

# APPLYING TRANSVERSE GRADIENT UNDULATORS TO SUPPRESSION OF MICROBUNCHING INSTABILITY

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## Abstract

The microbunching instability developed during the beam compression process in the linear accelerator (LINAC) of a free-electron laser (FEL) facility has always been a problem that degrades the lasing performance, and even no FEL is able to be produced if the beam quality is destroyed too much by the instability. A common way to suppress the microbunching instability is to introduce extra uncorrelated energy spread by the laser heater that heats the beam through the interaction between the electron and laser beam, as what has been successfully implemented in the Linac Coherent Light Source and Fermi@Elettra. In this paper, a simple and effective scheme is proposed to suppress the microbunching instability by adding two transverse gradient undulators (TGU) before and after the magnetic bunch compressor. The additional uncorrelated energy spread and the density mixing from the transverse spread brought up by the first TGU results in significant suppression of the instability. Meanwhile, the extra slice energy spread and the transverse emittance can also be effectively recovered by the second TGU. The magnitude of the suppression can be easily controlled by varying the strength of the magnetic fields of the TGUs. Theoretical analysis and numerical simulations demonstrate the capability of the proposed technique in the LINAC of an x-ray free-electron laser facility.

## INTRODUCTION

X-ray free-electron lasers (FELs) hold great promise as ultra-short, tunable, intensity radiation sources for advanced user applications and open up new frontiers of ultra-fast and ultra-small sciences at the atomic scale. In the recent years, the successful user operation of the first FEL facilities in soft and hard x-ray regimes announced the birth of the x-ray free-electron laser. In the x-ray FEL process, the required high intensity electron beams of sub-picosecond (sub-ps) length are usually obtained by compressing longer beams in magnetic bunch compressors at relativistic energies. The bunch compressor manipulates longitudinal phase space of the electron beam with a considerable energy chirp by introducing the dependence of a particle's longitudinal position on its energy. As the result, the bunch can be significantly compressed in the bunch compressor due to the momentum compaction factor therein. On the other hand, due to existence of the longitudinal space charge (LSC), the coherent synchrotron radiation (CSR) and structural impedance in the beam transportation and compression process, the initial small energy and density perturbation in the electron bunch can be amplified with a large gain factor in many cases, which results in the increase of the fragmentation of the longitudinal phase space

and dilute the emittance. This process of amplification is usually known as the microbunching instability, and it will seriously degrade the FEL performance thereafter.

The TGU is an undulator with a transverse gradient between the magnetic poles. In this paper, by investigating the transport matrix and the beam dynamics in a TGU, a simple and inexpensive technique based on transverse-to-longitudinal phase space coupling is proposed to suppress the microbunching instability of an electron beam. It is found that by adding a TGU right before the magnetic bunch compressor where large energy chirp exists, the gain of the microbunching instability developed in the electron beam can be effectively suppressed by the additional slice energy spread and the density mixing from the transverse spread introduced by the TGU. Meanwhile, the additional slice energy spread and the transverse emittance growth introduced by the first TGU can also be recovered very well by another TGU right after the compressor. Compared with the other techniques, this method is quite simple and could be easily applied to all existing FEL facilities in addition to a laser heater.

## METHODS

The TGU is an undulator with a transverse gradient between the magnetic poles. The original idea of the TGU was a tool to prevent gain degradation in FEL oscillators due to the large energy spread of the electron. Recently, the idea has been applied to laser-plasma accelerator-driven high-gain FELs for the design of compact x-ray FEL devices. The TGU can be realized by canting the poles of a regular undulator and the gradient is usually made in the horizontal direction. Because the electrons at different horizontal positions feel different magnetic fields, the path length of an electron traversing a TGU depends on its transverse coordinates at the entrance of the TGU, as the result, the transverse-to-longitudinal phase space coupling is introduced. Ignoring the vertical effects, the first-order transport matrix of the TGU in  $(\vec{x}, x', \vec{z}, \delta_\gamma)$  phase space can be derived as

$$\mathbf{R}_{\text{TGU}} = \begin{bmatrix} 1 & L_T & 0 & \tau L_T/2 \\ 0 & 1 & 0 & -\tau \\ \tau & \tau L_T/2 & 1 & -\tau^2 L_T/6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where  $L_T$  is the effective length of TGU and  $\tau$  is the strength of TGU. In the TGU transport matrix (1), one can see that the effective elements  $R_{31}$  and  $R_{24}$  are of the same value but in opposite signs.

Assuming a TGU is placed right before the first dipole of a magnetic bunch compressor where the energy chirp is

large, because of the transverse-to-longitudinal coupling introduced by the TGU, it turns out that the electrons at the different transverse locations in the bunch have different path lengths traversing the TGU, which results in the redistribution of the longitudinal beam phase space and increases the longitudinally slice energy spread, or called heating. In the following, we start our investigation for the beam with linear energy chirp. To study the behavior of the microbunching instability in the presence of the TGUs in a reasonable way, the density perturbation in one wavelength is divided into multiple slices. And because the wavelength of the microbunching instability is usually much shorter than the bunch length, the assumption of the uniform longitudinal density distribution within a beam slice is employed in the following discussion.

To investigate the behavior of the microbunching instability, the density perturbation in one wavelength is divided into multiple slices. Because the wavelength of the microbunching instability is usually much smaller than the bunch length, the assumption of the uniform longitudinal density distribution within a beam slice is employed in the following discussion. If a linear energy chirp is added on the electron beam with Gaussian energy distribution before the TGU (TGU1) before bunch compressor (BC), the longitudinal phase space distribution of the beam particles within a thin slice reads

$$f_0(z, \delta_\gamma) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma} \exp\left[-\frac{(\delta_\gamma - h\gamma_0 z)^2}{2\sigma_\gamma^2}\right]. \quad (2)$$

Here we define  $z$  the longitudinal coordinate of a beam particle within the bunch, where  $z=0$  represents the beam center, and  $z>0$  is behind the beam center.  $I_0$  is the longitudinal beam current,  $\gamma_0$  is the relativistic central beam energy,  $\sigma_r$  is the initial uncorrelated energy spread,  $\delta_\gamma = \Delta\gamma(\gamma_0)^{-1}$  is the energy divergence of a particle,  $h = d\delta(\gamma dz)^{-1}$  is used for quantifying the beam energy chirp. After passing through the first short TGU with period length  $\lambda_u$ , period number  $N_u$ , transverse gradient  $\alpha$  and central undulator parameter  $K_0$ , electrons at different horizontal positions  $x$  will see different  $K$  values, where  $K(x) = K_0(1 + \alpha x)$ , which results in different path lengths and converts the longitudinal coordinate into

$$z_f = z + \tau x + \frac{\tau L_T}{2} x' - \frac{\tau^2 L_T}{6} \delta_\gamma, \quad (3)$$

where  $\gamma$  represents the energy of the particle and  $L_T = N_u \lambda_u$  is the length of TGU. Defining  $\tau = L_T K_0^2 \alpha (2\gamma^2)^{-1}$  the gradient parameter of TGU for particle energy  $\gamma$ , one can easily see that  $\tau$  is essentially the  $R_{31}$  element in the transport matrix of TGU (1).

Without losing generality, a beam of Gaussian distribution in  $x$  and  $x'$  without correlation was employed in our study. After passing through TGU, the distribution of the beam particles within a longitudinal thin slices becomes

$$f_0(z, x, \delta_\gamma) = \frac{I_0}{(2\pi)^{3/2} \sigma_\gamma \sigma_x \sigma_{x'}} \times \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \exp\left(-\frac{x'^2}{2\sigma_{x'}^2}\right) \times \exp\left[-\frac{(\tilde{\delta}_\gamma - h\gamma_0 z - h\gamma_0 \tau x - h\gamma_0 \tau L_T x'/2)^2}{2\sigma_\gamma'^2}\right], \quad (4)$$

where  $x$  is the horizontal position of a beam particle and  $x=0$  is in the center of the beam. To simplify the problem,  $\tilde{\delta}_\gamma = \delta_\gamma(1 + \frac{h\gamma_0 \tau^2 L_T}{6})$  is defined. For a sufficiently thin beam slice, we make the assumption that all the particles within the slice have the same longitudinal coordinate  $z$ . It is found in equation (4) that the horizontally correlated energy spread is converted into longitudinally uncorrelated energy spread with the energy chirp  $h\tau$ , which increases the slice energy spread of the beam before compression. As a result, the gain of the microbunching instability during compression is reduced. For simplification, after the integration along the horizontal axis, we obtain the energy distribution at  $z=0$  without horizontal dependency

$$\begin{aligned} f_0(\tilde{\delta}_\gamma) &= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' f_0(z=0, x, \tilde{\delta}_\gamma) \\ &= \frac{I_0}{2\pi \sigma_\gamma \sigma_x \sigma_{x'}} \frac{\sigma_\gamma \sigma_x \sigma_{x'}}{2} \\ &\quad \times \sqrt{\frac{2\pi}{\sigma_\gamma'^2 + (h\tau\gamma_0 \sigma_x)^2 + (h\gamma_0 \sigma_{x'} \tau L_T/2)^2}} \\ &\quad \times \exp\left[-\frac{\tilde{\delta}_\gamma^2}{2[\sigma_\gamma'^2 + (h\tau\gamma_0 \sigma_x)^2 + (h\gamma_0 \sigma_{x'} \tau L_T/2)^2]}\right] \\ &\quad \times \text{erf}(X) \Big|_{x=-\infty}^{x=\infty} \\ &= \frac{I_0}{\sqrt{2\pi[\sigma_\gamma'^2 + (h\tau\gamma_0 \sigma_x)^2 + (h\gamma_0 \sigma_{x'} \tau L_T/2)^2]}} \\ &\quad \times \exp\left[-\frac{\tilde{\delta}_\gamma^2}{2[\sigma_\gamma'^2 + (h\tau\gamma_0 \sigma_x)^2 + (h\gamma_0 \sigma_{x'} \tau L_T/2)^2]}\right], \end{aligned} \quad (5)$$

where  $\sigma_\gamma' = \sqrt{\sigma_\gamma^2 + (h\tau\gamma_0 \sigma_x)^2 + (\frac{h\tau\gamma_0 \sigma_{x'} L_T}{2})^2}$ , which can be much larger than the original slice energy spread  $\sigma_\gamma$ .

Another important capability of TGU is to introduce the longitudinal mixing from the transverse spread. Following the same method in reference [1–3], based on equation (5) and including the contribution from the horizontal beam distribution, the final gain of the microbunching instability after the passage through the bunch compressor with TGU taken into account reads

$$\begin{aligned}
G_f &= G_0 \exp\left(-\frac{k^2 R_{56}^2 \sigma_y^2}{2\gamma_0^2}\right) \exp\left(-\frac{k^2 \tau^2 \sigma_x^2}{2}\right) \\
&\times \exp\left(-\frac{k^2 \tau^2 L_7^2 \sigma_x^2}{8}\right) \exp\left(-\frac{k^2 \tau^4 L_7^2 \sigma_y^2}{72\gamma_0^2}\right) \\
&\approx G_0 \exp\left(-\frac{k^2 R_{56}^2 \sigma_y^2}{2\gamma_0^2}\right) \exp\left[-\left(\frac{k^2 \tau^2 \sigma_x^2}{2} + \frac{k^2 \tau^2 L_7^2 \sigma_x^2}{8}\right)\right],
\end{aligned} \tag{6}$$

where  $G_0 = k|R_{56}|I_0 Z(k)(\gamma_0 I_A Z_0)^{-1}$ , with  $Z_0 = 377 \Omega$  the free-space impedance and  $Z(k)$  the longitudinal impedance of wavenumber  $k$ . In equation (9), the terms on the right hand side provides us the way for suppressing the microbunching instability. In the equation, one can see that the gain of the microbunching instability can be damped by the two factors introduced by the transverse-to-longitudinal coupling throughout the TGU: one is the additional slice energy spread in the presence of significant beam energy chirp, another is the smearing of the microbunches caused by the longitudinal mixing from the transverse spread.

The discussions above enlighten us to implement the TGUs into the LINAC for the suppression of the microbunching instability. The layout of the proposed lattice is shown in Figure 1. In the lattice, two TGUs with the opposite gradient parameters are symmetrically placed on both sides of the magnetic bunch compressor.

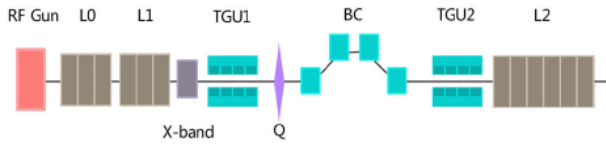


Figure 1: Lattice layout of TGU suppression scheme.

## SIMULATION STUDIES

In this section, we will show the proof of principle and possible performance of the proposed double-TGU technique for the suppression of microbunching instability illustrated in Figure 1. As usual, an X-band radio-frequency (RF) structure is employed before the first TGU to compensate the second-order nonlinear components in the longitudinal phase space to avoid the undesired growth of transverse emittance and energy spread. For simplicity purpose, we just look at one compression stage. L1 and L2 are two accelerating sections to provide and compensate the energy chirp to the beam before and after the bunch compressor. Without losing generality, the nominal beam and LINAC parameters were employed in the simulation and are shown in Table 1. Moreover, since the length scale in which the structural impedance is effective is much longer than that of microbunching wavelength, we may neglect the effects from the linac wakefields in the simulation without compromising accuracy.

Table 1: Main Beam and LINAC Parameters

Parameter	Value
Bunch charge (nC)	1.0
Beam energy before BC (MeV)	245
Bunch length (FWHM) before BC (ps)	7
Peak beam current before BC (A)	110
Slice beam energy spread (rms) before BC (keV)	3
Linac length up to BC (m)	17.3
$R_{56}$ of BC (mm)	-72
Beam compression ratio	9

The simulation starts right before TGU1 and ends after TGU2. the Gaussian beam in the six dimensional phase space is employed, and the density modulation are added on the beam profile before entering into TGU1 with peak-to-peak amplitude of 10%, and 50 microns in wavelength. The particle tracking code ELEGANT [4] is used to do the simulation in linac and a 3-D algorithm based on the fundamentals of electrodynamics is employed to do the simulation in TGU. The simulation starts before TGU1 where the beam energy is about 245 MeV, the peak current is about 110 A. As mentioned above, a variable-gap TGU (TGU1) with 10 periods of 10 cm period length,  $B_0 \approx 2.0 T$  and transverse gradient  $\alpha = 66 m^{-1}$  is adopted before compression. The electron beam is then compressed in the ratio of 9 in the magnetic bunch compressor. After that, another TGU (TGU2) with the same parameters as TGU1 except the opposite direction of the transverse gradient is used to recover the transverse emittance and the energy spread.

In Figure 2, it shows the longitudinal phase spaces of the central part of the beam at the exit of TGU2 with the energy chirp removed when TGUs are on and off. In the figure one can clearly see that the micro-structures are smeared out significantly with the scheme we proposed.

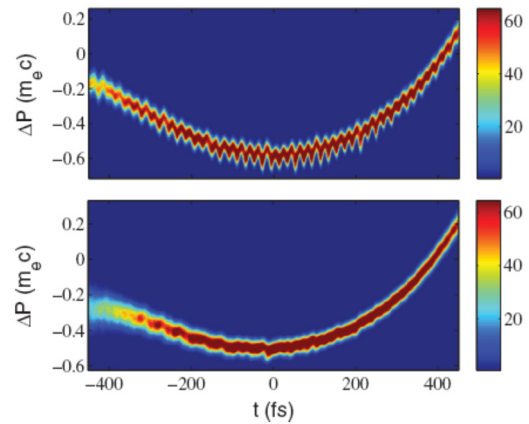


Figure 2: Longitudinal phase space (time-momentum) of the central part of the beam with the energy chirp removed at the exit of TGU2 with TGUs off (upper) and on (lower).

## CONCLUSIONS

In this paper a simple scheme for suppression of the microbunching instability by TGUs in the LINAC of a FEL facility was proposed and studies were carried out in details both analytically and numerically. In our discussion, the first-order transport matrix and the thin lens approximation of quadrupoles were employed which are valid in most of the cases. The theoretical analysis shows that the TGU was able to suppress the instability by two factors: the additional slice energy spread and the longitudinal mixing from the transverse spread without changing the direction of the beam propagation. By applying two TGUs symmetrically before and after the bunch compressor, the microbunching instability can be effectively suppressed by those two factors and the beam quality can also be restored very well after all by carefully choosing the TGU parameters. In the simulation, the typical parameters of a mid-energy electron LINAC was employed to demonstrate the feasibility and the efficiency of the TGU scheme, and the result indicates that the scheme we proposed is able to suppress the microbunching instability significantly with the well-preserved transverse emittance and no notable additional jitter and slice energy spread introduced. Moreover, because the TGU scheme does not need external RF power, laser and chicane systems, etc., it has the advantages over the other schemes in terms of high efficiency, less complexity and better jitter tolerance. As a novel method, the TGU scheme opens us a new way to improve the performance of the x-ray free-electron laser, and can be a good candidate for the microbunching instability control in addition to a laser heater.

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## REFERENCES

- [1] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov, "An analytical description of longitudinal phase space distortions in magnetic bunch compressors", *Nucl. Instrum. Methods Phys. Res., Sect. A* **483**, 516 (2002).
- [2] Zhirong Huang and Kwang-Je Kim, "Formulas for coherent synchrotron radiation microbunching in a bunch compressor chicane", *Phys. Rev. ST. Accel. Beams* **5**, 074401 (2002).
- [3] J. Qiang, C. E. Mitchell, and M. Venturini, "Suppression of Microbunching Instability Using Bending Magnets in Free-Electron-Laser Linacs", *Phys. Rev. Lett* **111**, 054801 (2013).
- [4] Y. Wang and M. Borland, Pelegant: "A Parallel Accelerator Simulation Code for Electron Generation and Tracking", *AIP Conf. Proc.* **877**, 241 (2006).