DESIGN OF MUON COLLIDER LATTICES*

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Abstract

A Muon Collider (MC) promises unique opportunities both as an energy frontier machine and as a factory for detailed study of the Higgs boson and other particles (see e.g. ref. [1]). However, in order to achieve a competitive level of luminosity a number of demanding requirements to the collider optics should be satisfied arising from short muon lifetime and relatively large values of the transverse emittance and momentum spread in muon beams that can realistically be achieved with ionization cooling. Basic solutions which make possible to achieve these goals with realistic magnet parameters are presented.

INTRODUCTION

There are a number of requirements that are either specific or more challenging in the case of a muon collider:

- Low beta-function at IP: β^* values of a few millimeters are considered for muon colliders in the c.o.m. energy range 3-6 TeV.
- Small circumference C to increase the number of turns (and therefore interactions) the muons make during their lifetime.
- High number of muons per bunch: $N_\mu \sim 2\cdot10^{12}$ and higher is envisaged.
- Protection of magnets from heat deposition and detectors from backgrounds produced by secondary particles.

Other requirements are specific to either high energy MC or the Higgs factory and will be analysed in the subsequent sections.

Though short muon lifetime, ~ 2000 turns, creates a number of problems, but there is a positive side of it: such short time is not enough for high-order resonances to manifest themselves. Other sources of diffusion like IBS or residual gas are also too weak to produce a halo, so if the muon beams were pre-collimated e.g. at 3σ before injection into the collider ring their distribution is likely to stay bounded by a close value. This would relax the requirements on the dynamic aperture and on the efficiency of the halo removal from the ring.

LATTICES FOR HIGH-ENERGY MC

In order not to lose much in luminosity due to the hourglass effect (see e.g. ref. [2]) the bunch length should be small enough: $\sigma_z \leq \beta^*$. With longitudinal emittance expected from the final cooling channel this will render the momentum spread $\sigma_p/p \sim 0.001$ which is an order of magnitude higher than in hadron colliders like Tevatron or LHC. Therefore a high energy MC must have a large momentum acceptance and - to obtain small σ_z with a reasonable RF voltage - low momentum compaction factor $|\alpha_c| \sim 10^{-5}$.

Also, for beam energies $E > 2$ TeV there is a peculiar requirement of absence of straights longer than ~0.5 m in order not to create "hot spots" of neutrino radiation [3]. As a consequence quadrupoles must have a dipole component to spread out the decay neutrinos. This creates difficulties with β^*-tuning sections which must allow for β^* variation in a wide range without breaking the dispersion closure.

In the following subsections we consider different parts of the ring.

Interaction Region (IR)

The dipole component in the IR quadrupoles may play a positive role sweeping away from the detector the charged secondary particles created by decays in incoming muon beam. To maximize this positive effect the strongest quadrupole in the Final Focusing (FF) multiplet – which is usually the second one from the Interaction Point (IP) – should be defocusing.

Another important requirement to the FF multiplet is that the last quadrupole was also defocusing in order to reduce the "dispersion invariant" generated by the following strong dipole used in the chromaticity correction scheme discussed later.

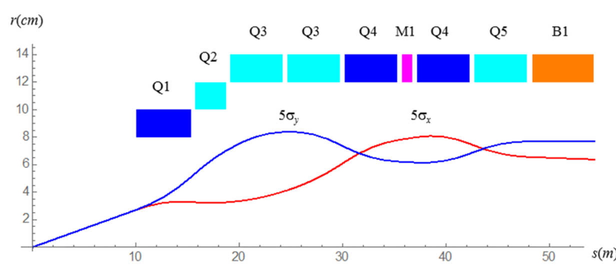

Figure 1: 6 TeV IR magnet apertures and 5σ beam envelopes for $\beta^* = 3$ mm and normalized emittance $\varepsilon_{\perp N} = 25$ (π) mm·mrad. Defocusing magnets are shown in cyan.

Table 1: 6 TeV IR Magnet Parameters

Parameter	Q1	Q2	Q3	Q4	Q5
ID (mm)	160	200	240	240	240
G (T/m)	200	-125	-100	103	-78
B_{dipole} (T)	0	3.5	4.0	3.0	6.0
L (m)	5.3	3.0	5.1	5.1	5.1

To satisfy both requirements simultaneously the multiplet should be either a doublet or a quadruplet. The first option was used in the $E_{c.o.m.} = 1.5$ TeV design [4]. For higher energies it is advantageous to use the second option which allows for much smaller β^*.

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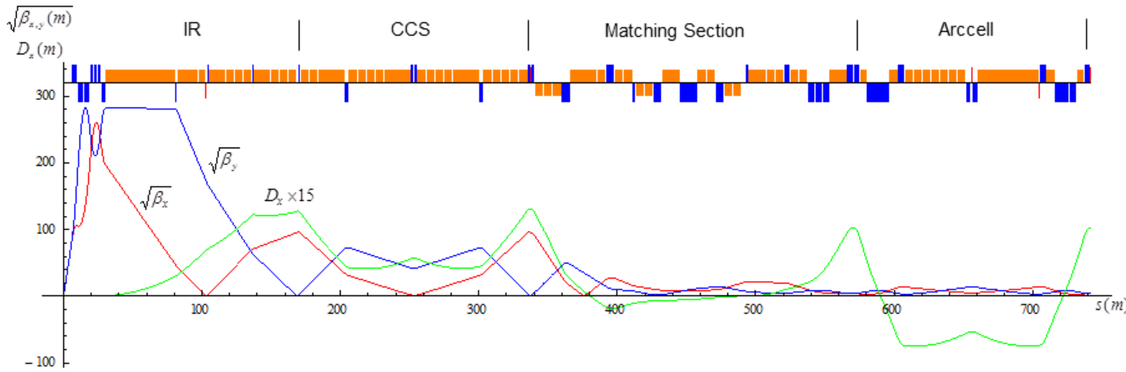

Figure 2: Layout and optics functions from IP to the end of the first arc cell in 3 TeV MC for β^* = 5 mm.

Figure 1 shows the beam sizes and FF magnet apertures in the preliminary design of a Ec.o.m.= 6 TeV MC based on HTS technology expected to achieve 16 T pole tip field in quadrupoles and 20 T field in dipoles. The magnet parameters are given in Table 1. The design of lower energy MC assumed the existing Nb3Sn magnet technology.

Chromaticity Correction

Requirements of large momentum acceptance and small β^* make correction of the Interaction Region (IR) chromaticity a challenging problem. It can be solved using Chromatic Correction Sections (CCS) [5]. The original approach is to employ two CCS – one for horizontal and the other for vertical correction – on each side of the Interaction Point (IP). Each CCS has a pair of sextupoles separated by *-I* transformation to achieve cancellation of spherical aberrations so the original scheme can be called a 4- sextupoles scheme.

However, the CCS are sources of significant chromaticity themselves so that the required integral strength of the two sextupoles in a CCS is higher than with a single sextupole correction. The ensuing detrimental effect on the dynamic aperture (especially vertical) can be alleviated by adding weak compensating sextupoles at some (small) distance from the main sextupoles [6].

Another problem is the optics sensitivity to magnet field errors and misalignments which is getting worse with increased number and strength of elements at high beta locations. To reduce such sensitivity a 3-sextupole scheme was proposed in [4], where the vertical chromaticity is corrected with a single sextupole placed at a low horizontal beta-function location to reduce aberrations (see Fig. 2), while the horizontal chromaticity is still corrected with a CCS since smallness of β_y at a normal sextupole location is beneficial but does not suppress horizontal aberrations.

Arcs and Tuning Section

Each arc consists of 6 so-called Flexible Momentum Compaction (FMC) cells of the type described in [4] which allows for independent control of all important parameters: tunes, chromaticities, momentum compaction factor and its derivative with momentum.

The matching section includes a chicane with adjustable bending field which does not perturb the orbit outside and changes the total orbit length only slightly. It allows for β^* variation in wide range (3 mm-3 cm in the case of the 3 TeV MC) without breaking the dispersion closure [7].

3 TeV MC Lattice Performance

With tunes $Q_x \approx Q_y \approx 19.1$ the stable momentum range exceeds $\pm$ 0.6 %. Figure 3 shows the 2048 turns dynamic aperture in the plane of initial particle coordinates at IP x_{in}, y_{in} for indicated values of constant δ_p calculated with beam-beam interaction off (solid lines) and on (dashed line) using MADX PTC_TRACK routine and MAD8 TRACK LIE4 option respectively.

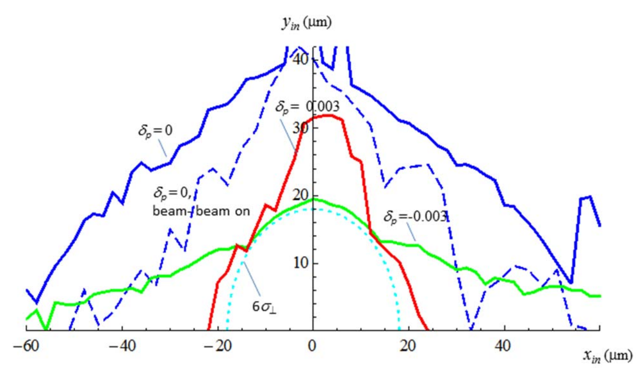

Figure 3: The 3 TeV MC dynamic aperture in the plane of initial particle coordinates for $\beta^* = 5$ mm. Solid and dashed blue lines: $\delta_p = 0$ without and with beam-beam interaction, green and red lines: δ_p = -0.003 and δ_p = 0.003 respectively without beam-beam interaction. Dotted line shows 6σ beam ellipse.

These results show that the three-sextupole chromaticity correction scheme can provide sufficient dynamic aperture and momentum acceptance. Still there was a problem with off-momentum dynamic aperture in the presence of beam-beam interaction.

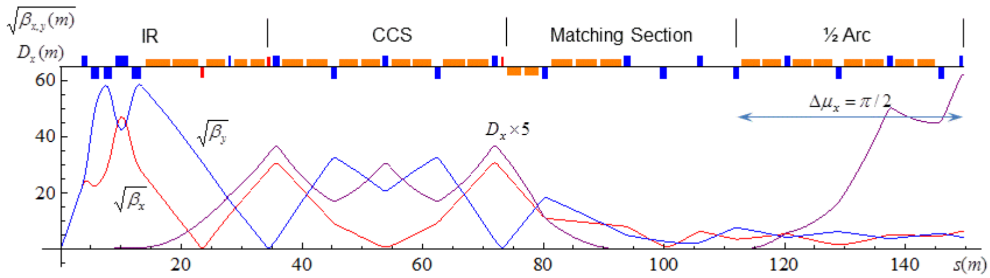

Figure 4: Layout and optics functions in half ring of the Higgs factory for β*=2.5 cm.

However, the dynamic beta effect reduces β* from 5mm to less than 3mm, so β* in the bare lattice can be increased to alleviate the above-mentioned problem.

HIGGS FACTORY LATTICE

There is a number of advantages of muon collider as the Higgs factory [8], among them a high cross-section of the Higgs boson production in the *s*-channel and the possibility of obtaining a sufficiently low muon beam energy spread to directly measure the Higgs boson peak width which is expected to be ~4MeV.

With low energy spread being a priority, the ionization muon cooling can be stopped at the minimum longitudinal emittance – before the final cooling stage which is mostly an emittance exchange – leaving the transverse emittance relatively high (see Table 2). As a consequence quite small values of the beta-function (a few cm) at the Interaction Point are required to achieve sufficiently high luminosity resulting in a large beam size in the Final Focus quadrupoles.

There is a number of effects which can increase energy spread, among them: microwave instability, longitudinal beam-beam effect, path-lengthening due to transverse oscillations.

These effects can be mitigated by a large value of the momentum compaction factor. The latter effect is proportional to chromaticity so that the chromaticity correction is still needed despite extremely low momentum spread.

The necessity of chromaticity correction and of the beam size management in the Final Focus quadrupoles justify the employment of the same solutions as in the high energy MC case: a quadruplet FF and CCS. The optics functions in half ring (starting from IP) are shown in Figure 4 for β*=2.5 cm. Note that with this IR design, β* can be varied from 1.5 to 10 cm by changing the gradients in matching sections without perturbing the dispersion function. The momentum acceptance of the ring exceeds ±0.5%, the dynamic aperture in absence of errors is about 8 sigma. The systematic field errors in the FF quadrupoles reduce the latter by a factor of 2 so that correction of these errors as well as of the fringe-fields is necessary [9].

Table 2: Muon Collider Design Parameters

Parameter	HF	High Energy MC		
Collision energy, TeV	0.126	1.5	3.0	6.0*
Repetition rate, Hz	15	15	12	6
Average luminosity / IP, 10^{34}/cm^2/s	0.008	1.25	4.6	11
Number of IPs	1	2	2	2
Circumference, km	0.3	2.5	4.34	6
β*, cm	1.7	1	0.5	0.3
Momentum compaction factor, 10^{-5}	$7.9{\cdot}10^3$	-1.3	-0.5	-0.3
Normalized emittance, (π)mm·mrad	200	25	25	25
Momentum spread, %	0.004	0.1	0.1	0.083
Bunch length, cm	6.3	1	0.5	0.3
Number of muons / bunch, 10^{12}	4	2	2	2
Beam-beam parameter / IP	0.02	0.09	0.09	0.09
RF frequency, GHz	0.2	1.3	1.3	1.3
RF voltage, MV	0.1	12	50	150

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