BEAM TUNING AND ERROR ANALYSIS OF A SUPERCONDUCTING LINAC *

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Abstract

Beam tuning and error analysis of a superconducting linac for heavy ion beams are introduced in this paper. System errors of the beam tuning are analysed numerically, which include random errors of cavities and magnets, and measurement errors of the absolute beam phase, beam bunch length, and transverse beam profiles. Statistical formulas are developed from tedious and time-consuming numerical simulations, and the studies provide advantage tools not only to analyse the errors of linac beam tuning, such as RF phase and amplitude tuning of superconducting cavity, longitudinal and transverse beam matching, but will also be helpful to the linac design with practical beam diagnostics and authentic accelerator lattices.

INTRODUCTION

In the beam tuning of a superconducting linac or other accelerators, imperfections and errors of the accelerator components are unavoidable, thus it can be important to understand the effects and to improve error tolerance in the accelerator design as well as in the practical beam tuning particularly for a high power machine. Usually, intensive numerical computation and simulations with sophisticated accelerator models are needed in the error analysis, which is tedious and can be extremely time-consuming. A simple statistical estimation, even a rough one, can be considered in most cases.

E.g. the effects of random misalignments of a focusing channel in a periodic lattice are analogous to the problem of random walk, and a simple statistical estimation of the amplitude of beam orbit distortions can be expressed as [1]:

$$\delta = \frac{\sigma \cdot \sqrt{N}}{\cos(\frac{\mu}{2})} \tag{1}$$

where, δ is the rms amplitude of coherent orbit distortion, σ is the rms alignment errors of the focusing elements, μ is the phase advance and N number of focusing elements.

As particle acceleration and RF defocusing effects exist, a linac lattice generally is not perfectly periodic, however, Eq. (1) is still a suitable estimation of the linac beam orbit distortion by misalignments of the focusing magnets, prior to any beam orbit corrections.

CAVITY PHASE AND AMPLITUDE

Cavity RF phase and amplitude tuning with beam is one of the primary tasks before operation of a linac. Phase scan and signature matching techniques [2] are widely applied in proton and heavy ion linac. Phase and amplitude tuning

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of the FRIB superconducting cavity [3] is used here merely as an example, as the method is also valid to other linac.
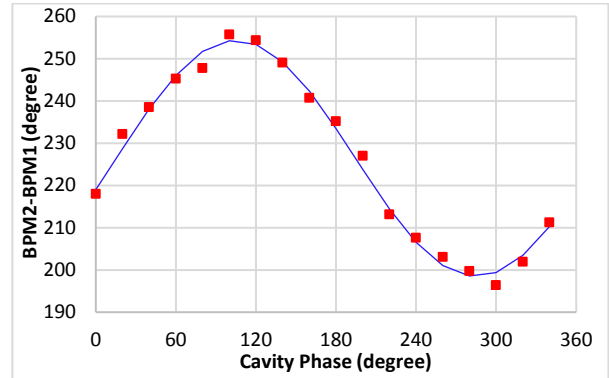

Figure 1: A cavity phase scan and the signature matching.

Figure 1 shows a cavity 2π phase scan and the time-of-flight signature matching in simulations with online model, for a beam energy approximately 17 MeV/u it is almost a pure sinusoidal curve. Errors of the tuning are mainly come from measurement errors of the absolute beam phase with beam phase monitor (BPM) pairs.

To estimate the error of phase scan with random BPM phase measurement errors, cavity RF phase and amplitude are treated as a phasor, the rms error of this phasor is:

$$\delta = \frac{\Delta}{A \cdot \sqrt{N}} \tag{2}$$

where, Δ is random BPM phase errors assumed in uniform distributions, A is the amplitude of BPM phase difference, and N number of measurements in the phase scan ($N \geq 3$).

E.g. in Figure 1, an uniform distribution of random BPM phase error of $\pm 2°$ is assumed, the amplitude of BPM phase difference is about $28°$ in the cavity 2π phase scan, number of the BPM phase measurements 18, and we have $\delta \approx 1.7\%$. It means that an rms amplitude error of this cavity is about 1.7%, and an rms phase error is nearly $1°$ in the tuning.

The amplitude of BPM phase difference in a cavity 2π phase scan can be expressed as:

$$A = \frac{QV_a}{mc^2} \cdot \frac{\omega L}{c \beta^3 \gamma^3} \tag{3}$$

where, Q is charge state of the beam particle, mc^2 is the rest energy, c is speed of light, β and γ relativistic parameters; V_a is acceleration voltage of the cavity, ω and L - angular frequency and drift distance of the BPM pairs.

Other errors in a beam time-of-flight measurement with BPM pairs, such as BPM alignment errors and calibration errors, can be summarized as errors of velocity and energy:

$$\frac{dv}{v} = \frac{dL}{L} + \frac{dt}{t}$$

$$\frac{dE}{E} = \gamma(\gamma + 1)\frac{dv}{v} \tag{4}$$

where, dL is BPM alignment error, and dt calibration error.

LONGITUDINAL BEAM MATCHING

Longitudinal beam matching is conducted with scans of buncher cavity amplitude and measurements of the bunch length with a bunch shape monitor (BSM) [3]. It can be expected that errors of the matching mainly come from the BSM measurement errors and the cavity RF errors. We studied both errors with intensive computer simulations.

Based on the results of the simulation studies, an rms error of the beam bunch length from the random BSM measurement errors can be expressed as:

$$\delta = \Delta \cdot \sqrt{\frac{3}{N}} \tag{5}$$

where, Δ is random BSM measurement errors assumed in uniform distributions, and N number of the measurements in the amplitude scan of the buncher cavity ($N \geq 3$).
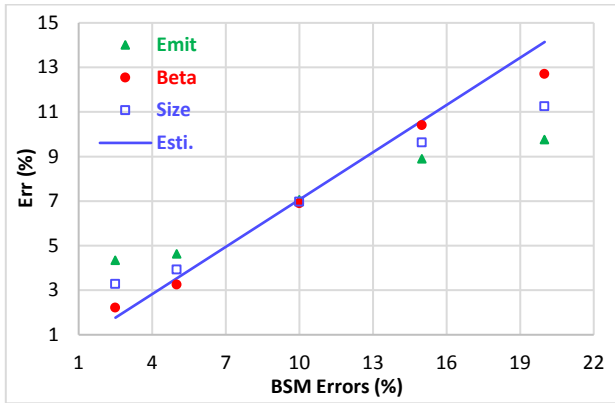

Figure 2: Beam bunch length rms errors vs random BSM measurement errors assumed in uniform distributions.

Figure 2 shows a simulation of the bunch length rms error (Size) vs random BSM errors in uniform distributions against the estimation (Esti.) using Eq. (5). Also shown in the figure are errors of longitudinal beam emittance (Emit) and beta function (Beta) in the simulation studies.

Bunch length error from RF errors is complicated as the lattice is not periodic, and the error also depends on from where the beam is reconstructed in the matching exercise. Specific simulation study with RF errors is still needed.

However, the total rms error of beam bunch length after longitudinal beam matching can be estimated by:

$$\delta = \delta_{BSM} + \sqrt{2} \cdot \delta_{RF} \tag{6}$$

where, δ_{BSM} is rms bunch length error from measurement errors only with a BSM – which may use Eq. (5), and δ_{RF} is rms bunch length error only from the cavity RF errors – which comes from simulation of the linac with RF errors.

In Eq. (6), the RF errors are dominated by random jitters which do not have any correlation, a longitudinal matching exercise is equivalent to a beam goes through a lattice with number of RF cavities doubled – the first time reconstruct injection beam, and the second time optimize the buncher cavity – both processes utilize the cavity RF models.

TRANSVERSE BEAM MATCHING

Because transverse beam matching is important to the operation of a high power linac, two different algorithms for the FRIB driver linac are developed: a wire scanner (WS) array with 4 or 5 WS stations for simultaneous beam profile measurements when a beam matching is needed and spaces are available, and multiple quadrupole scans with the beam profile measurements using a single WS station when the space is limited. In the linac a horizontal-vertical beam coupling exists, thus each WS station is equipped with a horizontal, a vertical, and a 45° diagonal wire [4].

In the error analysis of transverse beam matching, it is noted that the errors also depend on the lattice design and where the beam is reconstructed in the matching exercise. To simplify the problem, error of only the beam size is concerned and analysed in details. As in beam transport the mismatch factor is roughly proportional to the error of the maximum beam projection [5].

When there is no beam coupling, errors of the transverse beam matching will be the same as that of the longitudinal beam matching, in both horizontal and vertical planes, the beam size errors after a matching exercise with random WS measurement errors can be expressed as Eq. (5), where Δ is replaced by random WS measurement errors which is also assumed in uniform distributions. N is number of the beam profile measurements in the matching exercise: it is 5 when using the 5-WS array method, and 9 with the two-quadrupole scans method as which totally has 9 scan steps and associated beam profile measurements.

Beam size error from quadrupole magnet errors depends on where the beam is reconstructed, and it also depends on connections of the power supplies. When the quadrupole pairs are powered in series, error of phase advance is [6]:

$$\delta = 2\tan(\frac{\mu}{2}) \cdot \sqrt{\frac{N}{2}} \cdot \sigma \tag{7}$$

where, δ is rms error of the beam phase advance, σ is rms error of the quadrupole magnets, μ is the phase advance per cell and N total number of the quadrupoles.

If all the quadrupoles are powered independently, then the error of phase advance can be expressed as [6]:

$$\delta = \sqrt{\frac{1+sin^2(\frac{\mu}{2})}{cos^2(\frac{\mu}{2})}} \cdot \sqrt{N} \cdot \sigma \tag{8}$$

In intensive numerical simulations with only quadrupole errors, it is noted that error of the maximum beam size in a periodic lattice is equal to the error of phase advance, while error of the minimum beam size in the same periodic lattice is approximately twice that of the phase advance.

From the above studies, a mismatch from measurement errors with WS and errors of quadrupoles after a transverse matching without beam coupling can be expressed as:

$$\delta = \delta_{WS} + \sqrt{2} \cdot \delta_Q \tag{9}$$

where, δ_{WS} is rms beam size error from WS measurement errors only, and δ_Q is the beam size error from quadrupole errors only.

Without a beam coupling, Eq. (5) and Eq. (7) or Eq. (8) can be used for error analysis of transverse beam matching with the WS measurement errors and quadrupole magnet errors. However, the matching can much more complicated when strong beam coupling exist in the FRIB linac.

4 Beam Dynamics, Extreme Beams, Sources and Beam Related Technology

4A Beam Dynamics, Beam Simulations, Beam Transport

Because multiple solutions exist in the reconstruction of a coupled beam with the WS measurements even without any errors, a satisfactory transverse beam matching can be demonstrated with the reconstructed beam parameters that are completely different to those of the injection ones. Analysis of transverse beam matching with beam coupling exists may not be so straight forward with a practical global searching algorithms as which may intend to solve all the possibilities over the entire beam phase spaces, instead we have to limit the searching range within a local minimum so that a reasonable answer could be found and also can be directly benchmarked against the input beam parameters – a bad data is eliminated if the difference is beyond 3 sigma; as a consequence, an error analysis of the beam transverse matching could be conducted within a limited resource.

In the simulation studies with several thousand seeds of the 5-WS array and the two-quad scan transverse matching exercises, an rms error of the beam size from random WS measurement errors can be expressed as:

$$\delta = 2\Delta \cdot \sqrt{\frac{3}{N}} \qquad (10)$$

where, Δ is random WS measurement errors assumed in uniform distributions, and N is the number of the beam profile measurements ($N \geq 4$).

Table 1: Errors of Beam Parameters of 5-WS Array

WS (%)	Eq. (10) (%)	Simulation Studies		
		Err$_{AVG}$ (%)	Err$_{MIN}$	Err$_{MAX}$
±2.5	3.9	2.5	1.8	3.3
±5	7.7	8.3	5.1	12
±10	15	19	15	23
±15	23	26	22	30

Table 1 lists the results of simulation studies of the 5-WS array against Eq. (10), and Table 2 is those of the two-quad scans against the equation. Even though rms errors of the minimum and the maximum beam parameters (horizontal and vertical beam emittances and beta functions) scattered wildly, an average rms error of the beam size is reasonably close to Eq. (10). As the number of seeds is limited to each case (~500), statistical errors or uncertainties of 4% are expected in these simulation studies.

Table 2: Errors of Beam Parameters of Two-Quad Scan

WS (%)	Eq. (10) (%)	Simulation Studies		
		Err$_{AVG}$ (%)	Err$_{MIN}$	Err$_{MAX}$
±2.5	2.9	2.8	1.7	4.9
±5	5.8	6.2	3.1	11
±10	12	10	5.3	15
±15	17	14	12	17

Total errors of a beam transverse matching exercise from random errors of both WS measurements and quadrupole magnets when a beam coupling exists can also be estimated using Eq. (9), and to conduct the estimation, Eq. (10) can be applied for WS random measurement errors, and Eq. (7)

or Eq. (8) can be applied for the quadrupole errors which depends on the connections of the power supplies. Here, we need to point out one more time that Eq. (7) and (8) can be applied directly in the case of analysis of the maximum beam size, while in the case of the minimum beam size is concerned instead for the transverse beam matching, it must times two to have a correct solution.

In most of the above error analysis random jitters which have no any correlations are assumed the dominate error of the beam measurements as well as the beam elements, and under such conditions, to reduce errors of the beam tuning, it is always favourable to repeat and increase number of the beam measurements, and meanwhile to reduce number of involved beam elements in the beam matching exercises, as which helps to reduce errors and to improve precision of the experiments. However, in the case of systematic errors, it is well known that simply increase the number of beam measurements or repeat the beam experiments offer little help; to reduce systematic error of the matching, different approaches are necessary. Nonetheless, lengthy numerical simulations and computations may not very important to the analysis of systematic errors, as in principle, these errors could be recalibrated with different measurement methods or/and finely corrected with various beam based techniques.

Uniform measurement errors are assumed in the studies, as the actual distribution is unknown, however, in the cases of quasi-Gaussian distributions with rms errors, it can be easily converted by a factor of $1/\sqrt{3}$. It should also be noted that in the matching exercises beam profile measurements better be evenly distributed within 90° phase advance whenever possible to increase the sensitivity, or ambiguity appears and the errors could be worse than the predictions with the equations, Eq. (5) and Eq. (10).

CONCLUSIONS

Beam tuning and error analysis of a superconducting linac with random measurement errors and beam element errors are studied with intensive computer simulations, and simple statistical equations are developed to estimate the effects of those errors. These equations can be applied in estimations of errors in the beam tuning exercises, and can also be directly used in superconducting linac designs with practical beam diagnostic systems and realistic linac lattice in which errors and imperfections exist, and whenever the errors become important.

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