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A NOTE OF THERMAL ANALYSIS IN SYNCHROTRON RADIATION ACCELERATOR ENGINEERING

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Abstract

Thermal and thermomechanical analysis is one of the key process while designing accelerator components that may subject to synchrotron radiation heating. Even some closed-form solutions are available, and yet as to complex geometry numerical analysis such as finite element method (FEM) is commonly used to obtain the result. However due to its complexity of density distribution of the heat load, implementing such boundary conditions in the finite element method (FEM) model is relatively tedious.

In this report we provide a simplified, practical and more conservative method to apply heat load both for bending magnet and insertion device. In addition, a general purpose synchrotron radiation heating numerical modelling is also introduced, and a simple FEM model with EPU power heat load is also compared

INTRODUCTIONS

As to synchrotron accelerator radiation heat load issue, typically when it comes to analysing the designs of crotch absorber, fixed masks, photon stoppers…, etc., one finds that bending magnet (BM) and insertion device (ID) are two major heat sources. The synchrotron radiation (SR) is primary in Gaussian distribution in 1D or 2D. Unless one can compile embedded programming in the FEM tool (such as ANSYS), manually applying such non-constant power density on the nodes in the FEM model is a tedious work. On the other hand, the total heat flux input may be underestimated due to human error.

Due to high speed computer capability nowadays, we find that FEM modelling for this type of analysis, during material assignment element meshing as well as solving process are fairly straightforward. Instead, applying synchrotron radiation heat load distribution is the most time-consuming task among the entire modelling process. To speed up this process, we introduce two methodologies, a simplified and a realistic model for the analysis.

SYNCHROTRN RADIATION DISTRIBUTINOS

Bending Magnet

The power distribution function for the bending magnet can be found in [1]:

$$q\left[\frac{Kw}{mrad^2}\right]=5.425E[GeV]B[T]I[mA]f(\gamma\varphi)=q_o f(\gamma\varphi) \tag{1}$$

Where

$$f(\gamma\varphi)=\frac{1}{\left(1+\gamma^2\varphi^2\right)^{5/2}}\left(1+\frac{5}{7}\frac{\gamma^2\varphi^2}{\left(1+\gamma^2\varphi^2\right)}\right) \tag{2}$$

is the shape function of bending magnet, $\gamma=1957E$ is relativistic energy and φ is the vertical opening angle. Kim [2] suggested that the shape function in equation (2) can be approximated as Gaussian distribution as follows:

$$f(\gamma\varphi)=\exp\left(-\frac{\varphi^2}{2(\sigma_o/\gamma)^2}\right) \tag{3}$$

Where σ_o is the standard deviation and is found to be

$$\sigma_0=\frac{32}{21\sqrt{2\pi}}\approx 0.608 \tag{4}$$

To simplify the FEM modelling, we can approximate the above shape function to be a Heaviside step function $q(\varphi)=q_o(H(\varphi-\frac{\sigma_c}{\gamma})-H(\varphi+\frac{\sigma_c}{\gamma}))$.

As suggested in [3], the equilibrium beam half beam size σ_c is given as

$$\sigma_c=0.608\sqrt{\frac{\pi}{2}}\approx 0.762 \tag{5}$$

This assumption is valid because in general, the bending magnet beam size along vertical direction (Gaussian profile) is much smaller than that of the designed body itself. The benefit of utilizing this simplified power distribution are the following:

- Step function distribution gives more concentrated power distribution, which leads to more conservative thermal result.
- Easy to apply power to the FEM model. Only few nodes have to be meshed on the heating surface. This dramatically reduce the modelling time.

Insertion Device

As was given by [4] , power density of elliptical polarized undulator is

$$q[w/mrad^2]=0.0844E^4[GeV]I[A]\frac{L[m]}{\lambda_o^2[m]}\times f\left(k_x,k_y,\theta_x,\theta_y\right) \tag{6}$$

Where $f\left(k_x,k_y,\theta_x,\theta_y\right)$ is the shape function defined as

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$$f\left(k_x,k_y,\theta_x,\theta_y\right)=\int_{-\pi}^{\pi}\left[\frac{(k_x^2\cos^2\alpha+k_y^2\sin^2\alpha)}{\left(1+(k_y\sin\alpha-\gamma\theta_x)^2+(k_x\cos\alpha-\gamma\theta_y)^2\right)^3}-\frac{\left[\left(k_y^2-k_x^2\right)\sin 2\alpha-2k_y\gamma\theta_x\cos\alpha+2k_x\gamma\theta_y\sin\alpha\right]^2}{\left(1+(k_y\sin\alpha-\gamma\theta_x)^2+(k_x\cos\alpha-\gamma\theta_y)^2\right)^5}\right]d\alpha \quad (7)$$

As you can see, the generalized power distribution for ID is quite complicated to implement into FEM as boundary conditions. Even Sheng [5] has proved that it is not only possible to approximate above shape function into Gaussian type explicit function, but also a constant peak power distribution ones. The Gaussian type shape function $f\left(k_x,k_y,\theta_x,\theta_y\right)$ in (7) is found to be

$$\left(k_x\cos^2\theta+k_y\sin^2\theta\right)\exp\left(-\frac{\frac{k_x^2k_y^2}{k_x^2\cos^2\theta+k_y^2\sin^2\theta}\frac{1}{\left(2\beta-\beta^2\right)^2}\left(\sqrt{\frac{\theta_x^2}{\left(\frac{k_y}{\gamma}\right)^2}+\frac{\theta_y^2}{\left(\frac{k_x}{\gamma}\right)^2}}-1\right)^2}{2\sigma_o^2}\right) \quad (8)$$

Where θ is the polar angle in $\theta_x-\theta_y$ plane. The constant power distribution is in an elliptical donut shape with rim width $\frac{2\sigma_c}{\gamma}$. Note that its area is given as [5]

$$Area=\pi\left(\left(\frac{k_y}{\gamma}+\frac{\sigma_c}{\gamma}\right)\left(\frac{k_x}{\gamma}+\frac{\sigma_c}{\gamma}\right)-\left(\frac{k_y}{\gamma}-\frac{\sigma_c}{\gamma}\right)\left(\frac{k_x}{\gamma}-\frac{\sigma_c}{\gamma}\right)\right)=\frac{2\pi\left(k_x+k_y\right)\sigma_c}{\gamma^2} \quad (9)$$

And the effective half beam width σ_c is found to be

$$\sigma_c\approx 0.762\frac{(\beta^2+1)}{(\beta+1)},\ \beta=\begin{cases}\frac{k_x}{k_y} & \text{when } k_y>k_x\\ \frac{k_y}{k_x} & \text{when } k_x>k_y\end{cases} \quad (10)$$

It is interesting to note that for linear undulator $\left(k_y=0\right)$ or helical undulator $\left(k_x=k_y\right)$, σ_c will be exactly equal to that for bending magnet in (8).

An analytical comparison between Gaussian type power distribution and step function power distribution has been studied in [3].

IMPLEMENTATION OF ACTUAL SR POWER DISTRIBUTION IN FEM

We take Solidworks® and ANSYS® as example, since these two CAD and FEM tools are the most well-known software packages available. Solidworks is used as solid modelling tool to construct geometry and mesh, whereas ANSYS is for the FEM analysis.

In Solidworks, after solid model is built and meshed, we apply dummy heat load on the heating surface element where SR power strikes. Then the meshed model is exported as an ANSYS input file.

By default, the exported ANSYS thermal model file using SOLID87 *3-D 10-Node Tetrahedral Thermal Solid* element [6], there are specific surface nodes and heating surface number. With these geometry information, one can come up with an intermediate process (a script or a computer program) to read the geometry file (nodal coordinate, nodal number, element number, boundary conditions…, etc.) exported from Solidworks, the incident angle θ_{inc} is calculated for each surface load element by applying inner product SR source unit vector and unit normal vector $\tilde{n}$ of the heating surface.

To determine the corresponding θ_x and θ_y of an arbitrary centroid coordinate of the triangular surface element $\left(x,y,z\right)$ (determined by averaging out three vertex coordinates of the heating surface element), we simplify the geometry by, for instance, if the source coordinate of SR source is at $\left(x_o,0,z_o\right)$ and it strikes at corresponding surface coordinate $\left(0,0,0\right)$, then

$$\begin{cases}\theta_x\approx\left(\frac{\left(y-y_o\right)\sqrt{x_o^{\ 2}+z_o^{\ 2}}}{\left(x-x_o\right)x_o+\left(z-z_o\right)z_o}\right)\\ \theta_y\approx\left(\frac{-x_oz+xz_o}{\left(x-x_o\right)x_o+\left(z-z_o\right)z_o}\right)\end{cases} \quad (11)$$

And the power density applied on that heating surface will be

$$q[w/l^2]=\frac{0.0844E^4[GeV]I[A]}{l^2 10^{-6}}\frac{L[m]}{\lambda_o^2[m]}\sin\left(\theta_{inc}\right)\times f\left(k_x,k_y,\theta_x,\theta_y\right) \quad (12)$$

where l is the distance from source to the central coordinate of the heating surface $\left(x,y,z\right)$. Figure 1 illustrates the geometrical relationship.

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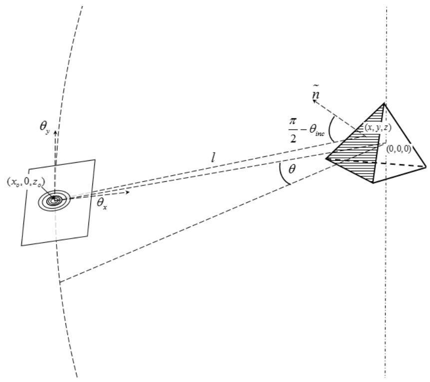

Figure 1: Illustration of one finite element heating surface element vs. heat source.

CASE STUDY

A typical EPU power thermal simulation is carried out and compared with step function type and Gaussian power distribution. As shown in Table 1.

Table 1: Simulation Parameters for EPU

Name	Symbol	Value
Beam energy	E	3[GeV]
Beam current	I	500[mA]
Distance from source	D	$10m$
Period	λ_o	4.8[cm]
Number of period	N	67
Horizontal deflection parameter	k_x	2
Vertical deflection parameter	k_y	4
Inclined angle	θ	8^o

A simple FEM model is used as a comparison case, the model is a $160\,[mm]\times 25\,[mm]$ copper block with $5\,[mm]$ water channel on the other side. The heating surface is flat and has 8^o inclined w.r.t. the ID source. For step function power density, as shown in Figure 2. The maximum temperature rise is found to be $252\ ^o\mathrm{C}$

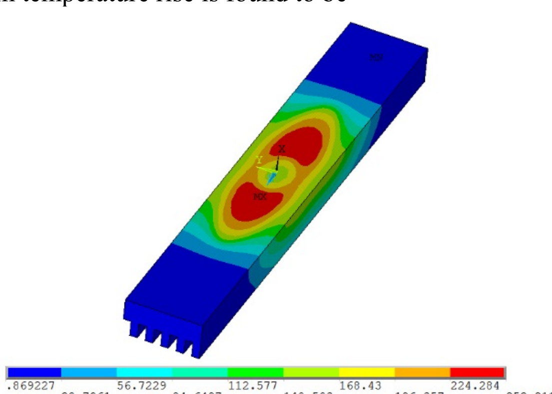

Figure 2: Temperature rise of the model with step function power distribution.

with the same model, as is shown in Figure 3, real power distribution using (12) is implemented. The maximum temperature rise is $212\ ^o\mathrm{C}$. These two maximum temperature results agree with our predictions that the temperature heated by step function power distribution in general is higher than that of Gaussian type.

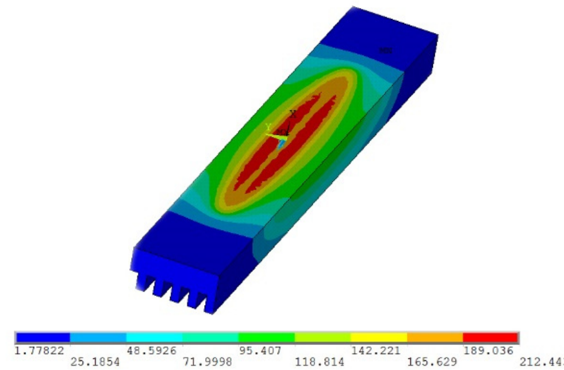

Figure 3: Temperature rise of the model with real EPU Gaussian power distribution.

DISCUSSION AND CONCLUSION

A simplified step function power heating is developed both for bending magnet and insertion device. Simplified power distribution is much more efficient to implement for FEM analysis, and yet it provides more conservative temperature result.

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