# LINEAR OPTICS CHARACTERIZATION AND CORRECTION METHOD USING TURN-BY-TURN BPM DATA BASED ON RESONANCE DRIVING TERMS WITH SIMULTANEOUS BPM CALIBRATION CAPABILITY*

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*Abstract*

A fast new linear lattice characterization / correction method based on turn-by-turn (TbT) beam position monitor (BPM) data in storage rings has been recently developed and experimentally demonstrated at NSLS-II. This method performs least-square fitting iteratively on the 4 frequency components extracted from TbT data and dispersion functions. The fitting parameters include the errors for normal/skew quadrupole strength and 4 types of BPM errors (gain, roll, and deformation). The computation of the Jacobian matrix for this system is very fast as it utilizes analytical expressions derived from the resonance driving terms (RDT), from which the method name DTBLOC (Driving-Terms-Based Linear Optics Characterization/Correction) originates. At NSLS-II, a lattice corrected with DTBLOC was estimated to have beta-beating of <1%, dispersion errors of ~1 mm, and emittance coupling ratio on the order of $10^{-4}$.

## INTRODUCTION

There are many existing linear lattice characterization / correction methods such as LOCO [1] and other newer methods [2-4] for storage rings to name a few. As an alternative, a new algorithm called DTBLOC [5, 6] has been recently developed to characterize the linear optics from TbT data, based on linear RDTs [7-10]. TbT and dispersion data from BPMs are the only required input data for this method, which makes this tool fast. Then, the real and imaginary parts (or amplitude and phase) of two frequency components in each plane need to be extracted from TbT data by any high-precision frequency analysis method. For NSLS-II with 180 BPMs, the observation parameters consist of 1440 values from 4 complex frequency components (8 real values) per BPM and 360 values from $\eta_x$ and $\eta_y$ per BPM and 2 values from tunes. The fitting parameters are the integrated strength errors in normal and skew quadrupoles as well as the 4 BPM errors (horizontal & vertical gain, roll, deformation [or crunch]) for each BPM. The fitting is performed by SVD using a Jacobian matrix with analytical expressions based on RDTs.

## ANALYTICAL EXPRESSIONS

In this section, we will only show the outline of the derivation and the final result of the analytical expressions used in DTBLOC to compute the Jacobian matrix of the TbT frequency components with respect to the magnet strength errors and BPM errors. A more detailed derivation can be found in [5].

The complex Courant-Snyder variables $h_{x,-}$ and $h_{y,-}$ at turn $N$ and at a location $s$ in a ring can be expressed as a function of resonance driving terms (RDT) $f_{jklm}^{(s)}$ [8]:

$$h_{x,-}(s,N) = \hat{x} - i\hat{p}_x = \sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{s,x,0})} -2i\sum_{jklm} j f_{jklm}^{(s)}(2I_x)^{\frac{(j+k-1)}{2}}(2I_y)^{\frac{l+m}{2}} \times e^{i[(1-j+k)(2\pi\nu_x N+\psi_{s,x,0})+(m-l)(2\pi\nu_y N+\psi_{s,y,0})]}, \quad (1)$$

$$h_{y,-}(s,N) = y - i\hat{p}_y = \sqrt{2I_y}e^{i(2\pi\nu_y N+\psi_{s,y,0})} -2i\sum_{jklm} l f_{jklm}^{(s)}(2I_x)^{\frac{(j+k)}{2}}(2I_y)^{\frac{l+m-1}{2}} \times e^{i[(k-j)(2\pi\nu_x N+\psi_{s,x,0})+(1-l+m)(2\pi\nu_y N+\psi_{s,y,0})]}, \quad (2)$$

where the linear RDTs are defined as

$$f_{2000}(s) = \frac{\sum_w(-\Delta b_2 L^w)\beta_x^w e^{i\left(2\Delta\phi_x^{w,s}\right)}}{8\left(1-e^{2\pi i(2\nu_x)}\right)},$$

$$f_{0020}(s) = \frac{\sum_w(+\Delta b_2 L^w)\beta_y^w e^{i\left(2\Delta\phi_y^{w,s}\right)}}{8\left(1-e^{2\pi i(2\nu_y)}\right)},$$

$$f_{\substack{1001\\1010}}(s) = \frac{\sum_w(+\Delta a_2 L^w)\sqrt{\beta_x^w\beta_y^w}e^{i\left(\Delta\phi_x^{w,s}\mp\Delta\phi_y^{w,s}\right)}}{4\left(1-e^{2\pi i(\nu_x\mp\nu_y)}\right)}.$$

From Eqs. (1) and (2), for an uncoupled lattice with focusing errors only ($\Delta b_2 L \neq 0, \Delta a_2 L = 0$), we can derive analytical expressions for $\hat{x}$ and $\hat{y}$ TbT positions in terms of 2 sine-cosine pair terms with 2 different frequencies as a function of RDTs responsible for $\beta$-beating (2000 and 0020). Then adding coupling errors on top of this lattice, we can obtain another set of expressions for $\hat{x}$ and $\hat{y}$ TbT positions as a function of all the linear RDTs (2000, 0020, 1001, and 1010). After approximation assuming small magnet strength errors, which results in small RDT values, and using the relationship $x = \sqrt{\beta_x}\hat{x}$ and $y = \sqrt{\beta_y}\hat{y}$ between the normalized coordinates $(\hat{x}, \hat{y})$ and the phase-space coordinates $(x, y)$, we can express the magnitudes and phases for the primary and secondary frequency components $f_{x1}, f_{y1}$ and $f_{x2}, f_{y2}$ of $(x, y)$ TbT positions as the following:

$$|f_{x1}| = \sqrt{\left\{C_x^{(x)}\right\}^2 + \left\{S_x^{(x)}\right\}^2}, |f_{x2}| = \sqrt{\left\{C_y^{(x)}\right\}^2 + \left\{S_y^{(x)}\right\}^2},$$
$$|f_{y1}| = \sqrt{\left\{C_y^{(y)}\right\}^2 + \left\{S_y^{(y)}\right\}^2}, |f_{y2}| = \sqrt{\left\{C_x^{(y)}\right\}^2 + \left\{S_x^{(y)}\right\}^2}, \quad (3)$$

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$$\begin{aligned}\angle f_{x1} &= \Phi_x' + \phi_{x0}(s) + \Delta\phi_{\Delta\nu_x}(s) + \tan^{-1}(-S_x^{(x)}/C_x^{(x)}),\\ \angle f_{x2} &= \Phi_y' + \phi_{y0}(s) + \Delta\phi_{\Delta\nu_y}(s) + \tan^{-1}\left(-S_y^{(x)}/C_y^{(x)}\right),\\ \angle f_{y1} &= \Phi_y' + \phi_{y0}(s) + \Delta\phi_{\Delta\nu_y}(s) + \tan^{-1}\left(-S_y^{(y)}/C_y^{(y)}\right),\\ \angle f_{y2} &= \Phi_x' + \phi_{x0}(s) + \Delta\phi_{\Delta\nu_x}(s) + \tan^{-1}\left(-S_x^{(y)}/C_x^{(y)}\right),\end{aligned} \tag{4}$$

where $\Phi_x'$ and $\Phi_y'$ are initial phases of TbT motions, $\phi_{x0}$ and $\phi_{y0}$ are the phase advances without errors, and

$$\Delta\phi_{\Delta\nu_x}(s) = 2\pi\sum_w\left\{\mathrm{sgn}(s-w)\cdot\frac{1}{2}\left(\frac{\beta_x^w}{4\pi}\right)\cdot(\Delta b_2 L^w)\right\},$$

$$\Delta\phi_{\Delta\nu_y}(s) = 2\pi\sum_w\left\{\mathrm{sgn}(s-w)\cdot\frac{1}{2}\left(-\frac{\beta_y^w}{4\pi}\right)\cdot(\Delta b_2 L^w)\right\},$$

$$\begin{aligned}C_x^{(x)} &= \sqrt{2I_x\beta_{x0}}(1+4\Im\{f_{2000}\}),\\ C_x^{(y)} &\approx 2\sqrt{2I_x\beta_{y1}}(\Im\{f_{0110}\}+\Im\{f_{1010}^V\}),\\ S_x^{(x)} &= -\sqrt{2I_x\beta_{x0}}(4\Re\{f_{2000}\}),\\ S_x^{(y)} &\approx 2\sqrt{2I_x\beta_{y1}}(\Re\{f_{0110}\}-\Re\{f_{1010}^V\}),\\ C_y^{(x)} &\approx 2\sqrt{2I_y\beta_{x1}}(\Im\{f_{1001}\}+\Im\{f_{1010}^H\}),\\ C_y^{(y)} &= \sqrt{2I_y\beta_{y0}}(1+4\Im\{f_{0020}\}),\\ S_y^{(x)} &\approx 2\sqrt{2I_y\beta_{x1}}(\Re\{f_{1001}\}-\Re\{f_{1010}^H\}),\\ S_y^{(y)} &= -\sqrt{2I_y\beta_{y0}}(4\Re\{f_{0020}\}),\end{aligned}$$

$$\begin{aligned}\beta_{x1} &= \beta_{x0}\cdot(1+8\cdot\Im\{f_{2000}\})/(1+8\cdot\Im\{f_{0020}\}),\\ \beta_{y1} &= \beta_{y0}\cdot(1+8\cdot\Im\{f_{0020}\})/(1+8\cdot\Im\{f_{2000}\}).\end{aligned}$$

Finally, to include the effect of BPM errors (gains by $g_x$ and $g_y$, roll by $\theta$, deformation or crunch by $C$), the following relation can be used to convert the actual beam positions $(x, y)$ into the apparent beam positions $(\tilde{x}, \tilde{y})$:

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix} g_x & g_x(C+\theta)\\ g_y(C-\theta) & g_y\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}. \tag{5}$$

Then the final expressions for the apparent primary and secondary frequency components $\tilde{f}_{x1}, \tilde{f}_{y1}$ and $\tilde{f}_{x2}, \tilde{f}_{y2}$ yield the same expressions as Eqs. (3) and (4), except that all the variables $C_{x,y}$ and $S_{x,y}$ should be replaced with these corresponding variables with tilde:

$$\begin{aligned}\tilde{C}_x^{(x)} &= g_x\left\{C_x^{(x)} + C_x^{(y)}(C+\theta)\right\},\\ \tilde{S}_x^{(x)} &= g_x\left\{S_x^{(x)} + S_x^{(y)}(C+\theta)\right\},\\ \tilde{C}_y^{(x)} &= g_x\left\{C_y^{(x)} + C_y^{(y)}(C+\theta)\right\},\\ \tilde{S}_y^{(x)} &= g_x\left\{S_y^{(x)} + S_y^{(y)}(C+\theta)\right\},\\ \tilde{C}_x^{(y)} &= g_y\left\{C_x^{(x)}(C-\theta) + C_x^{(y)}\right\},\\ \tilde{S}_x^{(y)} &= g_y\left\{S_x^{(x)}(C-\theta) + S_x^{(y)}\right\},\\ \tilde{C}_y^{(y)} &= g_y\left\{C_y^{(x)}(C-\theta) + C_y^{(y)}\right\},\\ \tilde{S}_y^{(y)} &= g_y\left\{S_y^{(x)}(C-\theta) + S_y^{(y)}\right\}.\end{aligned}$$

The following final expressions for the amplitudes of the four frequency components are particularly insightful:

$$|\tilde{f}_{x1}| = g_x\sqrt{2I_x}\times\sqrt{\begin{aligned}&\left\{\sqrt{\beta_{x0}}(1+4\Im\{f_{2000}\}) + \sqrt{\beta_{y1}}(2\Im\{f_{0110}\}+2\Im\{f_{1010}^V\})(C+\theta)\right\}^2\\ &+\left\{\sqrt{\beta_{x0}}(-4\Re\{f_{2000}\}) + \sqrt{\beta_{y1}}(2\Re\{f_{0110}\}-2\Re\{f_{1010}^V\})(C+\theta)\right\}^2\end{aligned}},$$

$$|\tilde{f}_{x2}| = g_x\sqrt{2I_y}\times\sqrt{\begin{aligned}&\left\{\sqrt{\beta_{x1}}(2\Im\{f_{1001}\}+2\Im\{f_{1010}^H\}) + \sqrt{\beta_{y0}}(1+4\Im\{f_{0020}\})(C+\theta)\right\}^2\\ &+\left\{\sqrt{\beta_{x1}}(2\Re\{f_{1001}\}-2\Re\{f_{1010}^H\}) + \sqrt{\beta_{y0}}(-4\Re\{f_{0020}\})(C+\theta)\right\}^2\end{aligned}},$$

$$|\tilde{f}_{y1}| = g_y\sqrt{2I_y}\times\sqrt{\begin{aligned}&\left\{\sqrt{\beta_{x1}}(2\Im\{f_{1001}\}+2\Im\{f_{1010}^H\})(C-\theta) + \sqrt{\beta_{y0}}(1+4\Im\{f_{0020}\})\right\}^2\\ &+\left\{\sqrt{\beta_{x1}}(2\Re\{f_{1001}\}-2\Re\{f_{1010}^H\})(C-\theta) + \sqrt{\beta_{y0}}(-4\Re\{f_{0020}\})\right\}^2\end{aligned}},$$

$$|\tilde{f}_{y2}| = g_y\sqrt{2I_x}\times\sqrt{\begin{aligned}&\left\{\sqrt{\beta_{x0}}(1+4\Im\{f_{2000}\})(C-\theta) + \sqrt{\beta_{y1}}(2\Im\{f_{0110}\}+2\Im\{f_{1010}^V\})\right\}^2\\ &+\left\{\sqrt{\beta_{x0}}(-4\Re\{f_{2000}\})(C-\theta) + \sqrt{\beta_{y1}}(2\Re\{f_{0110}\}-2\Re\{f_{1010}^V\})\right\}^2\end{aligned}}.$$

The linear RDTs are all typically on the order of $10^{-2}$ or less. The roll $(\theta)$ and deformation $(C)$ errors are also on the same order. Thus, $\theta$ and $C$ have almost no impact on the primary frequency components $|\tilde{f}_{x1}|$ and $|\tilde{f}_{y1}|$. However, if the coupling RDTs are on the same order or less than $\theta$ and $C$, the secondary frequency components $|\tilde{f}_{x2}|$ and $|\tilde{f}_{y2}|$ will be substantially affected by these errors, which lead to apparent leakage of betatron motion from one plane to another plane, on top of the real coupled motion. As linear coupling is corrected, the coupling RDTs decrease while $\theta$ and $C$ stay constant. Therefore, it becomes increasingly important to take into account the roll and deformation errors as coupling is reduced.

## EXPERIMENTS

We have performed 2 experimental studies with NSLS-II storage ring to demonstrate the capability of DTBLOC. The first study was to characterize two different machine lattices using DTBLOC. The only difference between the two was whether all the ring skew quadrupole correctors were turned on or off. During the short interval between the two sets of measurements, the BPM errors should not have changed. Hence, DTBLOC should converge to the same values for the BPM errors, if the algorithm is valid. Figure 1 shows DTBLOC indeed found the BPM errors defined by Eq. (5) to be very close for the two cases.

Two interesting observations can be made from the estimated BPM errors. One is the ~10% outlier in the vertical gain estimates. This is believed to be caused by a hardware issue. LOCO and ICA also have found this outlier as well previously for different lattices [3]. The other is the rather large deformation errors of almost ~20 mrad RMS. As the impact of $\theta$ and $C$ on the observable secondary frequency components was explained earlier, the extraction of coupling RDTs from an experimental TbT data becomes very difficult with this level of deformation errors (explained in more detail in [5]). Therefore, it is important to include all 4 types of BPM errors in the fitting process in order to reduce coupling very well.

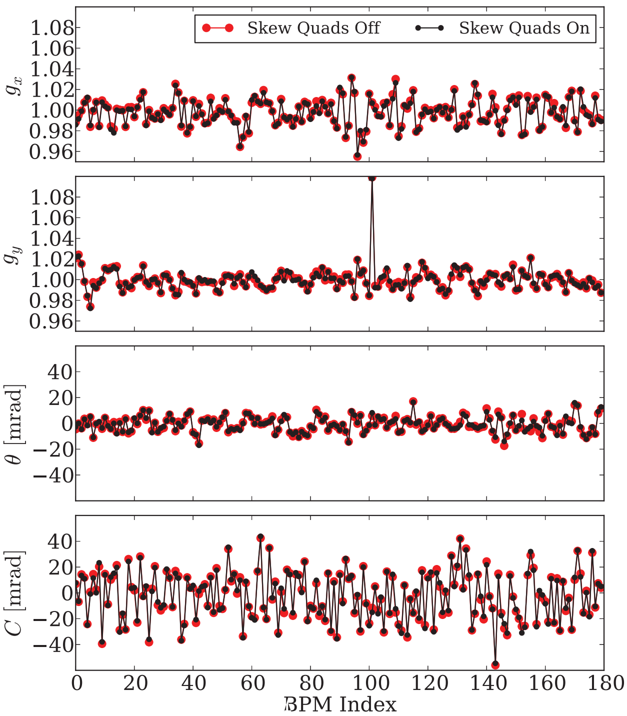


Figure 1: DTBLOC estimates of BPM errors for the 2 different cases "skew on" and "skew off".

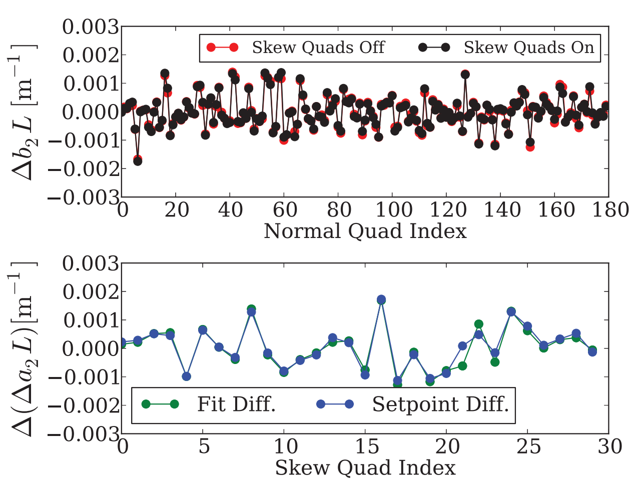


Figure 2: DTBLOC estimates of focusing ($\Delta b_2 L$) and skew quadrupole ($\Delta a_2 L$) errors for the 2 different cases "skew on" and "skew off".

Consistency checks for magnet strength error estimation can be also performed from this lattice characterization experiment. Since the normal quadrupole currents were not changed, the estimated focusing errors $\Delta b_2 L$ should not have changed, which is the case as shown in Fig. 2. As for the skew quadrupoles, since the setpoint changes for these magnets are known and their unit conversions between current and field are known, we can compare the expected change in the skew strengths and DTBLOC's estimated skew error change. The bottom plot of Fig. 2 shows good agreement of these changes.

Another study we performed with NSLS-II storage ring was to experimentally demonstrate that DTBLOC works as a fast linear lattice correction tool. Starting with a nominal lattice, two corrections suggested by DTBLOC were applied iteratively. Then all the normal quadrupoles were cycled and characterized again as the hysteresis for them is not negligible. The results of these corrections are shown in Table 1. The $\beta$-beat and the dispersion functions shown in the table are the values corrected by taking into account the estimated BPM error values. The $\beta$-beat was suppressed to below 1%. The dispersion errors were reduced to ~1 mm. The emittance coupling ratio was decreased to the order of 0.01%. Even though this ratio was estimated from the model constructed from the estimated magnetic strength errors, the lifetime reduction by a factor of 2.5 (from 31.6 hours to 12.6 hours) roughly agrees with the expected Touschek lifetime reduction of $\sqrt{10}$ (given the reduction of the estimated coupling ratio by a factor of ~10 and the bunch current being sufficiently high to be Touschek dominated). The values "N/A" for the coupling ratio in the table indicate that the coupling signal became too low to believe the estimated values.

Table 1: Linear Lattice Characteristics During Iterative DTBLOC Corrections (2 mA in 100 Buckets)

| | **Initial** | **Corr. #1** | **Corr. #2** | **Cycling** |
|---|---|---|---|---|
| RMS $\Delta\beta_x/\beta_x$ [%] | 4.3 | 0.8 | 0.4 | 0.5 |
| RMS $\Delta\beta_y/\beta_y$ [%] | 2.9 | 0.5 | 0.4 | 0.3 |
| RMS $\Delta\eta_x$ [mm] | 7.1 | 1.5 | 1.1 | 1.2 |
| RMS $\Delta\eta_y$ [mm] | 3.0 | 1.2 | 1.2 | 1.1 |
| Lifetime [hr] | 31.6 | 12.6 | 8.8 | 8.5 |
| Avg. $\epsilon_y/\epsilon_x$ [%] | 0.4 | ~0.04 | N/A | N/A |

## CONCLUSION

A new method of linear lattice characterization/correction is proposed using TbT data and dispersion data based on RDTs. Utilizing the derived analytical expressions for the observable frequency components in TbT data, a fast (~5 minutes instead of ~1 hour for LOCO at NSLS-II) linear lattice characterization/correction tool called DTBLOC has been implemented. Combined with the existing nonlinear lattice characterization method [11], DTBLOC will enable extraction of both linear and nonlinear RDTs from TbT data, thus allowing linear and nonlinear lattice modelling, within the single framework of the RDT formalism. DTBLOC successfully demonstrated experimentally its capability to estimate errors in linear magnetic elements and BPMs at NSLS-II storage ring, where DTBLOC has already been utilized to estimate and correct coupling errors induced by closed IDs, and is ready to generate a coupling feedforward table.

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