THE FIRST PARTICLE-BASED PROOF OF PRINCIPLE NUMERICAL SIMULATION OF ELECTRON COOLING

S. Abeyratne, B. Erdelyi
Department of Physics, Northern Illinois University, DeKalb, IL 60115

Abstract

Envisioned particle accelerators such as JLEIC demand unprecedented luminosities of $10^{34}\text{cm}^{-2}\text{s}^{-1}$ and small emittances are key to achieve them. Electron cooling, where a 'cold' electron beam and the 'hot' proton or ion beam co-propagate in the cooling section of the accelerator, can be used to reduce the emittance growth. It is required to precisely calculate the cooling force among particles to estimate the cooling time accurately. We have developed a novel code, Particles' High-order Adaptive Dynamics (PHAD), for electron cooling. This code differs from other established methods since it is the first particle-based simulation method employing full particle nonlinear dynamics. In this paper we present the first results obtained that establish electron cooling of heavy ions.

INTRODUCTION

Achieving a high luminosity of $10^{34}\text{cm}^{-2}\text{s}^{-1}$ requires a small emittance of the colliding ion beam. For this emittance reduction we employ electron cooling, which is the recommended cooling method for the JLEIC. In electron cooling, the ion beam (or proton beam) and the electron beam travel in the cooling section of the accelerator at the same average velocity. As a result of Coulomb collisions between electrons and ions, there is a friction force acting on ions. In order to model and simulate electron cooling, this friction force must be calculated precisely. Previous electron cooling simulations were phenomenological (e.g. BETACOOL, [1]) or parametrization of an average cooling force obtained by several simplifying approximations [2].

The construction cost of JLEIC is over \$1B. Electron cooling is a single point failure of JLEIC. Therefore, Coulomb force calculation, which is required to estimate cooling time, must be very precise in order to raise the confidence of the simulation.

Particle beams typically contain a large number of particles in the order of 10^{11}. The Coulomb force calculation using the exact method or pair-wise method is unrealistic for very large data sets. Therefore, the first challenge is to identify an approximate method to calculate the Coulomb forces efficiently and accurately. We have developed a new code and it is the first particle-based simulation method employing full particle nonlinear dynamics. The new code, Particle' high-order adaptive dynamis (PHAD), is used to simulate electron cooling.

In this code we use three major techniques to increase the efficiency and accuracy. We use the adaptive fast multipole method to calculate the Coulomb interactions between particles and its computational time and memory usage linearly scale with the number of particles($O(N)$). N is the number of particles. The runtime in pair-wise calculation method is $O(N^2)$ and we claim that the adaptive fast multipole method is efficient and accurate [3]. The Picard iteration based integrator [4] is used for time stepping and the Picard order can be used to control the level of accuracy. Strang operator splitting technique is used to increase the efficiency and to maintain the symplecticity [5].

We have developed the serial version and the parallel version of the PHAD code. Using the serial version, we gathered results for over one million time steps. The transverse emittance reduction of protons was observed.

ELECTRON COOLING SIMULATION

Three widespread approximate methods are the basis function method, the particle-mesh method and the hierarchical space decomposition method [6]. Among them, the hierarchical space decomposition method constitutes the fast multipole method (FMM), and we have shown [3, 7] that the adaptive fast multipole method is the most efficient and accurate way to calculate Coulomb forces. The FMM method asserts a priori accuracy. By increasing the FMM order, the accuracy can be increased.

We used a variable order Picard iteration-based integrator [4, 5] to study the propagation of particles with time. Each particle carries an individualized time step size and it is useful for investigating close encounters. The Picard order and the time step size are the two main factors of the integrator which are liable to accuracy of the near range force calculation. However, large Picard orders and too small time step sizes significantly affect the efficiency. Therefore, if small time steps are required, the Picard order must be decreased to maintain the efficiency. On the other hand, when the larger time steps are appropriate, the Picard order can be increased to maintain the accuracy.

Strang operator splitting method is used to solve the differential equations. This is a second order accurate operator splitting method. The particles in the beam experience slow varying forces and fast varying forces. The interaction between a particular particle with other particles in its neighborhood are the fast varying forces. Also, this particle experiences another slow varying force due to the mean field generated by the far away particles. Due to this nature, we need to introduce two time step sizes for fast and slow varying forces. The smaller time step size is determined by the distance between the evaluation point and the source point and the relative speed between them. In this case, each particle has its own time step size. The mean field force is calculated using the FMM. Since this mean field force is

assumed to be the slowly varying force, we can use large time step size for them. This set up allows to call the FMM only for the larger time steps. In other words, the number of FMM calls can be reduced and it saves time [8, 9].

The adaptive FMM, the Picard integrator and Strang operator splitting method are the constituents of the PHAD code.

In order to validate the accuracy of PHAD we compare the PHAD results with results obtained in the stand-alone N-body code. They showed an excellent agreement [8, 9]. The N-body code uses point-to-point force calculation method and the Picard integrator for time stepping.

The FMM code is written in C++ (data structuring) and COSY INFINITY 9.1 (Coulomb force calculation). COSY INFINITY 9.1 supports the MPI-based parallelization [10]. Intel Cilk Plus was used for shared memory parallelization and MPI for distributed memory parallelization. The results in the simulation were obtained with a partially parallelized PHAD.

PARAMETERS USED FOR THE SIMULATION

Protons or ions generated in the ion source are accelerated by the linac and injected into the booster ring of JLEIC [11]. The kinetic energy of proton is about 280 MeV at the injection. In this simulation we have chosen all parameters corresponding to a proton beam with kinetic energy of 280 MeV.

The average velocity of 280 MeV protons in the longitudinal direction is 0.64c ms^{-1}, where c is the speed of light. Beam cooling is essential in the booster ring in order to reduce the phase space volume and accumulate more protons. The length of the cooling section in the booster ring is 3m. The 'hot' proton beam and the 'cold' electron beam traverse the cooling section at a same average velocity (Fig. 1). Therefore, electron beam takes 3.6ns to pass the cooling section. When electrons move in the longitudinal magnetic field, they make a helical motion as shown in Fig. 1. The time taken for one revolution is about 35.77ps. Therefore, electrons make approximately about 100 revolutions in the 3m long cooling section. In order to account all close encounters, the period of 35.77ps can be divided into 10 small time steps of 3.56ps each. Therefore, the motion of electrons for 1000 time steps can be examined during one pass in the cooling section.

For this simulation we used 1100 particles, which comprised of 100 protons and 1000 macro-electrons (Table 1). The initial longitudinal momentum of protons and electrons were calculated by considering the fact that both types of particles travel at the same average longitudinal velocity of 0.64c, where c is the speed of light. The transverse momentum of electrons is zero. The transverse momenta of protons are listed in Table 1.

The cross section of the electron bunch is 0.005 m and that of the proton bunch is 0.01m. The bunch length is 0.1mm (Fig. 2). The estimated time step size is 3.6ps. Since we use a scaling factor for the time and the transferred time step size has units of 1/c, where c is the speed of light. Therefore, the time step size is equivalent to 0.001 m. The other parameter values are shown in Table 1.

Table 1: Parameters Used for Simulation 2

Element	Parameter	Unit	Value
proton	x-y radius	m	0.01
	mass	amu	1
	charge	e	1
	p_z	scaled with $\frac{1}{mc}$	0.83
	p_x	scaled with $\frac{1}{mc}$	0.1% of p_z
	p_y	scaled with $\frac{1}{mc}$	0.1% of p_z
macro-electron	x-y radius	m	0.005
	mass	amu	17.73
	charge	e	-32552.08
	p_z	scaled with $\frac{1}{mc}$	14.73
	p_x		0
	p_y		0
	current	A	1
	bunch length	mm	1
	B_z	T	1.5

Simulation 1

In the first simulation, we used a proton beam with extremely small initial transverse momentum, and the ratio between the longitudinal and transverse momentum, $\frac{p_{x,y}}{p_z}$ is $\approx 10^{-6}$. The longitudinal magnetic field is 1T. We gathered data for over one million time steps using the serial version of the PHAD code. According to Fig. 3 and Fig. 4, the transverse emittance of protons decrease very slowly with time. The emittances were calculated using the following formula.

$$\epsilon_{x,rms} = \sqrt{< x^2 > < x'^2 > - < xx' >^2}, \qquad (1)$$

where x is the position and $x' = \frac{p'_x}{p_z}$. p_x and p_z are respectively the horizontal and the longitudinal momentum. A similar expression can be derived for the vertical emittance, ϵ_y.

However, the electron bunch length increases with time. As a countermeasure for this issue we replaced the electron bunch with a 'cold' electron bunch after every 3m in our second simulation.

Simulation 2

In this simulation, we increased the transverse momentum of protons such that both horizontal and vertical momenta are 0.1% of the longitudinal momentum of protons. In addition, we refreshed the electron bunch after every 4700 time steps, which is equivalent to ~3m. According to Fig. 5, there is a emittance reduction of protons in both horizontal and vertical direction. Two horizontal lines are drawn as reference lines. For instance, with respect to the black line, the horizontal emittance indicated in red curve diminishes with time. Similarly, vertical emittances indicated in blue curve decreases with respect to the orange reference line. This simulation was performed with the MPI-parallel PHAD code.

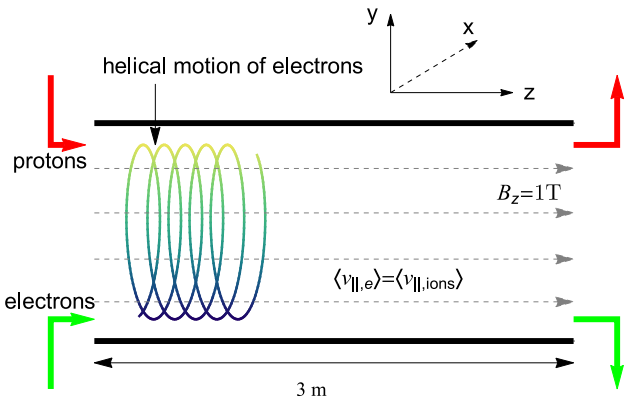

Figure 1: Cooling section in the booster ring.

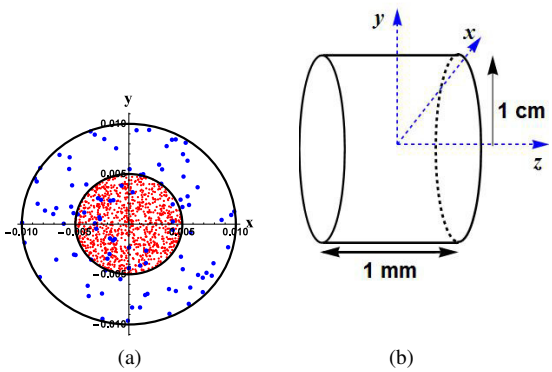

Figure 2: $x - y$ cross section and the bunch length of the particle bunch.

SUMMARY

We have developed new computational tools and a high performance computer code (PHAD) that allows, for the first time, a particle-based simulation of realistic electron cooling of heavy ion beams. The addaptive fast multipole method was used to calculate the Coulomb interaction force efficiently and accurately. Close encounters of particles were studied by means of the Picard iteration-based integrator. Strang operator splitting method enabled to maintain the symplecticity and to improve the efficiency. With optimum FMM order, Picard order and the time step size we can minimize the errors introduced by approximations and operator splitting. In $\sim$ 1% of the roughly estimated cooling time we observed $\sim$ 0.48% reduction of the transverse emittance. With the partially parallelized version of the PHAD code we gained a speedup of factor 4-5. We expect performance far better than the current version by implementing the fully parallelized version.

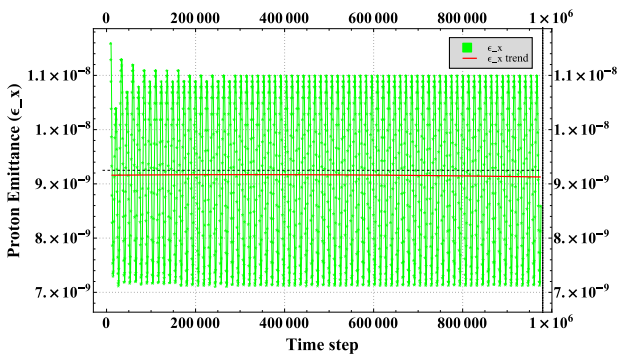

Figure 3: Horizontal emittance of protons. The red thick line is the trend line and the black dashed line is the reference line.

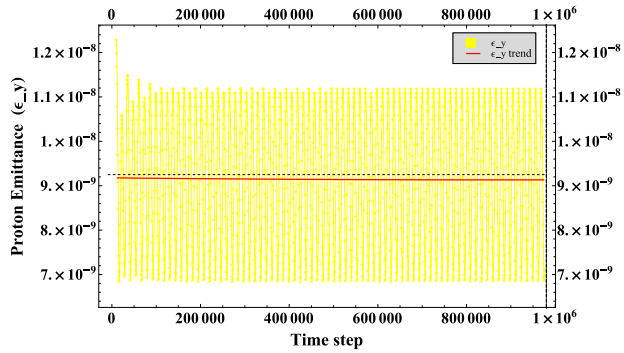

Figure 4: Vertical emittance of protons. The red thick line is the trend line and the black dashed line is the reference line.

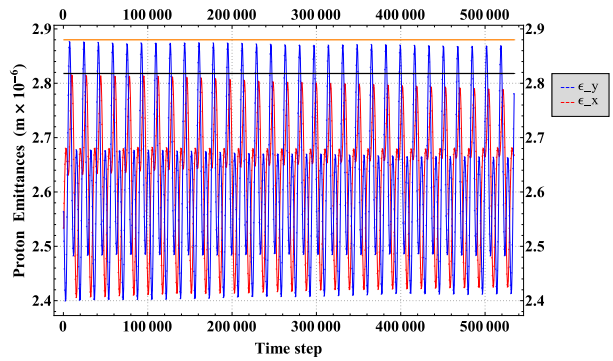

Figure 5: Transverse emittances of protons. Black and orange lines are reference lines for horizontal and vertical emitttances, respectively.

ACKNOWLEDGEMENT

This work was supported by the U.S. Department of Energy, Office of Nuclear Physics, under Contract No. DE-SC0008588.

REFERENCES

[1] A.O. Sidorin, I.N. Meshkov, I.A. Seleznev, A.V. Smirnov, EM Syresin, and GV Trubnikov, "BETACOOL Program for Simulation of Beam Dynamics in Storage Rings. *Nuclear Instruments and Methods in Physics Research Section A: Accel-*

erators, Spectrometers, Detectors and Associated Equipment, vol. 558, no. 1, pp. 325–328, 2006.

[2] A.V. Fedotov, D.L. Bruhwiler, A.O. Sidorin, D.T. Abell, I. Ben-Zvi, R. Busby, J.R. Cary, and V.N. Litvinenko, "Numerical Study of the Magnetized Friction Force," *Physical Review Special Topics-Accelerators and Beams*, vol. 9, no. 7, p. 074401, 2006.

[3] S. Abeyratne, S. Manikonda, and B. Erdelyi, "A Novel "Differential Algebraic Adaptive Fast Multipole Method," in *Proceedings of IPAC'13*, 2013, pp. 1055–1057.

[4] Ernst Hairer, Gerhard Wanner, and Christian Lubich, *Geometric Numerical Integration*, New York: Springer Berlin Heidelberg, 2006.

[5] H. Schaumburg and B/ Erdelyi "A Picard Iteration Based Integrator", in *Proceedings of NA-PAC'13*, 2013, pp. 210–212.

[6] S. Abeyratne and B. Erdelyi, "N-body Code to Demonstrate Electron Cooling, in *Proceedings of COOL15*, 2015.

[7] S. Abeyratne and B. Erdelyi, "Optimization of the Multipole to Local Translation Operator in the Adaptive Fast Multipole Method, in *Proceedings of NA-PAC'13*, 2013, p. 201–203.

[8] B. Erdelyi, S. Abeyratne, and H. Schaumburg, "A New High-Order Adaptive Dynamics Code with Applications to Electron-Ion Collider, in *Proceedings of EIC14*, 2014.

[9] B. Erdelyi and S, Abeyratne, "A Novel Code with High-Order Adaptive Dynamics to Solve the N-body Problem" in *Proceedings of HB2014*, 2014.

[10] M. Berz and K. Makino, COSY INFINITY Version 9.1 Programmer's Manual, 2011.

[11] S. Abeyratne, A. Accardi, S. Ahmed, D. Barber, J. Bisognano, A. Bogacz, A. Castilla, P. Chevtsov, S. Corneliussen, W. Deconinck, *et al.*, "Science Requirements and Conceptual Design for a Polarized Medium Energy Electron-ion Collider at Jefferson Lab," 2012.