# ASYMMETRIC FOUR-VANE RFQ*

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*Abstract*

A four-vane resonator is widely used in Radio Frequency Quadrupole (RFQ) accelerators. The field distribution in a long four-vane resonator can be easily perturbed by nearest dipole modes which are excited due to the local geometry errors. This paper describes the electromagnetic properties of a four-vane resonator with an introduced asymmetry between neighboring chambers. The asymmetry provides necessary separation of dipole modes keeping losses and field uniformity of the quadrupole mode similar to those in a conventional four-vane resonator. This feature of an asymmetric resonator is confirmed by analytical results from transmission line model as well as by CST Studio simulations.

## 4-VANE RESONATOR

A 4-vane resonator was the first radio frequency (RF) structure proposed for an RFQ. This structure is presented in the description of the RFQ invention and in the first publication [1]. The 4-vane resonator is a suitable choice for RFQs used for acceleration of high-intensity beams and for continuous wave accelerators due to the highest RF power efficiency and the most precise distribution of the quadrupole focusing electric field.

Most of RFQ resonators are long regular structures of several operating wavelengths. The properties of the resonators are defined by the properties of a multi-mode transmission line with proper boundary conditions. Both a lumped-circuit model [2] and a 3D computer simulation of a 4-vane resonator show the presence of three propagation modes. These propagation modes (an operating quadrupole mode $TE_{21}$ and two dipole modes $TE_{11}$) have close critical frequencies, and the main drawback of the conventional 4-vane resonator is that the frequencies of the operating quadrupole mode and dipole modes are close to each other. Usually, the frequency of $TE_{110}$ modes are getting even close to $TE_{210}$ due to the field equalizing undercuts at the ends of the electrodes.

This rapprochement makes the field distribution very sensitive to machining and alignment errors and causes the redistribution of RF power between the operating quadrupole $TE_{210}$ and parasitic $TE_{11n}$ modes. It leads to disturbance of voltage distribution in each chamber of the resonator and unbalance of electrode potentials over its cross-sections.

The separation of long 4-vane structures into short resonantly coupled sections decreases the disturbance effect caused by higher quadrupole $TE_{21n}$ modes [3], but not by the nearest dipole $TE_{11n}$ field. Also the resonant coupling of several 4-vane sections makes the mechanical design of an RFQ linac more complicated.

The ref. [4] explains the need for dipole-free frequency region around the operating frequency in order to provide stable operation of the resonator. This gap is defined as the difference between the operating frequency and the frequency of the closest dipole mode. The gap should be as wide as several percent of the operating frequency. In order to shift out dipole modes' frequencies suppressor loops [4] and stabilizing rods at the ends of the 4-vane resonator [5] are used. These methods allow changing of the boundary conditions at the end of the resonator differently for the dipole and quadrupole modes. It has also been argued that the installation of rods decreases the dipole perturbations considerably.

## ASYMMETRIC 4-VANE RESONATOR

Let us consider a conventional cloverleaf 4-vane resonator with cylindrical chambers of radius $R_{ch}$ and quadrupole tips, which is studied in ref. [2]. The spectrum of the resonator (frequencies of quadrupole and dipole modes) can be managed if one makes the equivalent inductances of the adjacent chambers not equal. This could be realized by using different radii for adjacent chambers, while keeping the radii of opposite chambers the same (see Fig. 1). Introduce a coefficient of asymmetry $k_{asym} > 0$, so the radii of the pair of large resonator chambers are $R_{ch1} = R_{ch} \cdot (1+k_{asym})$. The size of the another pair of chambers should be modified in such a way to keep the quadrupole mode frequency unaffected. Assuming that chamber specific inductance is linearly proportional to the cross-section area of a chamber, we get the radii of the small chambers:

$$R_{ch2} = R_{ch}(1+k_{asym}) / \sqrt{1+4k_{asym}+2(k_{asym})^2} \, . \qquad (1)$$

Figure 1 presents cross-sections of the resonators with $k_{asym} = 0$ and $k_{asym} = 0.30$. Computer simulation of the test resonator in the range $0.0 < k_{asym} < 0.30$ has been performed with CST Studio Suite [6]. The frequencies of the dipole modes (black and blue curves) and the quadrupole mode (green line) are shown in Fig. 2. The quadrupole mode is kept at the frequency of 350 MHz in the whole range of $k_{asym}$. The calculation was done for the test resonator with the following parameters: average radius $R_0 = 4$ mm, width of quadrupole tips $2R_e = 6.4$ mm, length of the vanes $L_{vane} = 3000$ mm. All results presented in the paper correspond to this test resonator.

It should be mentioned, that the asymmetry coefficient is not limited by 0.30 as shown in Fig. 2. For the asymptotic value $k_{asym} \to \infty$, we get $R_{ch2} / R_{ch} \to \sqrt{1/2}$ and the 4-vane resonator is transformed into the well-known 2H-resonator [7].

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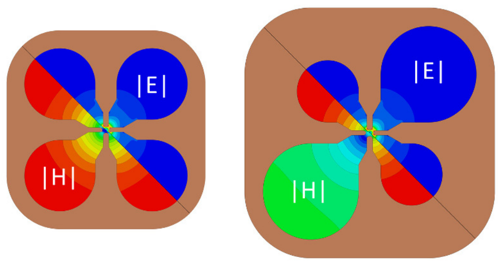


Figure 1: Cross-sections of the cloverleaf 4-vane resonator: symmetric and asymmetric ($k_{asym}$ = 0.30). Color represents the field magnitude.

One can see from Fig. 2, that the asymmetry of the chambers displaces two fundamental dipole modes differently. It can provide significant separation of these modes from the quadrupole one. At the same time simulation results show that the quality factor of the quadrupole mode doesn't change by more than 3% in a moderate range of asymmetries.

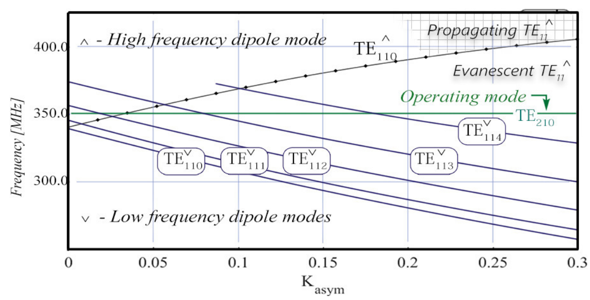


Figure 2: Resonant frequencies in the asymmetric 4-vane resonator.

## FIELD PERTURBATIONS

Small perturbations in the three-mode resonator model can be studied using the lumped circuit presented in Fig. 3.

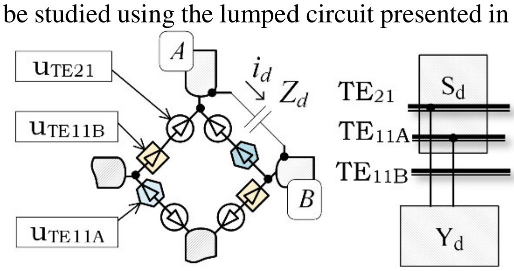


Figure 3: An equivalent lumped circuit of a perturbed part of the resonator and a correspondent node in the transmission line model.

Assume we have locally displaced tips of two adjacent quadrupole electrodes A and B (see Fig 3). Electrodes' displacement toward each other causes change of capacitance between the tips by a value $Z_d$. The 100 mm long section with 50 μm displacement of tips changes the capacitance by 0.04 pF or by 1.2% from the initial value. At the operating frequency this displacement is equivalent to the introduction of a reactive lumped element with impedance $\mathrm{Im}(Z_d)$ = -11.6 kΩ (see Fig. 3).

In general, the extended perturbed part of the resonator is described by a 6x6 scattering matrix [$Sd$]. But in our case of a short perturbed area, a conductance matrix [$Y$] with reduced order can be used. This matrix is clearly defined by elements of equivalent lumped circuit. For the case of Fig. 3 the matrix looks like:

$$[Y] = \frac{1}{Z_d}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

In addition to the reduced order of the matrix, due to the perturbation symmetry, coefficients $Y_{3,k}$ and $Y_{k,3}$ are equal to zero and the second dipole mode, which corresponds to these coefficients, doesn't take part in the redistribution of the RF energy. This matrix gives us the amplitude of the first dipole mode defined as:

$$U_2 = \frac{Z_m}{Z_m + Z_d} U_1, \tag{3}$$

where $Z_m$ – is the impedance of the dipole mode at the point of perturbation. Expression (3) shows that in order to reduce the amplitudes of dipole modes induced by random local geometry errors, the impedance $Z_m$ should be kept small along the whole resonator. From the transmission line theory, the value of $Z_m$ depends on the longitudinal coordinate $z$ as:

$$Z_m = \frac{Z_0\left(\Gamma + e^{2jk_z z}\right)\left(\Gamma + e^{2jk_z(L-z)}\right)}{2\left(-\Gamma^2 + e^{2jk_z L}\right)}, \tag{4}$$

where $L$ – resonator length, Γ – reflection coefficient at the ends of the resonator, $k_z = \mathrm{Im}(\gamma)$ – imaginary part of a complex propagation constant, $Z_0$ – impedance of the equivalent transmission line.

The numerator of formula (4) represents a periodical behaviour of impedance for propagating mode. With the denominator one could control the maximum of the local impedance along the resonator. When the value of the denominator approaches zero the maximum of the local impedance is going to infinity in a lossless system. The minima correspond to the denominator:

$$\left|-\Gamma^2 + e^{2jk_z z}\right| = \max = 2, \tag{5}$$

that requires to satisfy the condition $k_z L = n\pi$ for the electrical length of the resonator with ideal open boundary conditions. One can derive a minimum value of $|Z_m|$, which is equal to a half of the transmission line impedance $Z_m = Z_o/2$. The same value of $|Z_m|$ for non-propagating mode comes from (4) in the case of $Re(k_z) = 0$ and $Im(k_z) \neq 0$.

Figure 4 presents the distribution of $|Z_m|$ along the resonator in three most important cases: Curve 1 corresponds to resonance $TE_{114}$ ($k_{asym} \approx 0.16$), curve 2 corresponds to "safe resonator design" ($k_{asym} \approx 0.23$), curve 3 represents impedance of non-propagating dipole mode.

Due to splitting of frequencies of dipole modes, an extra free parameter – length of propagation for $TE_{11}$(1) mode appeares (see Fig. 2). This parameter can be used to attenuate the undesirable $TE_{11}$ mode. The behaviour of the propagation constant of dipole modes for the test resonator is shown in Fig. 5.


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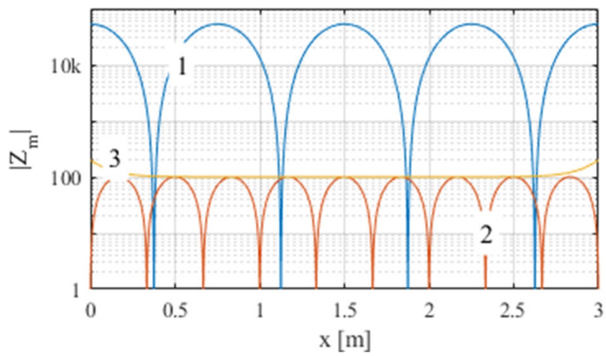


Figure 4: Impedance of the dipole modes along the resonator.

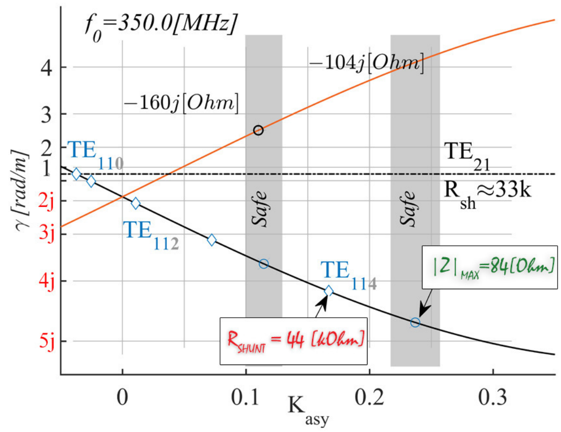


Figure 5: Propagation constants for different modes of asymmetric 4-vane resonator.

One can see, that both dipole modes are close to mode $TE_{112}$ at $k_{asym} = 0$. The asymmetry of resonator chambers shifts one of the dipole modes (red curve) from the propagation state into the non-propagation one. This mode goes through the $TE_{111}$ resonance, after that the impedance of this mode decreases and the propagation constant increases. At $k_{asym} > 0.08$ this mode doesn't affect the field distribution in the resonator any more.

Another dipole mode consequently goes through resonances $TE_{112}$, $TE_{113}$, etc., producing enough wide stopbands to provide safe operation of the resonator.

Figure 6 shows results of 3D simulation of field distribution distortions for operating quadrupole mode caused by local perturbation of the gap between adjacent quadrupole tips in the test resonator. The decreased gap is located in the middle of one of the large chambers. The solid red curves correspond to field distribution in the large chamber with reduced gap. The dashed red curves correspond to the field distribution in the opposite chamber. The blue curves correspond to the small chambers of the test resonator. The value of $k_{asym} = 0.125$ approximately matches the propagation constant of the lower dipole mode $\gamma \approx$ -3.5j, $k_{asym}$ = 0.30 – to $\gamma \approx$ -4.5j. One can see, that the local perturbation in a conventional symmetric 4-vane resonator induces significant field distortion in the perturbed chamber and in the opposite one. The maximum field error in this case is $\Delta E / E_0 \approx 17\%$.

The introduction of asymmetry between adjacent chambers with $k_{asym} = 0.125$ suppresses $\Delta E / E_0$ by a factor of two, and with $k_{asym} = 0.30$ suppresses $\Delta E / E_0$ to less than 4%.

Computer simulation has shown, that the same local perturbation in a gap between tips of the small chamber at $k_{asym} = 0.125$ or $k_{asym} = 0.26$ doesn't result in any visible distortions of the field distribution.

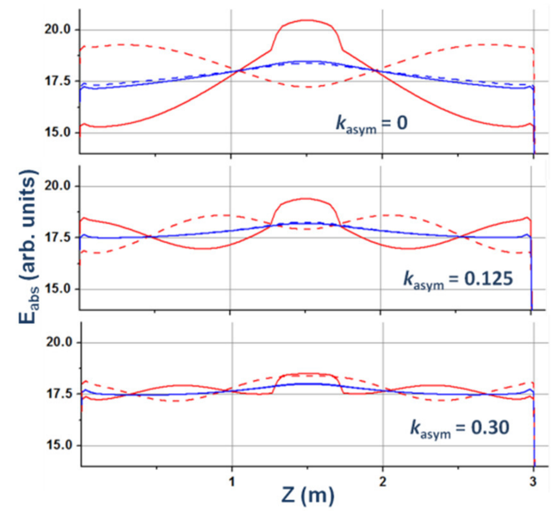


Figure 6: Perturbed field distribution in asymmetric 4-vane resonator.

## CONCLUSION

The presented results show that introduced asymmetry between adjacent chambers of a 4-vane resonator leads to a significant reduction in sensitivity of the field homogeneity to manufacturing and alignment errors. The resonator asymmetry just slightly increases the RF power losses compared to a conventional 4-vane resonator with similar parameters.

In this paper we discussed the 350 MHz resonator with cylindrical chambers. However, the proposed method of modes' splitting can be applied to any chamber shape and operating frequency. It is even more efficient for long resonators where the field stability problem has a higher importance.

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