DYNAMICS OF INTENSE BEAM IN QUADRUPOLE-DUODECAPOLE LATTICE NEAR SIXTH ORDER RESONANCE*

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Abstract

The presence of duodecapole components in quadrupole focusing field results in excitation of sixth-order single-particle resonance if the phase advance of the particles transverse oscillation is close to 60°. This phenomenon results in intensification of beam losses. We present analytical and numerical treatment of particle dynamics in the vicinity of sixth-order resonance. The topology of resonance in phase space is analyzed. Beam emittance growth due to crossing of resonance islands is determined. Halo formation of intense beams in presence of resonance conditions is examined.

INTRODUCTION

An ideal quadrupole lens has a constant gradient across the aperture created by poles with infinite hyperbolic shape. Unavoidable deviations from ideal pole shape results in the appearance of higher order harmonics in the quadrupole field spectrum. The vector-potential of the magnetic field of a lens with quadrupole symmetry contains harmonics of the order $2(2m+1)$, $m = 0,1,2....$:

$$A_z = -[\frac{G_2}{2} r^2 \cos 2\theta + \frac{G_6}{6} r^6 \cos 6\theta + \frac{G_{10}}{10} r^{10} \cos 10\theta + ..], \quad (1)$$

where G_2 is the gradient of quadrupole lens, G_6 is the duodecapole component, G_{10} is the "20-poles" component. The vertical component of the magnetic field along abscissa is given by

$$B_y(x,0) = G_2 x + G_6 x^5 + G_{10} x^9 + ... \quad . \quad (2)$$

While traveling through a quadrupole lens of length D, particles receive a momentum kick, which contains linear and nonlinear parts:

$$\Delta \frac{dx}{dz} = -\frac{qD}{mc\beta\gamma}(G_2 x + G_6 x^5 + G_{10} x^9 ...) . \quad (3)$$

The presence of duodecapole harmonic in quadrupole field results in excitation of sixth-order resonance if phase advance of transverse oscillations per focusing period is close to 60°. Increase of beam losses near 60° phase advance was observed experimentally at SNS linac [1]. Minimization of duodecapole component requires specific pole shape of quadrupole lenses [2]. In present paper we estimate effect of 6th order resonance on beam expansion.

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HAMILTONIAN OF SIXTH-ORDER RESONANCE

Consider particle motion in a quadrupole channel with focusing period S in the presence of duodecapole components. The quadrupole channel is substituted by a continuous focusing channel with phase advance μ_o per period, which for FODO focusing structure is determined as

$$\mu_o = \frac{S}{2D}\sqrt{1 - \frac{4}{3}\frac{D}{S}} \frac{qG_2 D^2}{mc\beta\gamma} . \quad (4)$$

Presence of duodecapole component is introduced as an additional nonlinear momentum kick, which particle receives once per focusing period. In the adopted approximations, single particle motion is described by a matrix:

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = \begin{pmatrix} \cos\mu_o & \sin\mu_o \\ -\sin\mu_o & \cos\mu_o \end{pmatrix} \begin{pmatrix} x_n \\ p_n + \Delta p_n \end{pmatrix} , \quad (5)$$

where n is the number of focusing period, x is the particle position, p is the modified particle momentum, and Δp is the nonlinear kick due to presence of duodecapole component:

$$p = \frac{S}{\mu_o}\frac{dx}{dz}, \qquad \Delta p = \delta_5 x^5 . \quad (6)$$

Let us introduce action-angle variables through transformation:

$$x = \sqrt{2J}\cos\psi, \quad p = -\sqrt{2J}\sin\psi . \quad (7)$$

Normalized emittance of the beam bounded by the ellipse in phase space is related to introduced action value as

$$\varepsilon = 2J\beta\gamma\frac{\mu_o}{S} . \quad (8)$$

Analysis shows that the Hamiltonian describing slow motion near 6th order resonance is given by

$$H(J,\psi) = J\vartheta - \frac{\delta_5}{4}J^3 - \frac{\delta_5}{24}J^3\cos 6\psi , \quad (9)$$

where $\vartheta = \mu_o - \pi/3$ is the deviation from "resonance" angle 60°.

Figure 1 illustrates topology of phase space structure. A particle moves either along internal phase space trajectory or along external islands with larger amplitude. In the vicinity of point J_u, particle motion is unstable. The particle might either remain inside internal phase space trajectory, or be trapped into resonance separatrices.

FIXED POINTS AND ISLANDS SIZE

The expression for the Hamiltonian, Eq. (9), allows us to analyze particle motion in the vicinity of resonance islands and to determine basic characteristics of phase space patterns. Fixed points (stable and unstable) are determined by equations

$$\frac{dJ}{dn} = -\frac{\partial H}{\partial \psi} = -\frac{\delta_5}{4} J^3 \sin 6\psi = 0\,, \qquad (10)$$

$$\frac{d\psi}{dn} = \frac{\partial H}{\partial J} = \vartheta - \frac{3}{4}\delta_5 J^2 [1 + \frac{\cos 6\psi}{6}] = 0\,. \qquad (11)$$

Equation (10) has a solution $\sin 6\psi = 0$, or $\cos 6\psi = \pm 1$. Unstable points are determined by condition $\cos 6\psi_u = 1$, which gives for the action variable at unstable point

$$J_u = \sqrt{\frac{8}{7}\frac{\vartheta}{\delta_5}}\,, \quad \psi_u = \frac{\pi}{3}k\,, \quad k = 0,1,2,... \qquad (12)$$

Stable points are determined as $\cos 6\psi_s = -1$, which gives for the action variable

$$J_s = \sqrt{\frac{8}{5}\frac{\vartheta}{\delta_5}}\,, \quad \psi_s = \frac{\pi}{6} + \frac{\pi}{3}k\,, \quad k = 1,2,... \qquad (13)$$

For practical reasons, it is important to determine the maximal value of action variable, which defines expansion of phase space area comprised by the beam. The outer separatrix touches the inner one at the unstable point, where Hamiltonian, Eq. (9), has the value $H_u = 2J_u\vartheta/3$. The value of the Hamiltonian is approximately the same at the internal unstable trajectory, and at the external separatrices. Particle with the value of Hamiltonian H_u reaches the point $J_{\max}$ having $\cos 6\psi = -1$. Substitution of the value of the Hamiltonian at the unstable point into Eq. (9) gives an expression for determination of the value of $J_{\max}$:

$$J_{\max}\vartheta - \frac{5}{24}\delta_5 J_{\max}^3 - \frac{2}{3}J_u\vartheta = 0\,, \qquad (14)$$

which has the solution $J_{\max} = 1.54 J_u$ or

$$J_{\max} = 1.54\sqrt{\frac{8}{7}\frac{\vartheta}{\delta_5}}\,. \qquad (15)$$

Relative increase of amplitude of particle trapped into resonance is $x_{\max}/x_u = \sqrt{1.54} = 1.24$. Taking into account Eqs. (8) and (12), the beam emittance corresponding to oscillations within internal phase space trajectory is restricted by the value

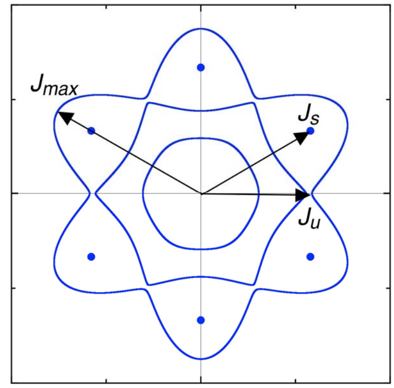

Figure 1: Topology of 6th order resonance.

$$\varepsilon_u = \sqrt{\frac{32}{7}\frac{\vartheta}{\delta_5}}\,\beta\gamma\,\frac{\mu_o}{S}\,. \qquad (16)$$

From Eq. (11), angular velocity in phase space drops from the value of ϑ in the center of phase space to zero at unstable point. Due to nonlinear dependence of betatron tune on amplitude of oscillation, distortion of beam phase space becomes significant for beam emittance $\varepsilon \approx 0.6\varepsilon_u$.

NUMERICAL SIMULATION of 6th ORDER RESONANCE

As an example, consider dynamics of single particle in FODO lattice with lens-to-period ratio $D/S = 1/3$, phase advance per period μ_o= 62.6° ($\vartheta = 0.0454$), and the value of duodecapole kick $\delta_5 = 0.254\,cm^{-4}$ (see Fig. 2). Because in FODO channel particle receives two nonlinear kicks per period, we approximate value of δ_5 from Eq. (3) as

$$\delta_5 = 2\frac{qG_6 DS}{mc\beta\gamma\,\mu_o}\,. \qquad (17)$$

Figure 3 illustrates dynamics of single particle calculated by matrix method, Eq. (5), and through direct integration of equations of motion in FODO lattice. Figure 3b illustrates stroboscopic image of phase space motion through plot of particle position in phase space once per period of FODO focusing channel. Phase space areas comprised by single particle in matrix mapping and in direct integration are close to each other.

Figure 4 illustrates emittance growth of the beam with initial KV distribution in the same focusing structure. As seen, presence of 6th order resonance results in degradation of phase space area and appearance of tails in beam distribution. In presence of space charge, effective betatron tune, μ, is lowered due to space charge repulsion:

$$\mu^2 = \mu_o^2 - k\frac{2I}{I_c(\beta\gamma)^3}(\frac{S}{R_e})^2\,, \qquad (18)$$

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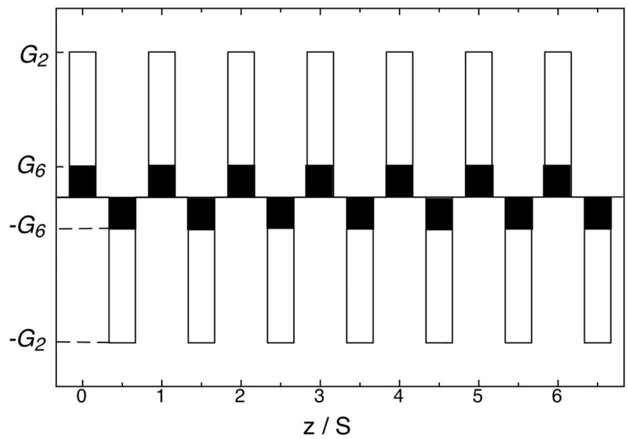

Figure 2: FODO channel with quadrupole G_2 and duodecapole G_6 field components.

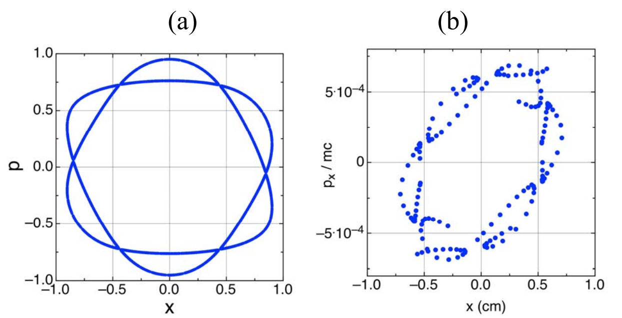

Figure 3: Dynamics of single particle in FODO focusing channel with $D/S=1/3$, μ_o=62.6°, $\delta_5 = 0.254\, cm^{-4}$: (a) matrix method, Eq. (5), (b) direct integration in FODO field.

where I is the beam current, $I_c = 4\pi\varepsilon_o mc^3 / q$= $3.13{\cdot}10^7 A/Z[Amp]$ is the characteristic current, R_e is the average beam radius, and $k=1...2$ is the coefficient depending on beam distribution. Due to non-uniform beam distribution, the value of depressed phase advance varies from μ in beam core till μ_o at the beam periphery. Figure 5 contains results of simulation of the beam in FODO lattice with $D/S=1/3$, μ_o= 86°, $\delta_5 = 8.6\, cm^{-4}$ for different beam distributions with depressed betatron tune μ below 60°. Due to spread of betatron tune, there are particles, which are trapped into resonance. Initial beam distributions are characterized by tails in phase space, which become longer after trapping into resonance creating significant beam halo.

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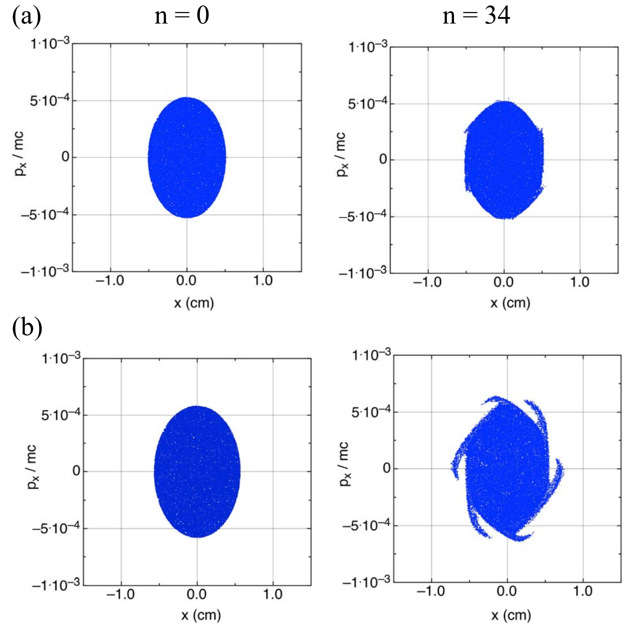

Figure 4: Distortion of the beam in FODO lattice: (a) $\varepsilon/\varepsilon_u = 0.5$, (b) $\varepsilon/\varepsilon_u = 0.6$.

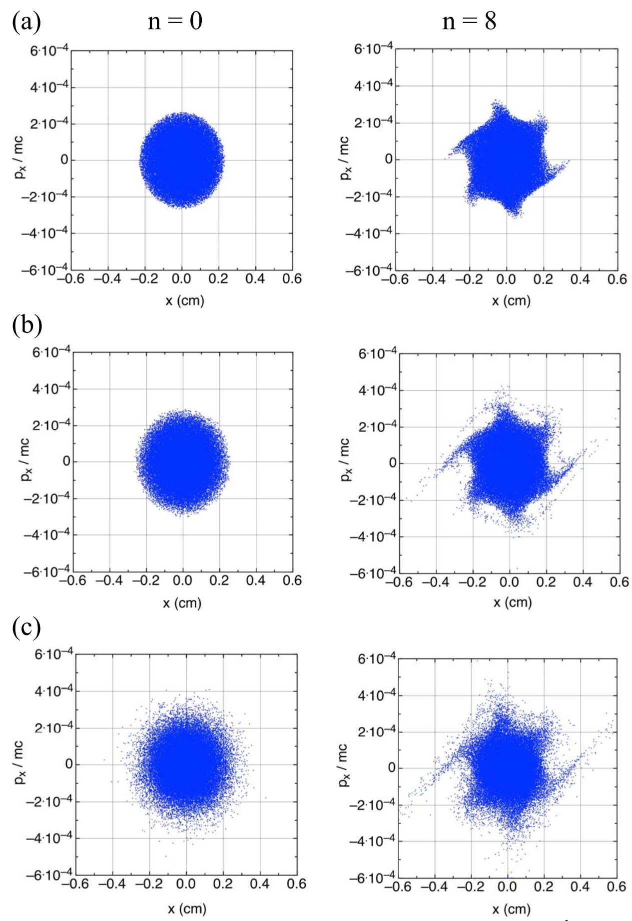

Figure 5: Dynamics of the beam in the vicinity of 6th order resonance for different beam distributions in the lattice with μ_o= 86° : (a) water bag, μ=58° (b) parabolic, μ=54°, (c) Gaussian, μ=38°.