

TRAVELING WAVE ACCELERATING STRUCTURE POWER INPUT CALCULATION WITH EQUIVALENT CIRCUIT METHOD *

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Abstract

Nowadays linac accelerating RF systems design is usually done by the finite difference method. It provides high accuracy of calculations and freedom in topology choosing, but may draw considerable amounts of computer resources with long calculation times. Alternative to this method, equivalent circuit method exists. The basic idea of this method is to build a lumped element circuit, which with certain approximation acts as an original accelerating cell. It drastically reduces the number of equations to solve. This method is long known but usually only used for the particular accelerating structures when speed of calculation is a key-factor. This paper describes an attempt to create more universal and user-friendly software application for calculating electrical field distribution in accelerating structures, provides mathematical equations this software is based on. The resulting application may be used for preliminary calculations of acceleration structures and help to determine cells electrodynamic parameters reducing overall design time.

INTRODUCTION

At present time finite differences method programs are the main tool for the accelerating structure calculations. Its advantages are relative universality and simplicity. These programs are applicable for all types of accelerating structures; same model can be used for different operation mode calculations; for the most part, these programs have friendly user interfaces. However, programs require large amount of time and machine resources. In contrast to finite differences method, the equivalent circuit method may be used for quick calculations of the sections with a large number of cells [1, 2]. There are some downsides to this method also. Firstly, the result of the structure optimization - optimal coupling coefficients and cells frequencies - does not provide information about the geometrical parameters of the structure: either finite difference method calculation or correspondence table are needed. Secondly, for two different cell type or even for a single cell operating in different modes equivalent circuits are needed. In this light it is obvious that the equivalent circuit method can not replace the finite difference method, but can be used to obtain reference parameters for the subsequent accelerating structure optimization, thus reducing calculation time. Particularly useful this program seem to be in application to calculation of the long structures or structures with a constant field gradient.

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EQUATIONS FOR ACCELERATING CELLS

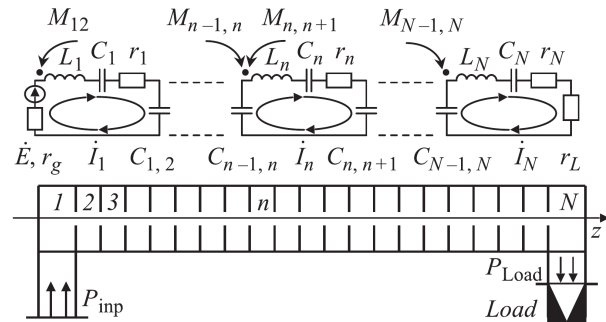


Figure 1: The equivalent circuit of the accelerating structure.

In Fig. 1 the equivalent circuit of the accelerating structure is shown. The equations describing this circuit can be represented in matrix form:

$$|A| \cdot |X| = |B|$$

Here, X is normalized cell field amplitude vector; right side of the equation represents particle current load and the input generator power. Matrix A is cell connection matrix: elements $a_{i,j}$, $i \neq j$ characterize connections between the cells i and j. This equation may be solved analytically with some simplifications but it would restrict application of the code to a very specific accelerating structure schemes. Alternatively, relying on the numerical methods, one may obtain solutions for various accelerating structure schemes incorporating different cell types. For example, addition of the waveguide connection between cells i and j would require only the replacement of the rows i and j in the matrix A. Previously obtained analytical solution would not be applicable in this case. Numerical solution allows characteristics calculation of the sections with variable geometry (with a constant gradient), for which analytical solution is very complicated, as well as carrying out calculations of cells tolerances ensuring efficient operation of the structure. Additionally, method of equivalent circuits is applicable for characteristics calculation of biperiodic accelerating structures.

Cells with Waveguide Connection

In Fig. 2 the equivalent circuit of the cell with electric and magnetic coupling and dumping load is shown. This circuit could be used for pure electric coupling cells if magnetic coupling coefficient is set to zero and vice versa. Kirchhoff equations and the input impedance equations for these cells are well known and described in [3].

This scheme can be made even more versatile by adding input generator power and transmission line connections

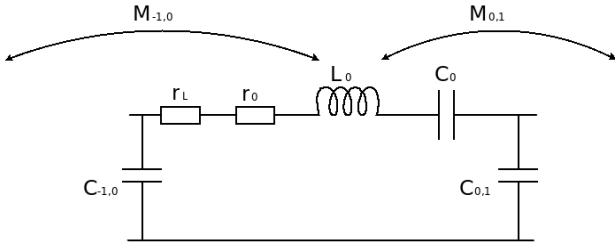


Figure 2: The equivalent circuit of the accelerating structure of the cell.

to other cells. This structure would be useful for calculations of hybrid structures, comprising standing and traveling wave sections, as well as accelerating structure - generator interaction study. Equivalent circuit for cell with waveguide connections is shown in Fig. 3. In this circuit transmission lines are equivalently replaced by quadripole.

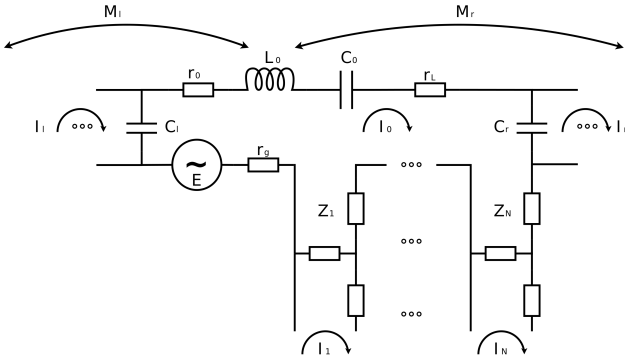


Figure 3: The equivalent circuit of the accelerating structure with waveguide connections.

The equation of the cell with generator power input, dumping load, electric and magnetic couplings and N waveguide connections:

$$\begin{aligned} & \left(\frac{f^2 K_l^H}{2f_l f_0} - \frac{K_l^E}{2} \right) \dot{X}_l + \left(1 - \frac{f^2}{f_0^2} + j \frac{f}{f_0} \frac{1 + \chi_{\Sigma a}}{Q_0} \right) \dot{X}_0 + \\ & + \left(\frac{f^2 K_r^H}{2f_r f_0} - \frac{K_r^E}{2} \right) \dot{X}_r + j \xi_{01} \dot{X}_1 \sqrt{\frac{\chi_1 f^2}{Q_0 Q_1 f_0 f_1}} + \dots + \\ & + j \xi_{0N} \dot{X}_N \sqrt{\frac{\chi_N f}{Q_0 Q_N f_0 f_N}} = j 4 \sqrt{\pi} \frac{f^2}{f_0} P_g \frac{\chi_g}{Q_0} \exp j \phi \end{aligned}$$

$$\xi_{ij} = \frac{\exp(-\gamma_{ij}) + \exp(-\gamma_{ji})}{1 - \exp(-\gamma_{ij}) - \exp(-\gamma_{ji})}$$

$$\chi_{\Sigma i} = \chi_g + \chi_L + \chi_1 \tau_{i1} + \dots + \chi_N \tau_{iN}$$

$$\tau_{ij} = \frac{1 + \exp(-2\gamma_{ij})}{1 - \exp(-\gamma_{ij}) - \exp(-\gamma_{ji})}$$

f_i is a resonant frequency of the cell, Q_i is its quality factor, K_i^e and K_i^h are coupling coefficients for the electric

and magnetic fields, χ_i are coupling coefficients with waveguides, γ_i are waveguides propagation constants, P is an input power, \dot{X}_i is a complex cell field amplitude.

The expression for the input impedance of the cell:

$$\begin{aligned} \hat{Z}_{in} = \chi_g \left\{ 1 + \chi_{\Sigma 0} - \chi_g + j Q_0 \left[\frac{f}{f_0} + \frac{f_0}{f} + \left(\frac{f_0}{f} \frac{K_l^E}{2} - \right. \right. \right. \\ \left. \left. \left. - \frac{f}{f_l} \frac{K_l^H}{2} \right) \frac{\dot{X}_l}{\dot{X}_0} + \left(\frac{f_0}{f} \frac{K_r^E}{2} - \frac{f}{f_r} \frac{K_r^H}{2} \right) \frac{\dot{X}_r}{\dot{X}_0} - \right. \right. \\ \left. \left. - \xi_{01} \sqrt{\frac{\chi_1}{Q_0 Q_1} \frac{f_0}{f_1} \frac{\dot{X}_1}{\dot{X}_0}} - \dots - \xi_{0N} \sqrt{\frac{\chi_N}{Q_0 Q_N} \frac{f_0}{f_N} \frac{\dot{X}_N}{\dot{X}_0}} \right] \right\}^{-1} \end{aligned}$$

This equation allows to carry out calculation and analysis of section input impedance in relation to parameters of cells and connecting waveguides.

IMPLEMENTATION

Accelerating structure performance calculation scripts are written in the programming language R; web user interface (WUI) is implemented with the help of shiny library [4]. It allows working with them remotely: via local network or Internet. Free version of the hosting shiny server only supports one simultaneous working session, but the paid version of the server provides simultaneous operation and additional functionality.

User Interface Overview

UI of the macros consists of the three main tabs: input tab, settings tab and modules tab. The input data tab displays a table of input data, tools for loading new data in .csv format and simple editing functions. Settings tab displays some output options and the description of the code. Modules tab contains sub-tabs of available calculation modules.

Solver Modules

Currently available four modules are: UvsCell-Solver, Dispersion, Zin-Solver, SetTrWave.

UvsCell-Solver module calculates accelerating structure cell electric field distribution by directly solving the system of equations. An example of this output module is shown in Fig. 4 (Hereinafter, calculations are made for the hybrid accelerating structure with the negative dispersion traveling wave).

Dispersion module calculates dispersion characteristic of selected cell as if it is included in section with the remaining unexcited cells. Coupling coefficient for the cell is taken as an arithmetic average of the neighboring coupling coefficients. An example of the module output is shown in Fig. 5.

Zin-Solver module calculates input impedance of accelerating structure or reflection from it in frequency range. Sample output from this module is presented in Fig. 6 and Fig. 7.

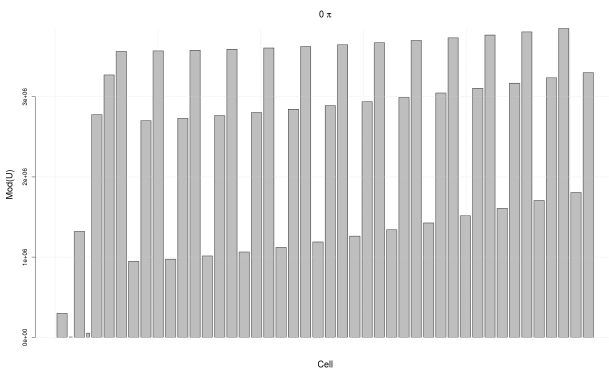


Figure 4: Distribution of the electric field in the accelerating section.

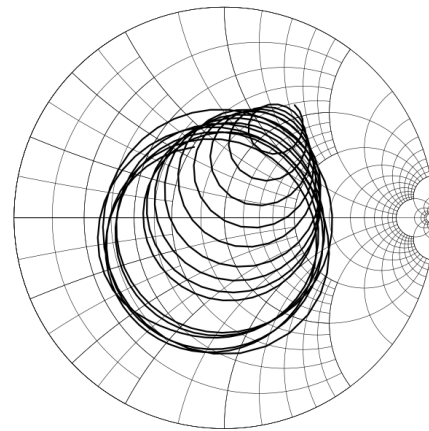


Figure 7: Input impedance of accelerating section.

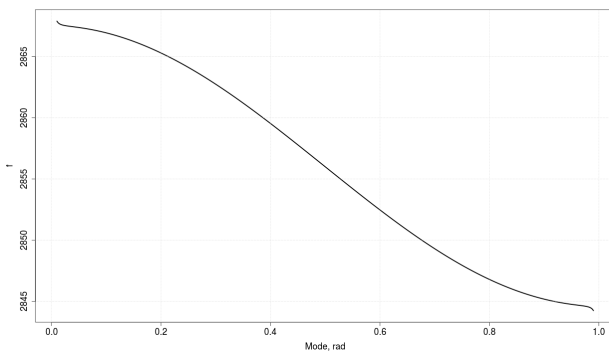


Figure 5: Dispersion characteristic of the cell.

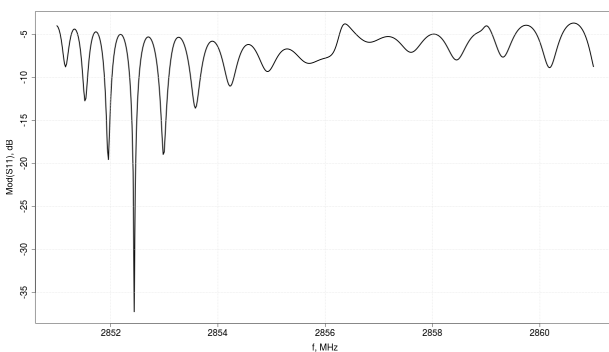


Figure 6: Band characteristics of accelerating section.

SetTrWave module is a macro for optimization of the traveling wave section. For a given input data using the Dispersion module it sets the resonance frequency of the mode $\pi/2$ section cells. Then, using known analytical equations, coupling coefficients and resonance frequencies for input power cells are calculated. There is no graphic output data from the macro.

CONCLUSION

In this study, the analytical expression for accelerating structure cell with waveguide connections amplitudes of the electric fields as well as the input impedance equations. Based on the derived expressions the code for calculating the electromagnetic characteristics of accelerating structures based on the method of equivalent circuits was written. In future a number of additional features are planned for implementation: the possibility of calculating the electric field distribution, taking into account statically given current in the cells; calculation of the transient processes in the cell; mismatch estimation based on a known distribution of the electric field; the expansion of the library of available cell types for higher modes.

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