

STABILIZATION METHODS FOR FORCE ACTUATORS AND FLEXURE HINGES

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Abstract

In the framework of the design of an adaptive optics x-ray mirror bender a stabilization system for ultra-low spring-constant force springs actuators and flexure hinges with nanometre resolution has been conceived. For this purpose its force resolution is better than 10^{-2} N. Since the actuator must preserve the force at this level and provide both pulling or pushing force the spring force was initially transmitted by a lever arm which was pivoting about a bearing articulation but its friction is up to the level of the required force. We present a novel method to compensate the stiffness of the corrector spring as well as a stabilized flexure, substituting the bearing, allowing to obtain a system able to introduce a variable force on a which its force insensible to movements of the application point in a range of about 1 mm and a frictionless, torque-free articulation in a range of a 1° . The method is based on a combination of magnets with the flexure, in a way that the elastic force exerted by the flexure is compensated by the force of the magnet. Preliminary results show that it is possible to stabilize the torque exerted by the flexure below 0.005 N·m, in a range of 1 degree.

INTRODUCTION

The Nanobender [1,2], Fig. 1, is an instrument conceived to bend X-Ray mirrors to the required focusing curvature and also correct the optics surface errors by means a force actuator. The role of the actuator is to apply a force to introduce a controlled deformation of the mirror substrate that compensates the surface errors of the mirror.

From the deformation model, it turns that to allow compensating surface errors below one nanometer, the forces have to be tunable in a range of up to 20 N or more, and with a resolution of 10^{-2} N or better. To preserve the induced deformation, and thus the corrected figure, the forces applied by the corrector must be stable also within that range.

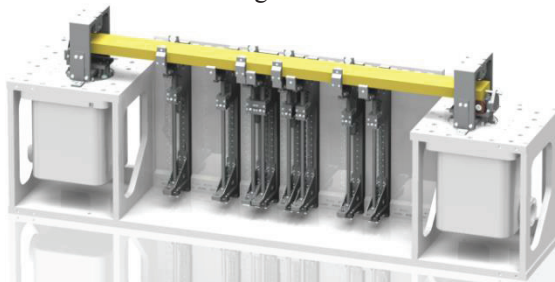


Figure 1: Nanobender.

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For a properly designed actuator, free of friction and parasitic forces, the main source of potential instability comes from mirror transport requirements. Since the mirror has to be moved from the metrology bench to the beamline, or even to a different laboratory, mirror systems must be prepared for transport.

In order to transport safely the mirror, the actuator contact to the mirror must be released. Considering that actuators are applying stress to the mirror, a shock on the transport crate could provoke that some actuator exceeded transitorily the fracture limit of the mirror. Releasing the contact of the actuator from the mirror implies that the repeatability of their relative position is limited to a few tens of microns.

Moreover the original corrector includes and articulation solved by a deep groove ball bearing, that although it has very low friction behaviour could be completely solved by means an also stabilized flexure applying the same compensating principle as for the force mechanism.

The actuator we propose incorporates a mechanism that preserve the force exerted by the actuator for small changes of such position. The working principle of such mechanism is described in this paper.

The paper is divided in two main sections: First the force compensating mechanism for the force actuation and second the stabilized flexure for the articulation.

STABLE-TUNABLE FORCE ACTUATOR

Description

The actuator is sketched in Fig. 2. This corrector have been patented [3]. In this case a pushing actuator is considered. The mechanism can be divided in the four following parts:

1. The frame of the actuator, which is the part of the actuator that supports the other parts of the mechanism, and is mounted on the main frame of the bender.
2. The transmission chain, which includes the lever arm and the contact to the mirror, is the part that transmits the forces exerted by the spring and the magnets, which have different application points, onto a single contact point to the mirror. The transmission also has articulations that absorb all the parasitic components of the forces, which would affect the performance of the system. Although the transmission chain is articulated, it is in practice static, because the reaction force induced

by the mirror equilibrates the forces exerted by actuator components.

3. The spring and its adjustment system, which allows regulating the force exerted by the actuator. One end of the spring is connected to the lever arm, where it applies a traction force. The other end of the spring is connected to the nut of a lead screw., driven by a stepper motor. Since the lever arm position does not change. The elongation of the spring can be adjusted by changing the position of the nut; this is, by turning the lead screw. This allows, therefore, adjusting the force exerted by the actuator. The spring adjustment system also includes fixations, guides and articulations to minimize friction and parasitic components of the force.
4. The magnet-puller and its adjustment system. It consists in a couple of magnets, set up in attraction, separated by a small gap each other. One of the magnets is attached to the lever arm, and the other magnet attached to the frame, by a fixation system that allows for adjusting its position. This allows adjusting the gap, and correspondingly, the pulling force this part of the system applies to the lever arm.

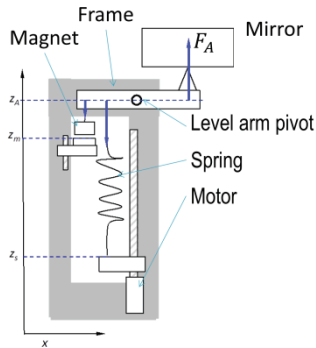


Figure 2: Layout of papers.

Both the magnetic structure and the spring mechanism are pulling the lever arm, and the total force is the contribution of the two mechanisms. On the other hand, if one considers a variation of the position of the mirror, for instance assuming it moves up, the spring will be less stretched, and therefore will exert a smaller force. The gap in the magnet structure will also decrease but in this case, the force will increase. By properly choosing the spring constant and the magnet strength it is possible to find a range where both contributions compensate each other and the total force is independent of the position of the mirror (see Fig 3). This is analysed in detail in the following section.

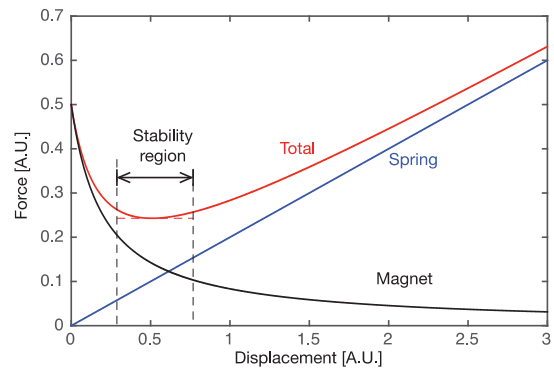


Figure 3: Illustration of the stabilization principle. The spring force has a linear dependence on the mirror displacement, while the magnetic force is given by an inverse law. The sum of the two forces has a minimum around which the force is independent of the displacement.

Analysis

The force applied by the lever arm on the mirror (F_A) is given by the following expression:

$$F_A = \frac{r_S F_S + r_m F_m}{r_A} \quad (1)$$

Here, F_S and F_m are the forces made by the spring system and the magnet system respectively. r_S , r_m , r_A are the distances of the application points on these forces to the pivot point. For simplicity, we have considered that all the forces are parallel each other (vertical) and orthogonal to the lever arm, and that the application points are aligned at the horizontal line defined by the pivot point. Figure 2 provides the sign convention considered in the equation, since both r_S and F_S are negative, their product is positive, and since r_A is positive, the resulting force contribution of the spring to the mirror is positive. The same argument applies to the magnet system.

The force exerted by the spring is proportional to the elongation of the spring, it can be written as follows.

$$F_S = -k_S(z_A - z_S - L_S) \quad (2)$$

Here L_S is the length at rest of the spring, counted from the attachment to the lever arm, to the attachment to the nut

The force made by the magnet system is inversely proportional to some power of the gap between the magnet and the metal block (d). The precise exponent depends on the geometry of the magnet. However for a diameter much larger than the gap between magnet and metal block, the exponent is close to one. Therefore the force can be written as.

$$F_m = -\frac{F_{max}}{d/d_m + 1} \quad (3)$$

F_{\max} and d_m are constants that parameterize the force dependence with distance.

F_{\max} is the maximum force made by the magnet (for gap zero), and d_m is the gap for which the force has decreased to one half of the maximum. The expression is valid only for positive gaps, In that case F_m is negative according to the sign convention given in Figure 2.

An example of values for F_{\max} and d_m are 8 N, and 0.5 mm respectively, which correspond to disk-shaped magnets, with diameter 15 mm, height 1 mm, and magnetization N35.

Spring-like Description of the Magnet

In order to provide a comprehensive description of the stabilization mechanism, it is worth expressing the magnetic force in terms of a spring constant. This can be done in a small region by making use of the Taylor expansion of the magnetic force, as follows:

$$F_m = -\frac{F_{\max}}{d_0/d_m+1} + \frac{F_{\max}}{d_m(d_0/d_m+1)^2}(d - d_0) \quad (4)$$

This can be expressed more compactly as:

$$F_m = F_{m0} - k_m(d - d_0) \quad (5)$$

Where:

$$F_{m0}(d_0) = -\frac{F_{\max}}{d_0/d_m+1} \quad (6)$$

and

$$k_m(d_0) = -\frac{F_{\max}}{d_m} \frac{1}{(d_0/d_m+1)^2} \quad (7)$$

k_m has units of spring constant, and unlike elastic systems, it takes negatives values. It can be adjusted by properly selected the point d_0 , between $-\frac{F_{\max}}{d_m}$ and zero.

Force Stability

By combining expressions (1), (2) and (5), one can write the total force as:

$$F_A = -\frac{r_S}{r_A} k_S(z_A - z_S - L_S) + \frac{r_m}{r_A} F_{m0}(d_0) - \frac{r_m}{r_A} k_m(d_0)(z_A - z_m - L_m - d_0) \quad (8)$$

Here we have expressed the gap of the magnetic structure as a function of the coordinates of its ends, as given in Figure 3.1. One can re-arrange the terms as follows:

$$F_A = -\frac{r_S k_S + r_m k_m}{r_A} z_A + \frac{r_S}{r_A} k_S(z_S + L_S) + \frac{r_m}{r_A} F_{m0} + \frac{r_m}{r_A} k_m(z_m + L_m + d_0) \quad (9)$$

The expression has four main terms, which express the dependence on z_A , z_S and z_m , one can see that the dependence on z_A vanishes for:

$$r_S k_S + r_m k_m = 0 \quad (10)$$

When this condition holds, the force exerted by the actuator does not depend on the position of the actuation point. This is equivalent to a system with null stiffness.

By replacing k_m by its explicit expression one can find the magnetic gap that produces such condition.

$$d_0 = +\sqrt{\frac{r_m F_{\max} d_m}{r_S k_S}} - d_m \quad (11)$$

There is solution only for positive values of d_0 , this is:

$$\frac{F_{\max}}{d_m} > \frac{r_S}{r_m} k_S \quad (12)$$

This condition means, essentially, that the maximum equivalent stiffness of the magnet structure must be as big as the stiffness of the spring.

The gap between magnets can be adjusted by adjusting the position of the magnet attached to the frame, this is z_m . The system includes a dedicated mechanism for this adjustment. Tuning z_m affects the total force applied by the actuator.

Force Tuning

Once the magnetic gap is adjusted to cancel the dependence of force on z_A , one can tune the total force of the actuator by adjusting the spring elongation. This is done by moving the lower end of the spring (z_S). Note that adjusting z_S does not change the equilibrium between the spring constants of the spring system and of the magnetic system, so the force stability is preserved for the full range of forces available for the actuator.

Range of Stabilization

The spring-like approximation of the magnet structure is valid within a small region around d_0 , the error of the approximation is dominated by the quadratic term of the corresponding Taylor series.

$$\delta F_m = \frac{F_{\max}}{d_m^2 (d_0/d_m+1)^3} (d - d_0)^2 \quad (13)$$

Which after some manipulations can be written as:

$$\delta F_m = F_m(d_0) \frac{k_m(d_0)}{k_{m,max}} \left(\frac{d-d_0}{d_m}\right)^2 \quad (14)$$

So the error increases more rapidly when the magnet force is near its maximum, and when the maximum spring constant is constant.

The range of validity, for a given threshold is:

$$\delta z_A = d_m \sqrt{\frac{k_{m,max}}{k_m(d_0)}} \sqrt{\frac{\delta F_m}{F_m(d_0)}} \quad (15)$$

The first factor is the characteristic length of the magnets, the longer it is, the smoother the decay of the field. The second term is the ratio of the spring constant to the maximum achievable by the magnets. It indicates that to have a wide stabilization range, the spring should be weak in comparison to the magnet. Finally, the third term is just the ratio between the force tolerance and the force contribution of the spring.

Summarizing, one needs strong magnets and use their weak field part, in order to have a large stability range.

Experimental Set Up

Figure 4 shows a picture of the experimental setup done to test the working principle of the force correction. The setup is shown in Figure 4.1. In this setup, the force at the application point is measured by a high resolution load cell, capable of measuring a maximum force of 50 N, and with resolution better than 10-3 N. Changes in the position of the application point are simulated by steering a motorized linear stage. Rigidly connected to the load cell. The load cell is connected to the transmission, which collects the force exerted by the spring and by the magnet system. The gap between the magnets is adjusted by a manual linear stage, while the elongation of the spring is actuated by a motorized linear stage.

In order to test the stability provided by the system, we scan the position of the force application point by scanning the linear stage on the top, and record the force measured for the load cell for each position. The same experiment is repeated for different elongations of the spring, which are adjusted by steering the linear stage in the bottom in figure 4.

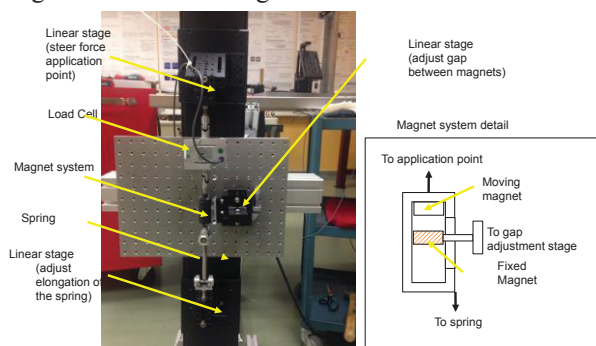


Figure 4: Picture of the experimental setup. A detail of the transmission system is given in the box at the right.

The corresponding results are given in the plot in in Fig. 5. There the deviation from the nominal force is represented as a function of the position of the application point. One can see that the deviation presents a minimum, and that in a range about 2.3 mm the force drift is below 0.02 N. Note also that the position of such minimum does not depend on the nominal force. This means that once the magnet gap is optimized, the

nominal force can be tuned by changing the elongation of the spring, while preserving the null-stiffness position.

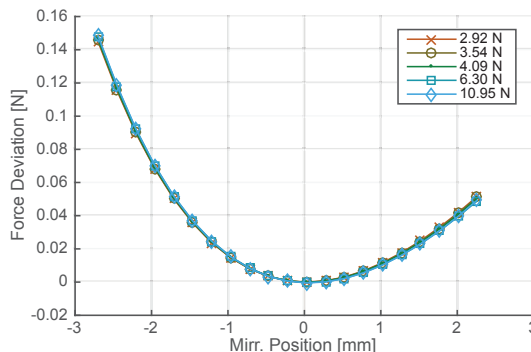


Figure 5: Force deviation from the nominal force as a function of the position of the force actuation point. Curves for different nominal forces, between 2.92N and 10.95 N, are represented. They all show a similar behaviour.

REDUCED RANGE, ZERO TORQUE & FRICTIONLESS ARTICULATION

Description

Flexures give a high performing solution when it is needed to achieve a short range and precise rotation as it give full rigidity in all axis, by its massive geometry, les in the movement one. Also thanks to its simplicity, and avoiding rolling elements as compared with bearings for instance, their thickness compared with the narrowing minimizes the rotation axis wobbling and they are naturally frictionless. Nevertheless they introduce a spring-like rotational torque which in some high accurate application may become a drawback. This torque is governed by the flexion theory equations on the narrowing cross section but as a flexure by definition is a reduced range articulation around its nominal stable position, or zero, thus could be simplified as a torsional spring, with a torque, angle dependent, of a constant rotational K value and of the turned angle.

Conceptual Design

The stabilized flexure is sketched in the Fig. 6. It has been patented [4]. It consists in a basic flexure made in four independent parts to facilitate the manufacturing as well as adjusting its rigidity. In this case the system is stabilized with two pairs of magnets but can be done as well with a single pair. The difference is that with two pair the flexure could be stabilized at its nominal free position or nominal zero.

The articulation has a fixed part or a base which hold two of the magnets and the fixed region of the flexures. It includes as well a rotating part which includes other two magnets and the bendable zone of the flexures. This flexure articulation is split into two different parts placed at each side, extreme, of the rotation axis. In

order to make it work the four magnet are paired and in attraction.

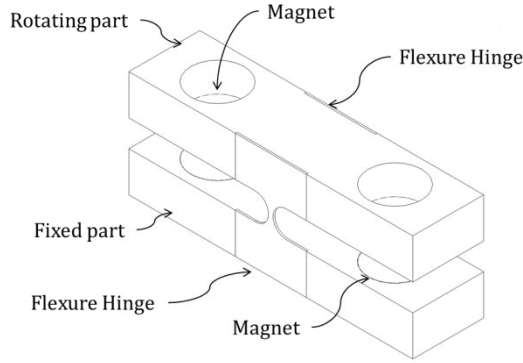


Figure 6: Stabilized flexure.

The articulation is dimensioned in such a way that the rigidity of the flexure is twice the equivalent magnet “equivalent” rigidity, which includes two set of them, but magnets, in attraction, could have a negative k value which could never be in the case of springs or spring-like system like a flexure. The flexure rigidity and the magnet would have the same rigidity, instead of half, in the case of designing it with a single set of magnets. The two magnetic arrangements they are trying to move the flexure from its free, or nominal zero position, but it preserves its free position as both magnets pairs are in opposite side of the flexure thus cancelling them the total resultant torque.

If the rotating part is twisted by means and external force or torque then the flexure is stressed increasing its resistance to the rotation with a returning torque. The gap in the two magnetic arrangements is increasing in one side and decreasing in the other, this make the attraction force is increasing in one arrangement and is decreasing at the other which make that the total flexure torque increase is compensated by the magnets torque variation thus no torque variation of the whole system. By properly choosing the spring constant, flexure spring constant, and the magnet strength, it is possible to find a range where both torques variations compensates each other and the total torque remains zero as at the free or nominal position. This means in this zero torque range the flexure will remain at the target angle from the zero.

Analysis

The total torque $T(\theta)$ introduced on the articulation is given by the following expression and plotted in the Fig. 7:

$$T(\theta) = M_{F_{m1}} - M_{\theta} + M_{F_{m2}} \quad (16)$$

Where $M_{F_{m1}}$ is the torque introduced by the left side flexure magnet; M_{θ} is the torque of the flexure and $M_{F_{m2}}$ is the torque of the right flexure magnet.

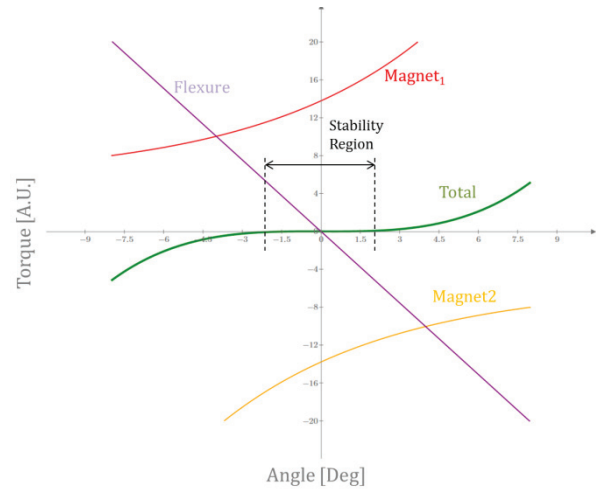


Figure 7: Graph of the stabilization principle. The flexure torque has a linear dependence while the magnetic law is given by an inverse law. The sum of the three torques is zero around its nominal free zero position and it is preserved zero in an angle range around this point where the torque is independent of the angle.

The torque exerted by the flexure is proportional to the rotation angle, and it can be written as follows:

$$M_{\theta} = k_{\theta}\theta \quad (17)$$

The magnet forces are although they have an angle interdependence they work in opposite sense, thus they are expressed like:

$$M_{F_{m1}} = F_{m1}D \quad (18)$$

and

$$M_{F_{m2}} = -F_{m2}D \quad (19)$$

Previous equation can be figure out from the system lay-out shown in next Fig. 8.

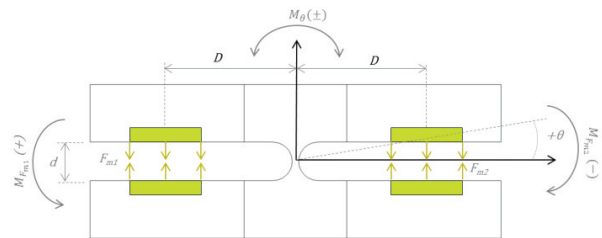


Figure 8: Sketch of the system dynamics.

In order to have the system equation dependent of a single variable, the angle θ , the magnet gap distance, and thus the magnet force equation, must be expressed in terms of this variable, thus the gap can be expressed as:

$$d_1 = d_0 - D \sin\theta \quad (20)$$

and

$$d_2 = d_0 + D \sin \theta \quad (21)$$

Where d_0 is the initial setting of the magnet gap where it achieve the required rigidity to make the system self-compensate the torques variation with the angle as described below:

Following equation 3 as the force actuator for the flexure F_{max} is the maximum force made by the magnet (for gap zero), and d_m is the gap for which the force has decreased to one half of the maximum. The expression is valid only for positive gaps, In that case F_m is Positive according to the sign convention given in Figure 8.

An example of values for F_{max} and d_m are 14,2 N, and 0.9mm respectively, which correspond to disk-shaped magnets, with diameter 8 mm, height 3 mm, and magnetization N45.

Each magnet have an opposite behavior, thus the system have two different F_m forces working on, F_{m1} and F_{m2} , which only have the same value at the nominal zero. An example of stabilized flexure with two flexures of 0,5 mm Aluminum sheet and Ø5 mm circles at the flexible point gives the following stabilization graph, Fig. 10:

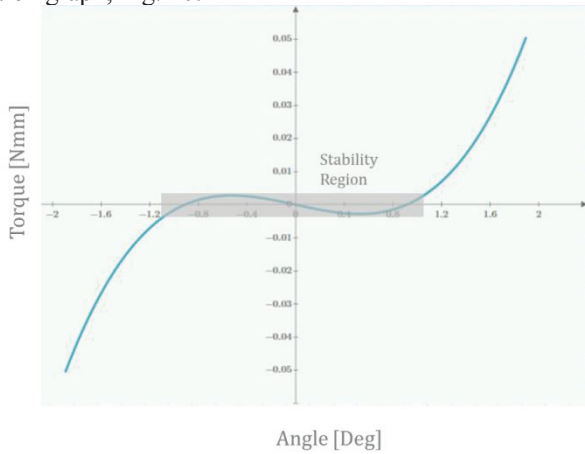


Figure 9: Calculation for specific flexure.

Torque Variation Compensation

The same spring-like mathematical expression of a magnet developed in previous section is applicable for the compensated flexure. By combining the expressions (16), (18), (19), (20), (21) and (5) the final torque can be expressed like:

$$T(\theta) = -2 k_m D^2 \sin \theta - k_\theta \theta \quad (22)$$

The expression has only two terms. k_m must take negative values to make the system work as will be demonstrated below.

The principle that makes the system have a self-compensated torque is that from a stable position, the nominal zero, whatever rotation in the working range

introduces a self-compensated Torque Variation which variation is zero thus achieving a new stable position in whatever position in the working range. Torque variation means, and seeing figure 3, just the derivative of this equation 12:

$$\frac{\partial T(\theta)}{\partial \theta} = -2 k_m D^2 \cos \theta - k_\theta \quad (23)$$

This derivate, equation is zero when there is a negligible variation of the torque, thus $\frac{\partial T(\theta)}{\partial \theta} = 0$ and given that $\cos \theta = 1$ for θ values around zero gives:

$$k_m = \frac{k_\theta}{-2D^2} \quad (24)$$

This equation means that the magnets stiffness must be as rigid as the flexure. The negative sign just shows that k_m could take negative values. The constant two just shows that this have two magnet set, if the system is set up with one magnet this constant 2 would not appear. As expected the distance of the magnet to the flexure center, D , plays a role in the rigidity as the force of the magnet by this distances is the torque exerted by the magnet sets, this means the stabilization could be adjusted by means d_0 & D .

The gap between magnets can be adjusted by adjusting the position of the magnet attached to the frame, this is d_0 . Tuning d_0 affects the total force applied by the actuator.

It is possible to arrive to the same conclusion by replacing k_m by its explicit expression at the equation 12 one can find the magnetic gap that produces such condition:

$$d_m \sqrt{\frac{F_{max} - 2 D^2 \cos \theta}{k_\theta}} - d_m = d_0 \quad (25)$$

There is solution only for positive values of d_0 , this is:

$$\frac{F_{max}}{d_m} > \frac{k_\theta}{-2 D^2} \quad (26)$$

CONCLUSIONS

With this actuator it is clear that a set up could be designed to introduce a controlled adjustable force and making it insensible to displacements of the force application force, thus a system which allows an accurate adjustment in a optics laboratory and then dismantle the mirror and remount it on site achieving the same correction fine tuning forces with in independence of the mechanical mounting errors.

The compensated flexure is able to stabilize a flexure in such a way that the linear torque introduced by the flexions of the hinge is completely cancelled in a articulation type, flexure, which it is intrinsically frictionless and thus obtaining the almost perfect articulation for a reduced range of about 1°. A final set up with the

flexure have to be mounted in the next steps to evaluate the proximity of a real compensated articulation to zero torque and its range bandwidth.

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