DEVELOPMENT OF THE METHOD FOR EVALUATION OF A SUPERCONDUCTING TRAVELING WAVE CAVITY WITH A FEEDBACK WAVEGUIDE

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Abstract

Euclid Techlabs is developing a superconducting traveling wave (SCWT) cavity with a feedback waveguide [1] and has demonstrated a traveling wave at room temperature [2] in a 3-cell SCTW cavity [3]. A special method described in this paper was developed for cavity evaluation. It is based on an S-matrix approach. The cavity tuning procedure based on this method is described.

INTRODUCTION

The accelerating gradient in RF cavities plays a key role in high energy accelerators [4], since the cost of the project is highly dependent on the overall length of the machine. Euclid Techlabs proposed [1] a superconducting traveling wave cavity with a feedback waveguide which can provide 20% - 40% percent higher accelerating gradient than conventional standing wave cavities with the same surface fields [5].

The first approach to a TW cavity was a single cell cavity. It was manufactured to prove the feasibility of cleaning the feedback waveguide. The cavity was cleaned at Argonne National Lab and tested at Fermilab at the vertical test stand in liquid helium. A 26 MV/m accelerating gradient was reached [5], which is comparable to Tesla single cell cavities with the same treatment. This opened the way to build a next generation TW cavity – a 3-Cell SCTW cavity [3] to demonstrate operation in the TW regime in a superconducting cavity with a high gradient. Two 3-cell SCTW cavities were built. Traveling wave operation was successfully achieved at room temperature [2]. The cavity during room temperature testing is depicted in Fig. 1.

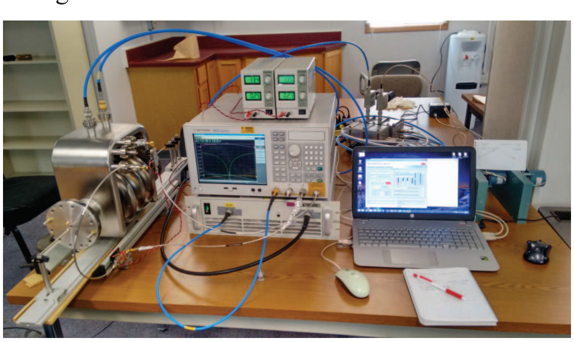

Figure 1: 3-Cell traveling wave niobium cavity room temperature test layout.

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Waveguide stiffening is required for cryogenic high power tests because of the small bandwidth during the test. The welding process is almost finished; cavity processing and testing will be started before the end of the year 2016.

A special model based on the S-matrix approach was used for cavity development. It is faster, easier and more accurate. In some cases, it is not possible at all to analyze the cavity with finite element modeling (FEM) software. The model can also include such features as sensitivity to microphonics, Lorentz force, and waveguide deformation by a tuning element. An analytical model was built and used first for traveling wave adjustment in 1 coupler ring. The analytical model was checked with FEM simulation of a full 3D model which showed the same result and thus, verified the analytical model. The same method was used for the 2 coupler superconducting traveling wave cavity analysis. It was also successfully compared to a full 3D FEM simulation.

COUPLER TRAVELING WAVE RING

The method of evaluation of a traveling wave cavity with a feedback waveguide was started from the simplest case of a rectangular waveguide ring with one coupler. This model does not have any corrugated parts for particle acceleration but derivation of the required parameters for traveling wave adjustment becomes simpler. Particle accelerating sections can be inserted afterwards. Reflections introduced into the ring can be canceled by placing a reflecting element $\lambda/4+N\cdot\lambda/2$ away from the corrugated section. The waveguide ring is depicted schematically in Figure 2.

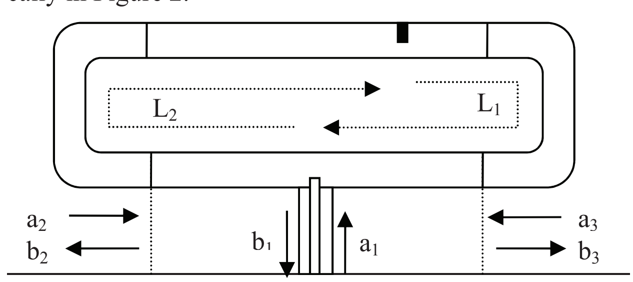

Figure 2: Schematic of the waveguide ring with a coupler and a reflection element.

The ring consists of a coax-waveguide transition, two 180 degree bends and a straight waveguide with a discontinuity. The analytical S-matrix forms of the elements can be found in [6]. Their S-matrices are shown below:

$$S_\Gamma = \begin{pmatrix} i\Gamma & \sqrt{1-\Gamma^2} \\ \sqrt{1-\Gamma^2} & i\Gamma \end{pmatrix}, \quad S_{coup} = \begin{pmatrix} \alpha & \delta & k \\ \delta & \alpha & k \\ k & k & \beta \end{pmatrix}$$

$$\alpha = \frac{\sqrt{1-2k^2}-1}{2},\ \beta = \sqrt{1-2k^2},\ \delta = \frac{\sqrt{1-2k^2}+1}{2}$$

A matrix equation of the ring can be composed according to the schematic in Figure 2 and S-matrices of the individual elements. The system of equations for the ring is shown below:

$$\begin{pmatrix} -1 & i\Gamma\cdot e^{-2L_2(iK_z+\gamma)} & 0 & \sqrt{1-\Gamma^2}\cdot e^{-L(iK_z+\gamma)} \\ \alpha & -1 & \delta & 0 \\ 0 & \sqrt{1-\Gamma^2}\cdot e^{-L(iK_z+\gamma)} & -1 & i\Gamma\cdot e^{-2L_1(iK_z+\gamma)} \\ \delta & 0 & \alpha & -1 \end{pmatrix} \cdot \begin{pmatrix} a_2 \\ b_2 \\ a_3 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -a_1 k \\ 0 \\ -a_1 k \end{pmatrix}$$

The traveling wave condition means that one of the waves is damped or equal to zero. The frequency of the traveling wave regime is determined by the ring length which should be equal to $L = N\cdot\lambda$. We are interested in the solution with small losses because it was found that the traveling wave does not exist in one coupler ring without losses unless beam loading is added in the model. Setting a_2 to zero requires two things. The first is that the length from the coupler to the discontinuity is equal to $L_1 = \lambda/8 + n\cdot\lambda/2$. The second one is that the reflection coefficient from the discontinuity is equal to:

$$\Gamma = \frac{1-e^{-2\gamma L}}{1+e^{-2\gamma L}}$$

The traveling wave solution does not depend on the coupling. The optimal coupling condition is $k = \sqrt{1-e^{-2\gamma L}}$ which was found setting b_1 to zero. The amplitude of the forward wave equals *1/k*, thus a small value of the coupling appears to be very attractive.

Validation of the Model by HFSS Simulation

The formulas obtained were checked by a 3D FEM simulation of the rectangular waveguide ring with a single coupler. The coupling coefficient was chosen to be *k*=0.2. Analytical S-parameters of the ring components were substituted by S-parameters simulated in HFSS. They were also adjusted for the required value of coupling, reflection and attenuation from the corresponding formulas. The traveling wave regime was obtained as seen from Figure 3.

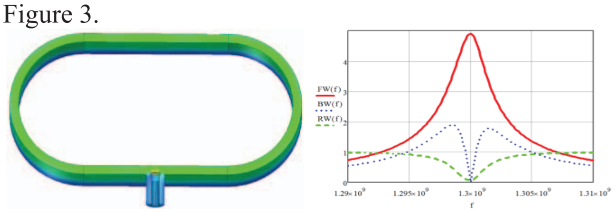

Figure 3: E-field complex amplitude distribution at 1.3 GHz in the adjusted waveguide ring on the left and corresponding amplitudes of the forward (solid line), backward (dot line) and reflected (dash line) waves on the right.

One can see from Figure 3 (right) the plot of the amplitudes of the forward, backward and reflected waves. This plot is obtained mathematically from the corresponding matrix solution. The traveling wave regime was obtained at a frequency, not initially equal to 1.3 GHz, but adjusted mathematically by multiplying corresponding matrix elements by $e^{-iK_z\cdot L}$. No 3D FEM simulation was required. The backward wave is highly damped at 1.3 GHz, while the forward one has a maximum which equals *1/k*. The reflected wave also has a minimum at the adjusted frequency. The 3D simulation of the full ring was done in HFSS with the corrected length. The simulation showed the presence of the traveling wave regime at the desired frequency (see Fig. 3 left) - the complex magnitude of the electric field distribution does not have nodes along the waveguide. The reflected wave is the only one possible to get from the simulation and it is the same as the reflection wave calculated mathematically. The forward and backward wave information is not obtainable from the 3D simulation.

COUPLER TRAVELING WAVE RING

As long as a TW is not obtainable in 1 coupler ring for the case without losses it cannot be used for a superconducting traveling wave (SCTW) cavity with a feedback waveguide without beam loading. That is why a 2 coupler solution was chosen for the 3-Cell SCTW cavity. As is well known, 2 couplers placed $\lambda/4 + n\cdot\lambda/2$ away from each other comprise a directional coupler. Thus, the traveling wave is provided by the couplers and any additional conditions are not required except reflection compensation along the ring. The analytical model was created to investigate traveling wave adjustment in a resonator with a feedback waveguide and 2 couplers. The model is based on S-matrix formalism and utilizes S-parameters calculated using finite element (FE) codes. The model is suitable for any resonator but will be discussed in detail for the example of the 3-cell SCTW cavity, depicted schematically in Figure 4.

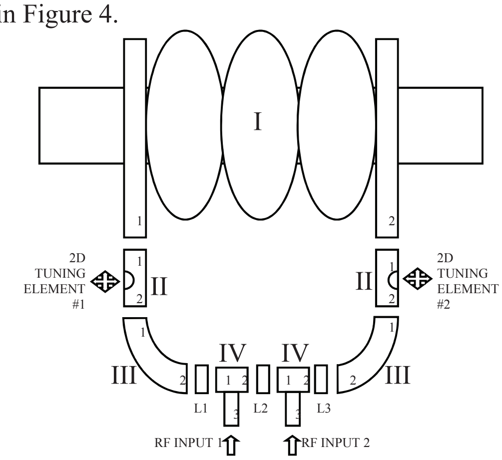

Figure 4: 3-Cell schematic for analytical model.

The 3-Cell SCTW cavity analytical model contains four main elements: a particle accelerating cavity (I), tuner (II), waveguide bend (III) and a power coupler (IV). Straight waveguide inserts (Li) can be added mathematically between those elements. The ring frequency can be adjusted mathematically by changing the electrical length ($\times e^{-iK_z \cdot L}$) of the inserts without any additional calculations in FEM software. Also, the position of the main elements along the cavity can be changed by the inserts. This is very helpful and does not require further FEM calculations. It is merely required to solve a matrix equation of the model to get information about circulating waves in the cavity. The matrix equation of the 3-Cell analytical model is shown below.

$$\begin{pmatrix}
-1 & 0 & S_{12}^{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{11}^{I} \\
0 & -1 & S_{22}^{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{21}^{I} \\
0 & S_{11}^{II} & -1 & 0 & S_{12}^{II} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & S_{21}^{II} & 0 & -1 & S_{22}^{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & S_{11}^{III} & -1 & 0 & S_{12}^{III} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & S_{21}^{III} & 0 & -1 & S_{22}^{III} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & S_{22}^{IV} & -1 & 0 & S_{21}^{IV} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & S_{12}^{IV} & 0 & -1 & S_{11}^{IV} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{22}^{IV} & -1 & 0 & S_{21}^{IV} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{12}^{IV} & 0 & -1 & S_{11}^{IV} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{22}^{III} & -1 & 0 & S_{21}^{III} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{12}^{III} & 0 & -1 & S_{11}^{III} & 0 \\
S_{21}^{II} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{11}^{III} & -1 & 0 \\
S_{11}^{II} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{12}^{II} & 0 & -1
\end{pmatrix} \times \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -S_{23}^{IV} \cdot A \\ -S_{13}^{IV} \cdot A \\ -S_{23}^{IV} \cdot B \\ -S_{13}^{IV} \cdot B \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The A and B terms in the excitation vector are the signals from RF inputs 1 and 2. They are complex numbers, i.e. they have both amplitude and phase and it is possible to change them for tuning purposes. S-parameters of the main elements were calculated in the operational frequency range. It is also possible to implement the S-parameters' dependence on any external effect which can affect them. For example, it can be external pressure variation (microphonics), Lorentz force or waveguide deformation by a tuner. All of these effects were integrated in the model. Thus, S-parameters in the model are functions of frequency and external parameters, such as pressure, acceleration gradient, and amplitude of waveguide deformation. The effect of external pressure variation at 2K was analyzed and it was found that it can be compensated by amplitude and phase redistribution (A and B signals) in the input cavity ports [7, 8]. Lorentz force compensation at 2K requires waveguide deformation with a 2D tuner [9].

Traveling Wave at Room Temperature

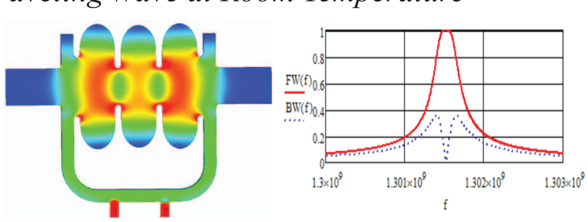

Figure 5: E-field complex magnitude distribution in the 3-Cell traveling wave cavity at room temperature conditions (left) and normalized amplitudes of forward wave (FW) and highly damped backward wave (BW), adjusted by the mathematical model.

The analytical model was used to predict the required parameters for the traveling wave excitation in the 3-Cell SCTW cavity for the successful room temperature test [2]. It was found that amplitude and phase variation of input signals was enough to highly damp a backward wave as shown in Figure 5 (right). Input signal parameters found from the model were used in the HFSS simulation and the TW regime was obtained at the same frequency.

CONCLUSION

The 1 coupler ring resonator mathematical model was created first. The traveling wave (TW) excitation conditions were found from the model. The backward wave suppression does not depend on the coupling and is not possible for the case without losses and beam loading. Analytical components of the model were replaced by S-parameters calculated by finite element modelling (FEM) software for the case with coupling k=0.2. The model was adjusted for 1.3 GHz and then the same solution was found in the simulation of the full 3D geometry.

Since the 1 coupler ring is not suitable for a superconducting cavity without beam loading because of the absence of losses, a 2 coupler ring was investigated and used for 3-cell superconducting traveling wave cavity development. The traveling wave can be excited by the couplers with proper amplitude and phase redistribution of input power. The traveling wave was adjusted in the cavity for room temperature (RT) conditions, i.e. before chemistry and 2K shrinkage. Input power redistribution was enough to tune the cavity at RT. Corresponding FEM simulations were made with the parameters found from the model. The traveling wave was obtained at the same frequency that was found from the model which proved the correctness of the applied method. The same result was obtained experimentally [2].

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