PROGRESS ON SKEW PARAMETRIC RESONANCE IONIZATION COOLING CHANNEL DESIGN AND SIMULATION

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Abstract

Skew Parametric-resonance Ionization Cooling (Skew PIC) is an extension of the Parametric-resonance Ionization Cooling (PIC) framework that has previously been explored as the final 6D cooling stage of a high-luminosity muon collider. The addition of skew quadrupoles to the PIC magnetic focusing channel induces coupled dynamic behavior of the beam that is radially periodic. The periodicity of the radial motion allows for the avoidance of unwanted resonances in the horizontal and vertical transverse planes, while still providing periodic locations at which ionization cooling components can be implemented. Properties of the linear beam dynamics have been previously reported and good agreement exists between theory, analytic solutions, and simulations. Progress on aberration compensation in the coupled correlated optics channel is presented and discussed.

INTRODUCTION

The Parametric-resonance Ionization Cooling (PIC) [1] concept aims to enable high energy muon physics by providing muon beam emittances that are an order of magnitude smaller than those achieved using conventional ionization cooling methods. PIC uses half-integer resonances in the linear optics of its cooling channel to provide simultaneous focusing in both transverse planes, and absorbers at the naturally periodic focal points to limit the angular spread of the beam. Skew Parametric-resonance Ionization Cooling (Skew PIC) [2,3] expands on this concept by utilizing coupling of the transverse motion to shift the canonical betatron tunes away from the resonant values that destabilize the beam in the uncoupled case. Previous work described the first implementation of the Skew PIC channel and presented simulation results demonstrating the rotational behavior at periodic absorber locations that is a hallmark of Skew PIC, as well as the use of a single quadrupole family to induce a parametric resonance in the Skew PIC channel.

To be suitable as the final 6D cooling stage of a high-luminosity muon collider, a Skew PIC channel must be able to control the p=200 MeV/c muons from upstream stages with initial RMS momentum spread Δp/p of 2% and initial RMS angular spread θ_{rms} of 130 mrad. The highly divergent nature of the particle distribution is very challenging for aberration compensation. Skew PIC may offer a solution by reducing the dimensionality of the aberration compensation problem to the radial dimension only, thereby reducing the number of compensating multipoles required. We present recent results discussing such nonlinear compensation, as well as discussions on coupling strength and implications for channel design.

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EFFECTS OF COUPLING STRENGTH

The PIC and Skew PIC concepts require the correlated optics condition to be met, i.e. the canonical betatron tune in one transverse plane is an integer multiple of that in the other transverse plane. In the PIC system, this property leads multipole fields to excite nonlinear resonances. Transverse coupling in a correlated optics system, as with Skew PIC, shifts the canonical betatron tunes away from the integer values that are problematic from the aberration compensation point of view. As previously noted in [2], the magnitudes of the curvature strength *K*, straight quadrupole strength *n*, and coupling strength *g* are related to the rotation angle along the circular trajectory a particle traces in the x-y or x'-y' phase space at special periodic points through the relation:

$$g = -\frac{1}{2}(k_1^2 - k_2^2)\sin 2\theta$$

where

$$k_{1,2}^2 = \frac{1}{2}\left(k_x^2 + k_y^2 \pm \sqrt{\left(k_x^2 + k_y^2\right)^2 + 4g^2}\right)$$

and

$$k_x^2 = K^2 - n$$
$$k_y^2 = n$$

Very large coupling strengths should be avoided to avoid the instability associated with a large rotation angle, where the dynamics can encounter a half-integer resonance. The dependence of the canonical betatron tune shift on the magnitude of the coupling strength is shown for one channel configuration in Figure 1. For this channel configuration, coupling strengths associated with tune shifts above Δν > 0.33 do not meet the coupled correlated optics requirement and the Skew PIC optics breaks down.

Stronger coupling may enable faster muon cooling by facilitating fast mixing of horizontal and vertical dynamic behavior. Conversely, weaker coupling may be more suitable due to constraints on the dispersion functions imposed by ionization cooling requirements. At the periodic absorber locations, the oscillating dispersion function generated by the coupled correlated optics must be small to minimize energy straggling of particles in the absorbers. A non-zero dispersion or first derivative of the

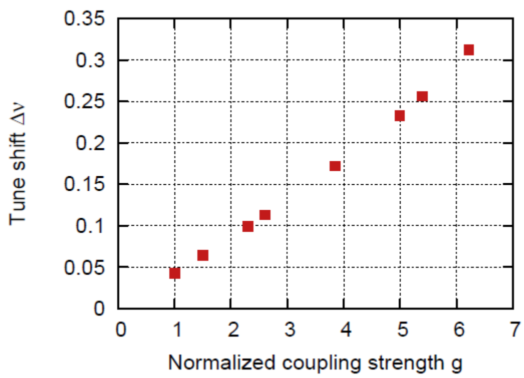


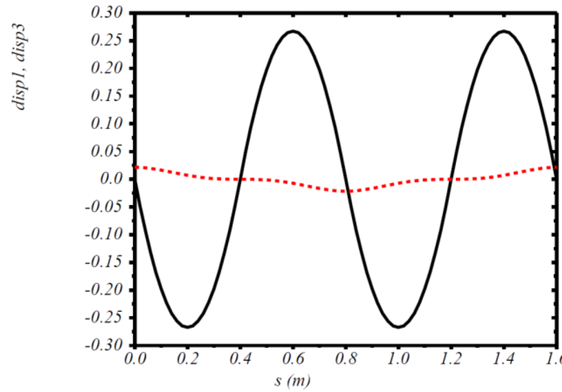
Figure 1: Tune shift dependence on coupling strength for one configuration of the Skew PIC channel.

dispersion function at these periodic absorber locations allows for emittance exchange, and dispersion maxima between absorber locations is ideal for compensation of chromatic aberrations. The dispersion functions for one coupling period of the Skew PIC channel with weak and strong coupling are shown in Figures 2a and 2b. The periodic absorber locations occur at 0 and 1.6 m. The horizontal dispersion function zeroes coincide with the periodic absorber locations, while the vertical dispersion function zeroes are completely out of phase. The magnitude of the vertical dispersion function increases with the coupling strength, suggesting the use of weaker coupling to limit the effect of beam smear at the absorber locations for effective ionization cooling.

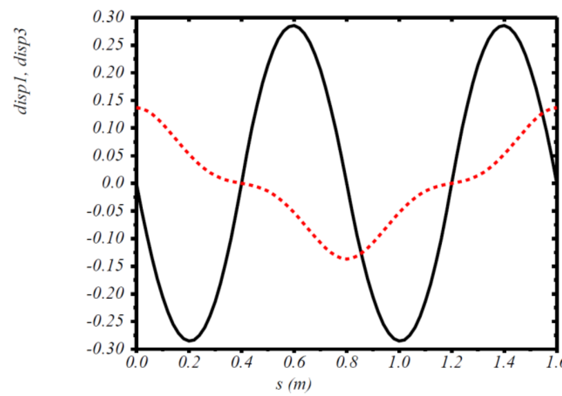
The coupled nature of the beam dynamics suggests that chromaticity compensation and correction of aberrations may be less straightforward than in typical uncoupled optics. Aberration compensation in the uncoupled PIC channel required many higher order multipoles to correct the beam smear at the periodic absorber locations, but these higher order multipoles induced higher order resonances that ultimately destabilized the beam. The coupled motion in Skew PIC reduces the independently-periodic motion in x and y planes to periodic motion in a single dimension r, the radial offset of a particle from the reference orbit, and the use of fewer higher order multipoles may be possible for the nonlinear compensation. To study the dynamic aperture of the Skew PIC channel, multipole components were added to the main sector bends and optimized for maximum transmission of highly divergent particle distributions.

SKEW PIC CHANNEL DYNAMIC APERTURE STUDIES

Because the equilibrium momentum and angular distributions of the muon beam at the entrance of a Skew PIC stage are rather large ($\Delta p/p = 2\%$ and $\theta_{rms} = 130$ mrad at p=200 MeV/c, respectively), the dynamic aperture of the channel must be able to accommodate all particles with minimal beam loss for efficient ionization cooling. The dynamic aperture of the Skew PIC channel was studied in MADX [4] using MINUIT [5] optimization



(a)



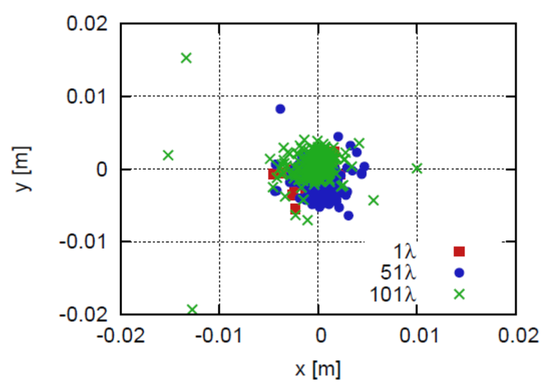
(b)

Figure 2: (a) Dispersion functions for weakly coupled channel. (b) Dispersion functions for strongly coupled channel. Absorbers would be placed at 0 and 1.6 m.

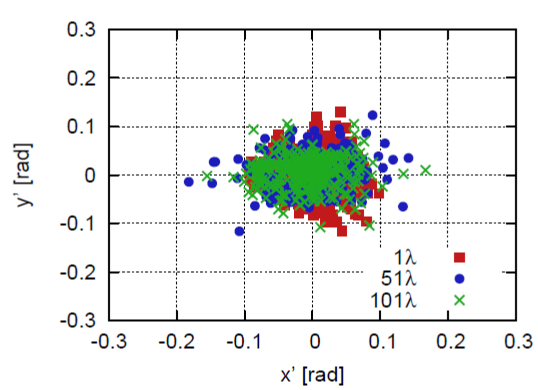
routines to minimize the radial offset of a given ensemble of particles through successive periods of the channel. Due to the nature of the magnetic field configurations and channel implementation, the most straightforward method of including higher order fields is to follow the phasing and periodicity of the curvature or coupling functions. This is achieved by adding the appropriate field component and magnitude to each unique sector bend in one period of the Skew PIC channel. To find optimal values for each multipole value, the multipole phase and periodicity were fixed over one period of the Skew PIC channel, and the multipole amplitude was varied to minimize the average radial offset at each periodic absorber location over a total channel length of 101 periods. The multipole forms analyzed in these dynamic aperture studies follow the phase and periodicity of the curvature function (K multipole); coupling function (g multipole); constant multipole (straight multipole); curvature function with phase shift of $\lambda_K/4$ (phase-shift K multipole); and coupling function with phase shift of $\lambda_g/4$ (phase-shift g multipole). Normal and skew forms were introduced separately.

Multipole optimization suggests that chromaticity compensation in the coupled correlated optics system becomes increasingly difficult as the coupling strength increases. For tune shifts below $\Delta\nu < 0.2$, particles with

2% momentum offset and θ_{max} up to 2 mrad survive through 101 channel periods with 100% transmission. For tune shifts above $\Delta\nu > 0.2$, 100% transmission over 101 channel periods was not possible for the sextupole forms implemented as described above. In one channel configuration with coupling strength corresponding to a tune shift of $\Delta\nu = 0.17$, particle distributions with θ_{max} = 37 mrad were well controlled using a normal K sextupole, normal g octupole, and skew phase-shifted g decapole harmonics, but dodecapole optimization failed with increased angular distributions. With reduced coupling strength corresponding to a tune shift of $\Delta\nu = 0.11$, recent results indicate that single sextupole and octupole harmonics easily control particles with θ_{max} up to 65 mrad, and two octupole harmonics (normal and skew) are necessary to increase the maximum angle to 82 mrad. Figures 3a and 3b show the x-y and x'-y' phase space in the plane of an ideal absorber location after 1, 51 and 101 channel periods for particles with $|\theta_{max}| = \pm 82$ mrad. 400 particles were initiated and the transmission is 100%. The large amplitude particles seen after 101 periods may be better controlled with higher order multipoles. The relative ease with which multipole solutions were found for the weak coupling case points to the use of weakly coupled channels for further optimization.



(a)



(b)

Figure 3: (a) x-y phase space in the plane of an ideal absorber location at 1, 51, and 101 Skew PIC channel periods. (b) x'-y' phase space at the same locations.

SUMMARY

Recent progress on Skew PIC simulations is presented. While strong transverse coupling may result in faster ionization cooling times, weak transverse coupling has the advantages of reducing large dispersion components at absorber locations that will contribute to energy straggling and beam smear, and easier compensation of chromaticity and other aberration effects. Multipole optimization for aberration compensation has resulted in a Skew PIC channel with angular acceptance of up to 82 mrad, and results are promising for further optimization for acceptance of muons with equilibrium angular spread θ_{rms} = 130 mrad expected from upstream stages in a high luminosity muon collider.

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REFERENCES

[1] V.S. Morozov, *et a*l., "Parametric-Resonance Ionization Cooling of Muon Beams," AIP Conf. Proc. 1507, 843-848, 2012.
[2] A. Afanasev, *et al.*, "Skew-Quad Parametric-Resonance Ionization Cooling: Theory and Modeling," in *Proc. IPAC15*, Richmond, VA, USA, May 2015, paper TUPHA013.
[3] A. Sy, *et al.*, "Muon Tracking Studies in a Skew Parametric Resonance Ionization Cooling Channel," in *Proc. IPAC15*, Richmond, VA, USA, May 2015, paper WEPJE015.
[4] MAD-X. `http://madx.web.cern.ch/madx`
[5] F. James and M. Roos, "Minuit: A System for Function Minimization and Analysis of the Parameter Errors and Correlations," *Comput. Phys. Commun.* 10, 343-367, 1975.