SEMI-3D BEAM-TRACKING CODE FOR ELECTRON INJECTORS USING BULK-TO-POINT CALCULATION TECHNIQUE FOR SPACE CHARGE FIELDS

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Abstract

A new semi-three-dimensional beam-tracking simulation code for electron injectors using bulk-to-point calculation technique for space charge fields is developed. The calculated space charge fields are not produced by a point charge but a doughnut which has the volume and whose cross-section is ellipsoid. Since the calculation noise which is usually caused by distributions of positions of point charge can be minimized, high accuracy calculation on emittance is realized with small number of electrons. Simultaneously, the calculation time becomes markedly shortened. In this paper, calculation examples for asymmetrical beams are demonstrated by the new code. The accuracy of emittance is also discussed.

INTRODUCTION

The emittance calculation technique is important in the design of electron injectors, particularly very low emittance electron sources such as X-ray free-electron lasers. There have been many analytical solutions [1–4] for beam dynamics, although it is difficult to accurately calculate practical bunch shapes and detailed emittance behavior. On the other hand, particle-tracking simulation codes [5–9] are useful for calculating the dynamics of complex bunch shapes of practical beams. However, the calculated emittances often depend on the number of particles.

These dependences are often caused by the calculation scheme of the space charge field which is produced by a point charge. This scheme makes calculation noise larger. Therefore, the author has developed the two-dimensional beam-tracking simulation code [10] using the calculation technique for space charge fields produced not by a point charge but by a bulk charge.

In this method, a short bunched electron beam is assumed to be an ensemble of several segmentation pieces in both the transverse and longitudinal directions. The trajectory of each electron, which is a point charge and located at each segmentation corner, is solved by the fourth-order Runge-Kutta method. When calculating space charge fields, they are assumed not to be produced by a point charge but by a doughnut that is separated by segmentation meshes. The shape of the entire bunch can be consequently calculated; thus, the emittances can be precisely calculated. These calculation techniques for space charge fields are similar to those used in Ref. [11] but nonlinearity of transverse fields can be calculated in this method. Therefore, highly accurate emittance calculations can be performed.

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Asymmetrical beams cannot be calculated by this two-dimensional code, however, calculations for these beams are often required in practical injectors. Therefore, the author upgrades this code to a semi-three-dimensional beam-tracking simulation code which is described in this paper.

OUTLINE OF 2D BEAM-TRACKING CODE

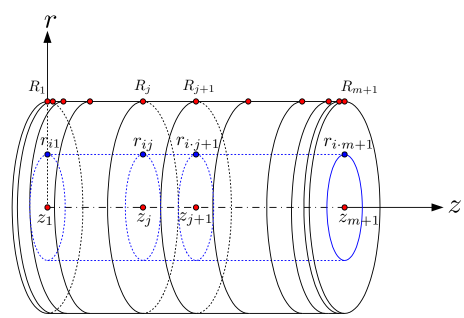

Figure 1: Bunch segmentation model used for the 2D code. This shows initial bunch configuration.

The Initial bunch segmentation model used for the 2D code is shown in Fig. 1. The bunch is divide into m slices in the longitudinal direction and n parts in the transverse direction. Each electron is located at each segmentation corner and tracked by the fourth-order Runge-Kutta method with sum of space charge fields produced by each segmentation doughnut. Charge density of each doughnut is uniform and charge of that is constant throughout calculations. $\left\langle r^2 \right\rangle$, $\left\langle r'^2 \right\rangle$ and $\langle rr' \rangle$ can be calculated from weighted mean values of the solutions for the each tracked electron; thus the normalized rms emittance that is defined by $\epsilon_r = \langle\gamma\rangle\langle\beta\rangle\sqrt{\langle r^2\rangle\langle r'^2\rangle - \langle rr'\rangle^2}$ can be calculated. In the rest of the paper, m and n are referred to as "segmentation numbers."

The space charge fields produced by the doughnuts can be calculated by subtracting the fields by an inner slice from the fields by an outer slice. These fields from the slice are functions of three variables, r_0/R_s, $2r_0/\gamma L$ and z_0/L [10], where r_0 is a position of the electron, γ, R_s and L are a relativistic factor, a radius and a longitudinal width of the slice respectively. By numerically calculating these fields and preparing as a three-dimensional mapping data file in advance, the fields can be referred in the tracking code by loading the file. Details of the 2D code are described in Ref. [10].

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UPGRADE TO SEMI-3D BEAM-TRACKING CODE

To upgrade from the 2D code to the 3D code, the bunched beam is segmented transversely in both the x and y directions, and the space charge fields are assumed to be produced by a doughnut whose cross-section is ellipsoid. The fields from the ellipsoidal doughnut become functions of four variables, r_0/R_s, $2r_0/\gamma L$, z_0/L and α, where R_s is a radius of the ellipsoidal slice in the x or y directions, αR_s is a radius in the other direction. By preparing a four-dimensional mapping data, the space charge fields can be calculated in the same manner as used in the 2D code, however, file size of the mapping data becomes large and it is impractical.

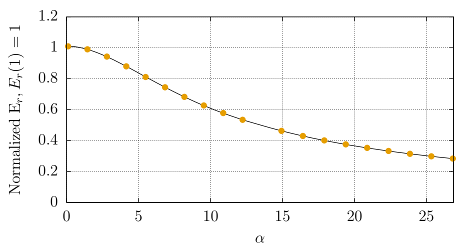

Figure 2: α dependence of the transverse space charge field. $r_0/R_s = 0.84$, $2r_0/\gamma L = 0.8$ and $z_0/L = 6.0$ for this example. Charge of the slice is constant with α.

Figure 2 shows an example of α dependence of the transverse space charge field produced by the ellipsoidal slice. The calculated points in Fig. 2 can be fitted by a function, $f(x) = A/\left[1+\left(\frac{x}{B}\right)^C\right] + D$, therefore, we can decrease the dimension of the mapping data file to three and the fields can be referred in the tracking code. This procedure is also valid for the longitudinal space charge fields.

For calculating the rms emittance of the ellipsoidal beams, $\left\langle x^2\right\rangle = r_d^2/2$, $\left\langle x'^2\right\rangle = r_d'^2/2$ and $\langle xx'\rangle = r_d r_d'/2$, where, r_d is a radius of the doughnut in the x direction; thus, the normalized rms emittance, ϵ_x, can be calculated.

Note that the energy of the doughnut must be uniquely determined in this segmentation model, however, radial velocity of an electron located at x_{ij} is obviously different from that of an electron located at y_{ij}. Therefore, the energy of the doughnut is defined as an average energy of electrons in the two directions.

3D-CALCULATION EXAMPLES AND ACCURACY

The calculations for the SPring-8 rf gun injector system are performed. The practical system consists of a single-cell S-band rf gun cavity with a copper cathode, two solenoidal coils and a 3-m long S-band accelerator structure. Here, the solenoidal coils are replaced by a quadrupole doublet for asymmetrical beam calculations. Table 1 shows parameters for the calculations.

Figure 3 shows the calculated transverse rms beam sizes using the semi-3D beam-tracking simulation code, along

Table 1: Parameters of the Calculations for the SPring-8 Rf Gun Injector System With a Quadrupole Doublet

Laser length (uniform distributed)	20 ps
Laser spot size (uniform distributed)	ϕ1.2 mm
Charge per bunch	0.2 nC
Max. electric field on the cathode	157.0 MV/m
Initial rf phase (when the head of the bunch is emitted)	sin 5 deg.
Initial emittance	0 mrad
Entrance of a quadrupole doublet	z = 0.4 m
Entrance of an acc. structure	z = 1.4 m
Energy at the exit of a gun cavity	3.7 MeV
Energy at the exit of an acc. structure	30.0 MeV

with those using the existing 3D particle-tracking simulation code [7] in which the point-to-point calculation technique is used for the space charge field calculations. The segmentation numbers for the semi-3D code are $m = n = 20$ and the number of particles for the existing code is 6×10^4. Image charge effects at the cathode are considered in the both codes. The beam is defocused at the exit of the gun cavity, and focused toward the accelerator structure by the quadrupole doublet. The beam size in the x direction becomes different from that in the y direction after the quadrupole doublet. The results calculated by the semi-3D code are almost the same as those calculated by the existing code.

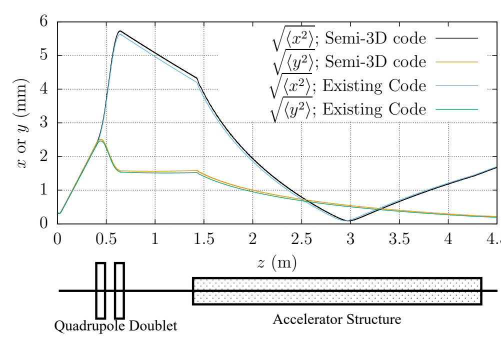

Figure 3: Time evolutions of beam sizes for the calculations in the SPring-8 rf gun injector system.

Figures 4 and 5 show the calculated bunch shapes at z = 4.5 m after the exit of the accelerator structure. Each dot on the black solid lines is an electron traced by the semi-3D code and the clouds of small dots are the particles calculated using the existing code. They agree with each other in both the x and y directions. The solid lines in Fig. 5, which are initially in order, intersect each other, however, these are correctly calculated [10].

Figure 6 shows the time evolutions of calculated normalized rms emittance. For calculating the fields of the quadrupole doublet in the both codes, they are assumed to be longitudinally uniform within the yokes of the doublet which are located from z = 0.4 m to 0.5 m and from z = 0.55 m to 0.65 m. The fringe fields are ignored. Therefore, the evolutions of the emittance are discontinued at the yokes of

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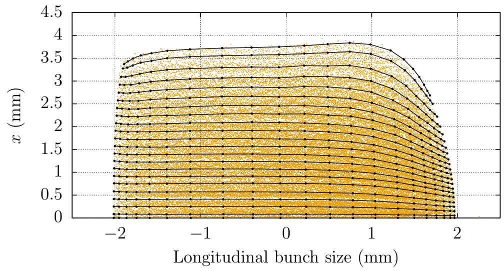

Figure 4: Bunch shapes in the x direction at the exit of the accelerator structure.

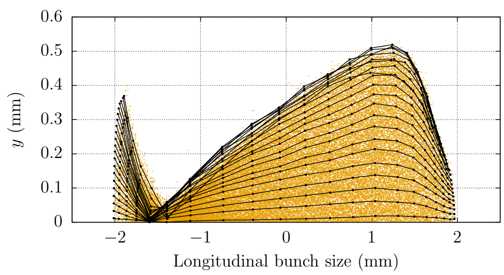

Figure 5: Bunch shapes in the y direction at the exit of the accelerator structure.

the doublet, however, the emittances agree almost perfectly in both the x and y directions. Note that these calculation examples are not optimized for realizing low emittance but for confirming accuracy of the semi-3D code for the asymmetrical beams.

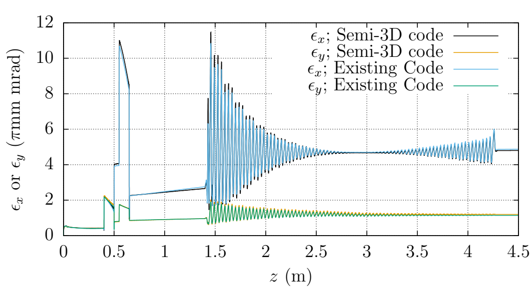

Figure 6: Time evolutions of normalized rms emittance for the calculations in the SPring-8 rf gun injector system.

Emittance dependence on the segmentation numbers in the semi-3D code and on the number of particles in the existing code are shown in Figs. 7 and 8 respectively. Emittance dependence in the semi-3D code is almost flat and weaker than that in the existing code since bulk-to-point calculation technique is used in the semi-3D code. The calculation time of the semi-3D code with the segmentation numbers of m=n=20 is 54 times faster than that of the existing code with number of particles of 6×10^4.

SUMMARY

The two-dimensional beam-tracking simulation code using bulk-to-point calculation technique for the space charge fields is upgraded to the semi-three-dimensional beam-

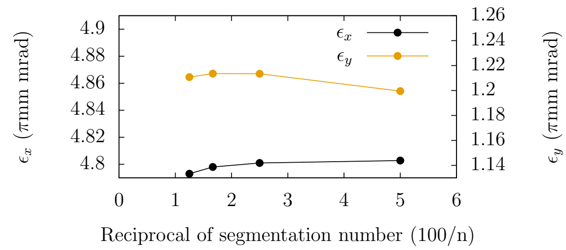

Figure 7: Emittance dependence on segmentation numbers in the semi-3D code. m and n are the same in each calculation. m=n=20 for the rightmost points and m=n=80 for the leftmost points. The emittances are the values at z = 4.5 m.

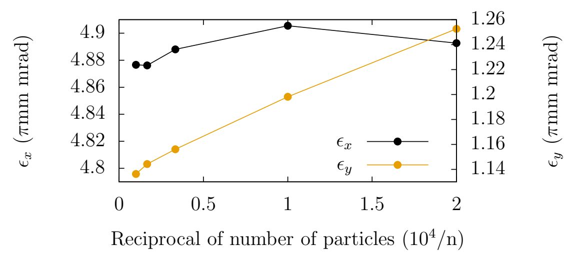

Figure 8: Emittance dependence on number of particles in the existing code. The number of particles is 5×10^3 for the rightmost points and 1×10^5 for the leftmost points.

tracking simulation code. The space charge fields from the ellipsoidal doughnuts are successfully calculated from the fields mapping data file which is numerically calculated in advance. The accuracy of emittance in the calculation examples for the asymmetrical beams used by the semi-3D code is better than those used by the existing code and the calculation time of the semi-3D code is much faster than that of the existing code. This semi-3D beam-tracking code is expected to be useful for calculating beam dynamics of electron injectors.

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