE DYNAMICS IN THE SUPERCONDUCTING ACCELERATORS FLASH AND EUROPEAN XFEL

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Abstract

At FLASH and the European XFEL accelerator superconducting 9-cell TESLA cavities accelerate long bunch trains at high gradients in pulsed operation. Several RF cavities with individual operating limits are supplied by one RF power source. Within the bunch train, the low-level-RF system is able to restrict the variation of the vector sum voltage and phase of one control line below 3E-4 and 0.06 degree, respectively. However, individual cavities may have a significant spread of amplitudes and phases. Misaligned cavities in combination with variable RF parameters will cause significant intra-pulse orbit distortions, leading to an increase of the multi-bunch emittance. An efficient model including coupler kicks was developed to describe the effect at low beam energies. Comparison with start-to-end tracking and experimental data will be shown.

INTRODUCTION

There are several ways for getting a proper description of the transverse beam dynamics in a RF accelerating structure, e.g. using tracking algorithms [1] or simplified analytic models [2]. Assuming knowledge of the electromagnetic field distribution and the initial conditions of the particles, tracking provides accurate solutions, even for very low particle energies. Since the track step has to be small compared to a cell length, many steps are required, which needs considerable computation time for simulations with high dimensional parameter scans. Established simplified analytic models on the other hand may calculate the beam transport by few matrix multiplications. However, they are based on assumptions, most importantly ultra-relativistic beams, which do not apply at most particle injectors. Thus, a major challenge is to set up a model for low particle energies $\gamma = [10 \ldots 200]$, which is simple enough in order to calculate its output within Milliseconds, yet able to reproduce key features of RF dynamics such as RF focussing and coupler kicks.

MODEL DEVELOPMENT

Our approach uses a combination of numerically calculated axial symmetric beam transport matrices and discretized coupler kicks, coefficients of which are derived via a Runge-Kutta tracking algorithm using a high precision 3D field map of the TESLA cavity. Once the parameterized coefficients for the beam transport matrices and coupler kicks are evaluated, the final model uses the matrix formalism to calculate the beam transport through an accelerating module consisting out of 8 cavities in the order of ms for 400 bunches.

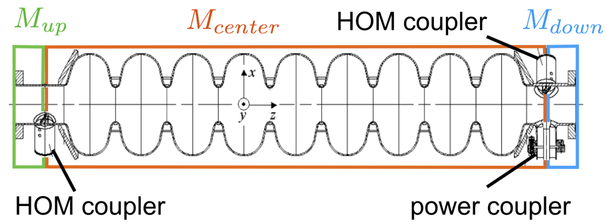

Figure 1: Longitudinal cross-section of a TESLA cavity. Highlighted are the HOM and power couplers and the transfer matrices as used for the model function.

3D-Field-Map

For the TESLA cavity two field maps are available. The axial symmetric field map describes the accelerating mode without geometric disturbances [3]. The 3D field map [4] describes this mode including the fields induced by both HOM and power coupler. Let $E_{[f/r]}^{[sin/cos]}$ being the sine and cosine like parts of the resonating electric field for the forwarded and reflected wave, respectively, and $A_{[f/r]}$ and $\phi_{[f/r]}$ amplitude and phase of the forward and backward wave from/to the fundamental mode coupler. The overall electric field component for the general case with given accelerating voltage V_0 and phase ϕ in respect to the beam can then be calculated with

$$E(t) = \Re\left[V_0/\overline{V}_r\, e^{i(\omega t+\phi)} \cdot \left(E_r^{cos} - i\Gamma \cdot E_r^{sin}\right)\right] \quad (1)$$

from the 3D field map provided by [4] for the pure decay mode, thus no incoming wave. $\overline{V}_r$ normalizes the field to the Eigenmode-solution of the field map. The voltage standing wave ratio

$$\Gamma = (A_r e^{i\phi_r} - A_f e^{i\phi_f})/(A_r e^{i\phi_r} + A_f e^{i\phi_f}) \quad (2)$$

describes the ratio between the difference of the forwarded and reflected wave in respect to the overall accelerating field. The magnetic component behaves analogously, using similar symmetry properties of the field components.

Beam Transport in Axial Symmetric RF Cavities

The change of transverse coordinates of a particle induced by an axial symmetric cavity can be written in terms of a matrix formalism. Using the Maxwell equations, a *quasi*-3D field map can be calculated from [3]. A Runge-Kutta algorithm is used to solve the equation of motion for one cavity for an ensemble of initial particles, entering the cavity at different offsets and angles. The calculation of the beam transport matrix then becomes a linear regression problem. The energy gain ΔE of a particle in the TESLA cavity is determined by the accelerating mode and is to a very good

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approximation independent of the coupler fields. It's dependency on the beam energy E_0, accelerating phase ϕ and gradient V_0 is very well described via

$$\Delta E = \left(a_1 - \frac{a_2 sin\,(\phi + a_3)}{E_0 - a_4} \right) \cdot V_0 \cdot cos\,(\phi)\,. \qquad (3)$$

for low beam energy with fitted coefficients a_i.

Coupler Kicks

Main and HOM couplers break the cavity's axial symmetry. In order to describe the transverse beam dynamics properly we use the axial-symmetric beam transport matrices and insert discrete kicks [5] at a certain location. The kick $\mathbf{k}$ on a bunch's centroid induced by a coupler can be expressed as

$$\mathbf{k} \approx \frac{\Delta E}{E_0} \cdot \left[\begin{pmatrix} V_{0x} \\ V_{0y} \end{pmatrix} + \begin{pmatrix} V_{xx} & V_{xy} \\ V_{yx} & V_{yy} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} \right] \qquad (4)$$

with E_0 being the initial energy, ΔE the energy gain per cavity and x and y the beam's transverse position at the coupler's position. The V_{ij} describe the normalized transverse deflection induced by the coupler and have to be found numerically. The full beam transport equation of one cavity becomes

$$\mathbf{u}_1 = M^{\mathrm{RZ}}_{\mathrm{down}} \cdot \mathbf{k}_{\mathrm{down}} \left(M^{\mathrm{RZ}}_{\mathrm{center}} \cdot \mathbf{k}_{\mathrm{up}} \left(M^{\mathrm{RZ}}_{\mathrm{up}} \cdot \mathbf{u}_0 \right) \right) \qquad (5)$$

where $M^{\mathrm{RZ}}_{\mathrm{i}}$ are the axial symmetric beam transport matrices between the corresponding reference points, so that $M^{\mathrm{RZ}}_{\mathrm{up}}$ describes the upstream beam transport between the entrance of the cavity and the first coupler, see Fig. 1. $\mathbf{k}_{\mathrm{up}}(\mathbf{u})$ evaluates the upstream coupler kick at the transverse coordinate $\mathbf{u} = [x, x', y, y']$. For low beam energy the V_{ij} depend on the mode of cavity operation, thus the real and imaginary part of the voltage standing wave ratio Γ, but also implicitly on the particle's initial energy E_0 and on both amplitude V_0 and phase ϕ of the accelerating gradient, spanning a parameter space $E_0 \times V_0 \times \phi \times \Re\Gamma \times \Im\Gamma$. The reason for this is the beam's trajectory dependence on these parameters, since the ultra-relativistic limit is not reached. The parameter fit is done as follows: At every point in this parameter space a reference particle's centroid distribution is created at the entrance of the cavity. The particle distribution at the exit of the cavity is obtained via tracking using the 3D-field map. In addition, the particle distribution in the center of the cavity is recorded. This gives two reference distributions, before and after each coupler region. Then the tracking is redone with the same parameters using the 1D-field map. This time, the particle distribution is recorded additionally at the coupler positions. Between each of these 5 reference points the axial symmetric beam transport matrices are calculated. For both couplers the reference distributions are compared with the output calculated with the linear beam transport using Eq. (5). A fitting routine was used to then find the V_{ij} which best describe the coupler kick of Eq. (4) using the ultra-relativistic limit [5] as a starting point. The global variation of the V_{ij} were found to be described via

$$\begin{aligned} V_{ij} = \quad & a_1 V_0 \cdot \Re\Gamma \cdot \Im\Gamma + a_2 V_0 \cdot \Re\Gamma + a_3 V_0 \cdot \Im\Gamma \\ + \quad & a_4 \Re\Gamma \cdot \Im\Gamma + a_5 V_0 + a_6 \cdot \Re\Gamma \\ + \quad & a_7 \Im\Gamma + a_8 \end{aligned} \qquad (6)$$

with

$$\begin{aligned} a_n(E_0, \phi) &= A_n(E_0) \cdot \cos\phi + B_n(E_0) \cdot \sin\phi + C_n(E_0), \\ A_n(E_0) &= \tfrac{y_n^A E_0 + z_n^A}{E_0 - w}, \qquad B_n(E_0) = \tfrac{y_n^B E_0 + z_n^B}{E_0 - w} \\ C_n(E_0) &= \tfrac{y_n^C E_0 + z_n^C}{E_0 - w} \end{aligned} \qquad (7)$$

The $\left[w, y_n^A, z_n^A, y_n^B, z_n^B, y_n^C, z_n^C\right]_{ij}$ are 49 constants for each coefficient V_{ij} and were found with a fitting routine.

MODEL VALIDATION

The developed model is compared to the results of a start-to-end tracking using ASTRA [1] and to experimentally derived data at FLASH. The evaluation limits are E_0=[5...150] MeV, V_0=[13...30] MV/m,ϕ=±30°, $\Gamma_{\Re,\Im}$ = ±3, u_0=±6 mm, u_0'=±6 mrad.

Comparison with ASTRA

The RF and beam input parameters were randomly created within the limits. The rms difference of the transverse position u for one cavity as a function of beam input energy is shown in Fig. 2 using different models for calculating the beam transport. At energies above 100 MeV the ultra-relativistic limit is by a very good approximation reached and the beam transport can be calculated according to Ref. [2] including coupler kicks according to Ref. [5]. Especially in the first cavities, however, it is important to use the fitted solutions for both the transfer matrices and the coupler kick coefficients V_{ij}. The rms difference between the developed model and ASTRA for the whole injector module using an initial beam energy of E_0 = 5.6 MeV is Δu_{RMS} =56 µm. Compared to the beam size of ≈1 mm this is a reasonable result.

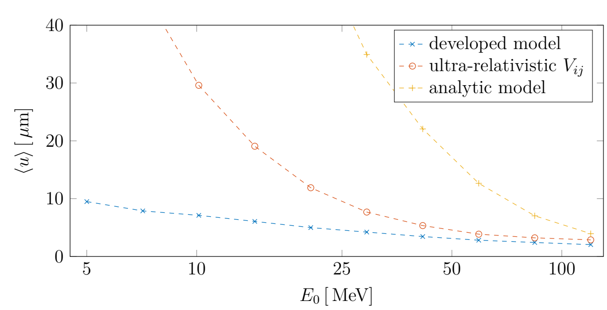

Figure 2: rms difference of the transverse position of different models compared to an ASTRA-tracking as a function of beam energy, each evaluated for one cavity. Plotted is the developed model (blue, Eq. (5)), numerical transfer matrices with ultra-relativistic V_{ij} (red, cf. [5]) and the analytic model (yellow, cf. [2]).

Comparison with Experimental Data

The model function is compared to experimental data recorded at FLASH with 400 bunches within one bunch-train. Each measurement is averaged over 100 pulses to deal with short-term jitter. The upper row of Fig. 3 shows the BPM readout at the first BPM downstream the injector module compared to the predicted orbit of the model function for the recorded beam input- and RF-parameters. No misalignments were assumed. The discrepancy between the predicted and the measured orbit is significant. The alignment of the GUN section in respect to the injector module is critical in determining the beam dynamics and is expected to be a significant source of error. In order to validate the numerical beam transport, experimental setups were chosen where the impact of misalignments are secondary. If the accelerating voltage is changed, the transverse off-axis fields change as well as the cavity's on-axis fields. If the change is slow enough to ensure a steady-state condition, Γ stays constant and coupler kicks vary of inferior order. Transverse misalignments may cause a transverse kick. A 3 kHz modulation was applied on the forward power on the previously shown reference setup assuring the accelerating voltage to be in resonance. Forward and reflected wave were measured and used for the model. The difference of BPM readouts and the difference of predicted readouts by the model function is plotted in the bottom of Fig. 3. This, so to speak, partial derivative of the transverse orbit on the accelerating voltage shows a reasonable agreement, since misalignments were not included in the calculation.

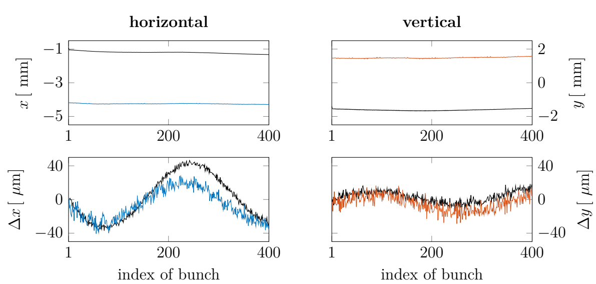

Figure 3: top: BPM readout (black) and predicted output (coloured) for the horizontal (left) and vertical (right) plane. bottom: Difference of the output when applying a 3 kHz modulation on the forward power. Plotted are the BPM readout differences (black) and the corresponding model evaluations (coloured). No misalignments were assumed.

In a second step the modeling of coupler kicks is studied. Caused by the limited bandwidth of the cavity, an increase of the modulation frequency of the forward power will lead to a smaller modulation amplitude of the overall accelerating voltage and to higher reflected power. The impact of main couplers compared to the overall transverse dynamics should therefore increase with higher modulation frequencies. Especially misalignments should for the most part cancel out. Main coupler kicks in the TESLA cavity mostly act in the horizontal plane. In Fig. 4 the differences from the reference setup are plotted for modulation frequencies of 3 kHz, 5 kHz, 50 kHz and 100 kHz for the horizontal plane. The model evaluation in the left column is calculated without coupler kicks. Comparison between the columns in the last two rows points out that the beam dynamics above a modulation frequency of several kHz is dominated by coupler kicks.

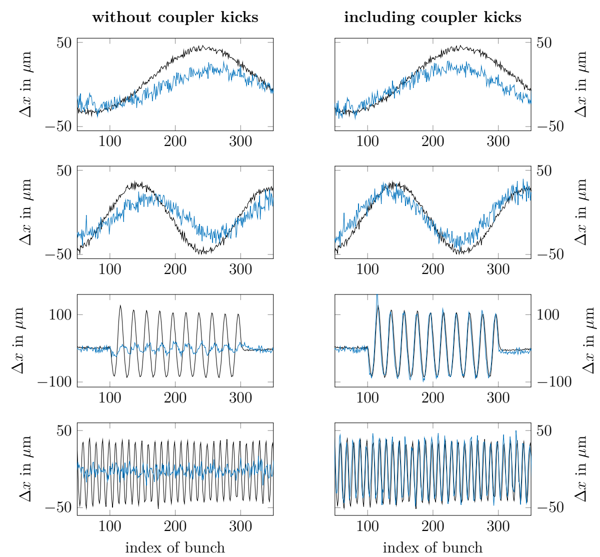

Figure 4: Difference of the output in respect to the reference (cf. Fig. [3], top) while applying modulations on the forward power. Plotted are the BPM readout differences (black) and the corresponding model evaluations (blue) for the horizontal plane. The beam transport calculations were done both including (left) and excluding coupler kicks (right).

CONCLUSION

An efficient model for the beam transport in a TESLA cavity at low beam energy was found. An analytical expression for describing coupler kicks in the injector module was given. Cross check was made against tracking and experimental data. It can be concluded that the presented model is both qualitatively and quantitatively able to reproduce the transverse beam dynamics at low beam energy.

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