TRACKING BASED COURANT-SNYDER PARAMETER MATCHING IN A LINAC WITH A STRONG SPACE-CHARGE FORCE

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Abstract

During the design of a hadron linac, matching at the interfaces of different structures or lattice periods is often performed with the linear approximation of the space-charge force. When space-charge is extremely strong, like in the low energy part of the proton linac of the European Spallation Source, such a matching method is not always good enough and could lead to a residual mismatch at the design level. To avoid this, a matching scheme based on iterations of tracking, thus including the full effect of the space-charge force, is developed. This paper presents the scheme itself as well as its application to the ESS linac.

INTRODUCTION

The European Spallation Source, current under construction in Lund, Sweden, will be a neutron source driven by a proton linac with an unprecedented 5 MW beam power. For such a high power linac, one of the most significant goals of the lattice design is to maintain high beam quality throughout the linac as well as to minimize beam losses as possible. One known cause of beam quality degradation is the mismatch of the Courant-Snyder (CS) parameters at interfaces of sections.

The normal conducting front-end of the ESS linac, which precedes sections with superconducting cavities, consists of an ion source, low energy beam transport, radio frequency quadrupole (RFQ), medium energy beam transport (MEBT), and drift tube linac (DTL) [1]. Housing a fast chopper, collimators, and various diagnostics devices, the MEBT has several functionalities [2] and one of them is to adjust the CS parameters at the DTL entrance to the values matched to the periodic structure of the DTL, the process referred to as *matching*. The matching must be performed not only at the design stage but also for the real machine, in which the beam parameters are not necessarily identical to the design values. Figure 1 shows a schematic layout of the MEBT. For the current baseline lattice, the matching to the DTL was done with the second and third buncher cavities and the last five quadrupoles.

The main simulation tool of the ESS linac is the TraceWin code [3]. The code performs the multiparticle tracking with a 3D space-charge routine, referred to as *PICNIC* [4], and also the fast *envelope calculation*, where only the beam centroid and RMS sizes are propagated under the linearized space-charge and cavity fields. During the design stage, the matching at the interfaces of the sections in the ESS linac had been performed based on the envelope calculation. This is because the contributions to the CS parameters from the nonlinearities are expected not to be significant but it was

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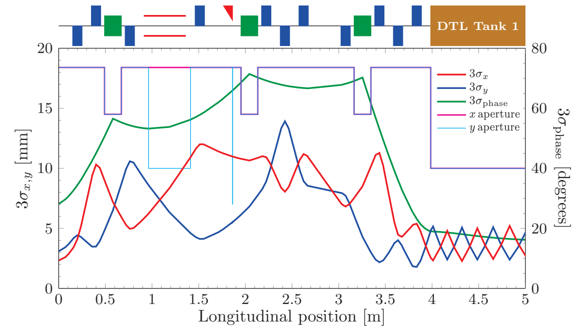

Figure 1: MEBT schematic with 3σ envelopes and apertures. The blue boxes above (below) the line denote focusing (defocusing) quadrupoles, the green boxes do buncher cavities, and the red lines and triangle do a fast chopper and its dump.

found that the MEBT was an exception to this condition. Table 1 compares the CS parameters at the MEBT exit calculated from the envelope calculation and tracking. We can see that all the plane has a discrepancy of 10% level in the mismatch parameter [5] and it is not ideal the lattice by design has such a level of residual mismatches. The causes of these mismatches are the strong space-charge in the MEBT due to the high current (62.5 mA) and low energy (3.62 MeV). For the longitudinal plane, the nonlinearity of the field of the buncher cavities has the same level of a contribution as the space-charge since the bunch length is as long as 60-70 degrees in 3σ at the locations of the buncher cavities (Fig. 1). Because of this situation, we developed a simple scheme to perform the matching with tracking. This paper discusses the scheme itself and sees the impact of the improved matching on the behavior of the beam in the DTL.

Table 1: CS Parameters at the MEBT Exit from the Tracking and Envelope Calculation (The errors for β and α are $\Delta\beta/\beta$ and $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ for each, with the Tracking case as the reference. $M_{x,y,z}$ is the mismatch parameter.)

Parameter	Tracking	Envelope	Error
β_x [m]	0.222	0.258	0.166
α_x	1.425	1.734	0.072
M_x	—	—	0.087
β_y [m]	0.784	0.896	0.143
α_y	−4.219	−4.876	−0.055
M_y	—	—	0.074
β_z [m]	0.413	0.389	−0.057
α_z	0.125	−0.103	−0.221
M_z	—	—	0.124

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MATCHING SCHEME

Principle

Our matching scheme is essentially the Newton's method. We suppose that $\mathbf{B}$ is a (column) vector for the CS parameters at the MEBT exit, $\mathbf{B} = (\beta_x, \alpha_x, \beta_y, \alpha_y, \beta_z, \alpha_z)^T$, $\mathbf{Q}$ is a column vector for the strengths of the quadrupoles and buncher cavities used for the matching, and $\mathbf{B}$ and $\mathbf{Q}$ are related with a nonlinear function $\mathbf{F}$ as $\mathbf{B} = \mathbf{F}(\mathbf{Q})$, where $\mathbf{F}$ is evaluated by the tracking for each set of $\mathbf{Q}$. We also suppose that $\mathbf{B}_f$ is the desired values and $\mathbf{Q}_0$ is the solution from the envelope calculation. Our goal is to find the $\mathbf{Q}$ which satisfies $\mathbf{B}_f = \mathbf{F}(\mathbf{Q})$ by using $\mathbf{Q}_0$ as the initial guess. First, we solve the following linearized equation for $\mathbf{Q}_1$:

$$\mathbf{B}_f - \mathbf{F}(\mathbf{Q}_0) = \mathbf{J}_0(\mathbf{Q}_1 - \mathbf{Q}_0)\,, \tag{1}$$

where $\mathbf{J}_0$ is the Jacobian matrix of $\mathbf{F}$ evaluated at $\mathbf{Q}_0$. Please note, when the dimension of $\mathbf{Q}$ is larger than that of $\mathbf{B}$, we solve this equation with the pseudoinverse method based on SVD. We repeat this process until $\mathbf{F}(\mathbf{Q}_n)$ becomes close enough to the target $\mathbf{B}_f$.

In practice, we apply a linear transformation to Eq. 1 so that the left hand side becomes

$$\Delta\mathcal{B}_0 = \left(\frac{\Delta\beta_{x,0}}{\beta_{x,f}}\,,\ \Delta\alpha_{x,0} - \alpha_{x,f}\frac{\Delta\beta_{x,0}}{\beta_{x,f}}\,,\ \cdots\right).$$

This is because we use $\Delta\beta/\beta$ and $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ (of all the plane) to evaluate the error and want the same weight is applied to these two parameters. The reason why these two parameters are adequate is discussed in the next section.

Error Parameters

When a beam line has a set of N focusing errors $\{q_i\}$, the errors in β and α are given by the following up to the first order of the errors:

$$\frac{\Delta\beta}{\beta} = -\sum_{i=1}^{N} q_i\beta_i \sin(2\psi_i) \tag{2}$$

$$\Delta\alpha = \sum_{i=1}^{N} q_i\beta_i\left[\cos(2\psi_i) - \alpha\sin(2\psi_i)\right]. \tag{3}$$

where β_i is β at the error q_i and ψ_i is the phase advance from the error q_i. Looking at Eqs. (2) and (3), when $\alpha \gg 1$, using $\Delta\alpha$ could lead to an overestimation of the error in α. Similarly, when $\alpha \ll 1$, using $\Delta\alpha/|\alpha|$ could cause an overestimation of the error in α. In addition, when $\alpha \gg 1$, both $\Delta\alpha$ and $\Delta\alpha/|\alpha|$ is highly correlated with $\Delta\beta/\beta$ and does not provide an independent information for the matching. From Eqs. (2) and (3), the parameter $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ satisfies

$$\Delta\alpha - \alpha\frac{\Delta\beta}{\beta} = \sum_{i=1}^{N} q_i\beta_i\cos(2\psi_i)\,. \tag{4}$$

We can see that, unlike $\Delta\alpha$ and $\Delta\alpha/|\alpha|$, this parameter has neither of the issues of the magnitude and correlation.

Table 2: Lattice Errors in MEBT and DTL [6] (MEBT quadrupoles also have 1% multipole errors at 15 mm up to duodecapolar components. The DTL Tank error is applied to all the elements in the tank. All the errors are in uniform distributions and the listed values are their amplitudes.)

Section	Element	$\delta x, \delta y$ mm	$\delta\theta_x, \delta\theta_y$ mrad	$\delta\theta_z$ mrad	$\delta E, \delta B$ %	$\delta\phi$ deg
MEBT	Quad	0.2	0	1.0	0.5	—
	Cavity	0.5	2.0	—	1.0	1.0
DTL	Quad	0.1	8.7	3.5	0.5	—
	Cell	0	0	—	1.0	0.5
	Tank	0.1	0	—	1.0	1.0

The pair $(\Delta\beta/\beta, [\Delta\alpha - \alpha(\Delta\beta/\beta)])$ also has a close relation with the mismatch parameter, which is defined as

$$M = \left\{1 + \frac{\Delta_M + [\Delta_M(\Delta_M + 4)]^{1/2}}{2}\right\}^{1/2} - 1\,, \tag{5}$$

where $\Delta_M = (\Delta\alpha)^2 - \Delta\gamma\Delta\beta$. In the lowest order,

$$M \simeq \frac{1}{2}\Delta_M^{1/2} \simeq \frac{1}{2}\left[(\Delta\beta/\beta)^2 + (\Delta\alpha - \alpha\Delta\beta/\beta)^2\right]^{1/2}. \tag{6}$$

This equation shows that these two parameters are effectively sin and cos terms of the mismatch parameters, supporting that the weight should be the same for these two parameters.

To test the above discussion, we simulated 1000 MEBTs with errors listed in Table 2 and checked the relations of $\Delta\alpha$, $\Delta\alpha/|\alpha|$, and $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ to $\Delta\beta/\beta$ at the MEBT exit (Fig. 2). Please note that, in this case, the calculations were all from the tracking the reference was the no error case. As expected, $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ is showing no issue of the magnitude and nor the strong correlation with $\Delta\beta/\beta$ in contrast to $\Delta\alpha$ and $\Delta\alpha/|\alpha|$.

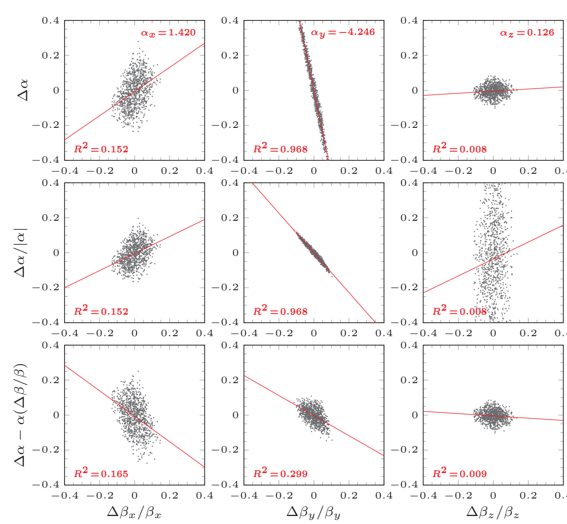

Figure 2: Relations of $\Delta\alpha$, $\Delta\alpha/|\alpha|$, and $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ to $\Delta\beta/\beta$ at the MEBT exit for 1000 MEBTs.

Matching Result

The simple scheme discussed in this section provided a solution with $\Delta\beta/\beta$ and $[\Delta\alpha - \alpha(\Delta\beta/\beta)]$ of all the plane around $\sim 10^{-5}$ after 4-5 iterations. The scheme started to become numerically unstable beyond this level but the level of $10^{-4\sim5}$ is practically good enough for the ESS linac. We also note that no solution was found when the matching was tried with the last four quadrupoles and the seventh quadrupole had to be included for a successful matching.

BEAM SIMULATION

In this section, we compare the beam quality in the MEBT and DTL when matched with the envelope calculation and with our scheme. Figure 3 compares the emittances and halo parameters [7] from the two cases, showing only the last half of the MEBT and DTL Tank 1 to see behavior of the beam in the vicinity of the interface. Please note that these the errors are not taken into account in these calculations. Seeing the emittances shown in the left column, reductions of the *beating* effect, which is an indication of the mismatch, is clear especially for the longitudinal plane. The reduction of the beating effect is also clear for the halo parameters. In this case, the situations among three planes are similar. The fact that the beating effect is still visible in the halo parameters even after the matching is improved indicates the difference in dynamics for the core and outer part of the beam.

As the final test, we compared the two cases by taking into account the standard set of the static errors listed in Tables 2 as well as the errors in the beam into the MEBT [6], estimated from the errors in the RFQ. Figure 4 shows the distributions, out of 1000 simulations, for the relative differences of the emittances (left) and for the absolute differences of the halo parameters (right). The emittances and halo parameters are the ones at the DTL exit and the differences were made with respect to the no error case. As we saw in Fig. 3, the improvement of the matching does not have a significant impact on the emittances of the transverse planes. We can see some improvement for the longitudinal emittance, but the change is only a small shift of the mean and the upper limit remains the same. For the halo parameters of the transverse planes, the reduction of the upper limit is clearly seen. In contrast to the no error case in Fig. 3, there is no significant different made for the longitudinal halo parameter in this case.

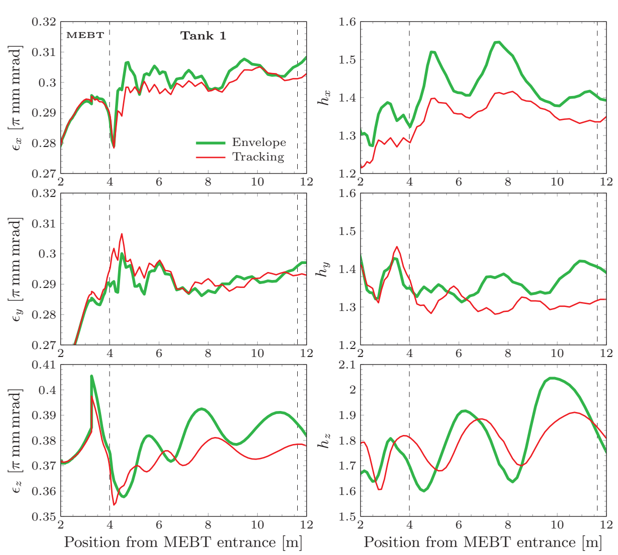

Figure 3: Emittances (left) and halo parameters (right) in the last half of the MEBT and DTL Tank 1.

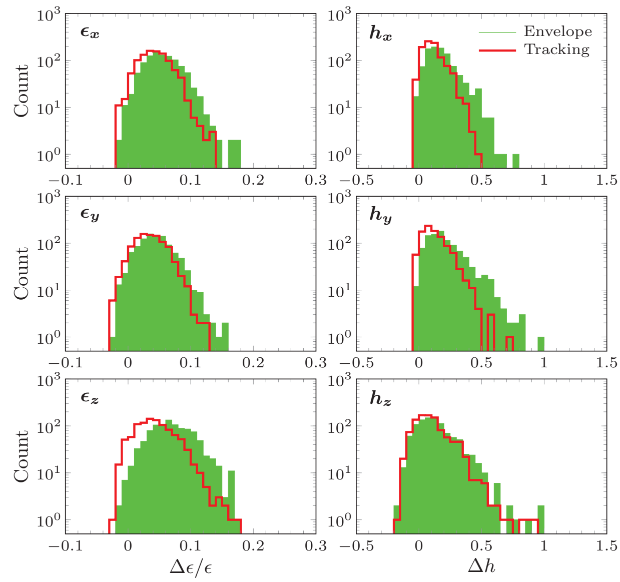

Figure 4: Distributions of the emittances (left) and halo parameters (right) at the DTL exit, with respect to the no error case, for 1000 simulations.

Given that an emittance growth of a few percent or an increment of the halo parameter of a few tenth is not too significant, it is clear that the tracking based matching improved the behavior of the beam in the DTL but it is unlikely that this improvement alone makes a significant impact to the real machine, such as reducing the losses in the downstream superconducting section. As seen in Fig. 4, the effects of the lattice errors are much larger than that of the mismatch.

CONCLUSIONS

A matching scheme based on tracking simulations, taking into account the nonlinear effects of the space-charge and cavity fields, was developed to improve the matching at the interface of the MEBT and DTL in the ESS linac and verified to work. Combined with measurements of diagnostics devices, a similar iterative approach may work for the matching in the real ESS linac and this possibility will be studied as a next step.

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