THE ROLE OF ADAMI INFORMATION IN BEAM COOLING*

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Abstract

We re-consider stochastic cooling as type of information engine using the Adami definition of information [1]. We define information as data which can permit the cooling system to predict the individual trajectories better than purely random prediction and then act on that data to modify the trajectories of an ensemble of particles. In this study we track the flow of this type of information through the closed system and consider the limits based on sampling and correction as well as the role of the underlying model.

INTRODUCTION

The view of stochastic cooling as a type of information engine goes back to Simon Van Der Meer [2] the inventor of stochastic cooling, who cast it as a form of the famous Maxwell's demon, which is currently understood as type of information engine.

If we recall James Maxwell came up with a famous thought experiment that challenged the ideas enshrined in the second law of thermodynamics, specifically the idea that for a system of particles in thermal equilibrium, where all the fast and slow moving particles were completely mixed, no more work could be extracted. Maxwell imagined a box containing this distribution with a wall dividing it into two sides (see Fig. 1). In the wall there was a door, which was controlled by some demon that would open the door only for fast moving particles and keep it shut for the slow particles. In this way, over time, all the fast particles would come to reside on one side of the box, leaving the slow particles on the other. In this situation, a heat engine could be run from the differential in temperature, and thus extracting work in violation of the second law of thermodynamics.

For many years Maxwell's demon challenged the understanding of entropy and the second law. Later, statistical mechanics were worked into the existing thermodynamic framework and entropy was understood as representing the possible states of a given system. Finally, entropy made its way into the new field of information theory when Shannon equated the statistical definition of entropy with information [3]. Maxwell's Demon began to be understood as a class of information engines. An information engine is currently understood as a system which can turn information into work. Here information gathered by the demon concerning the velocity of each particle represented a rise in entropy. This is because this information needed to be stored on some physical medium whose initial entropic state had to be considered. So, for example, a magnetic tape, which stores information as zeros and ones, needed to be first initialized to be all zeros. This initialization placed the tape into a lower entropic state, which was then given up as information was recorded. In the end, the work required to reset this memory would consume more energy than was extracted, thus preserving the second law. This is known as Landauer's erasure principle [4]. More recently a physical realization of a type of Maxwell demon machine has been created using a photon circuit [5]. However, the implications of Maxwell's Demon are still somewhat unresolved and debated.

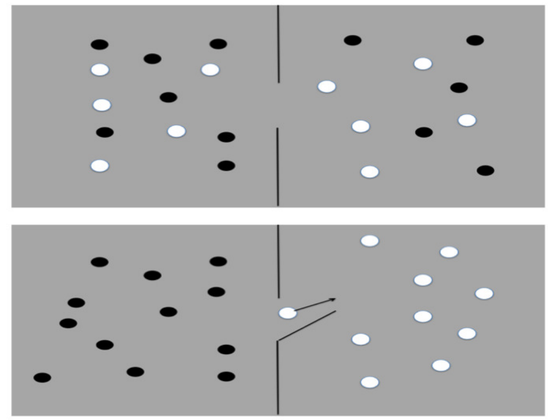

Figure 1: Fast (white) and slow (black) particle in equilibrium (top). Maxwell's Demon opens door to sort particles of different speed (bottom).

ADAMI INFORMATION

Of course the stochastic cooling system doesn't come anywhere near violating the second law of thermodynamics, since the energy consumed by the amplifiers, and kickers clearly introduce external energy to help lower the entropy of the cooled beam. However as Simon Van Der Meer recognized, the set-up is actually very similar. In the classical stochastic cooling system, there exists a pickup and kicker which operates somewhat like the demon, in that the demon (the cooling system) takes a measurement and then based on that performs an operation. In the case of stochastic cooling the demon kicks offending particles back into a lower orbit (see Fig. 2). What is important here is that this system uses 'information' together with externally supplied energy to lower the entropy and increase the order of the system, which in this case it is the beam. This is different from other standard cooling systems in that usually it involves an exchange of energy in the form of gases, liquids or particles mixing with a cool bath (as in electron cooling) that achieve the reduction of entropy. In the stochastic cooling case information about the predicted behavior of the system is a key component, without it, this type of cooling couldn't occur. Also the better one's information is the more cooling possible.

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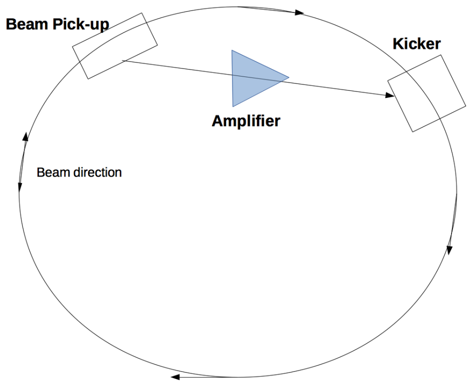

Figure 2: Typical stochastic cooling layout with beam going clockwise in ring. The beam position is sampled at the pickup, amplified and sent to the kicker to correct the beam.

The key role of information motivates a better and more precise definition of information. If we proceed using the textbook Shannon definition of information we quickly discover that it is inadequate to describe the nature of the information used in this system. This is because this definition which relates entropy to information is really a measurement of uncertainty and not a measure of what we would commonly understand as being 'known'.

The problem with the classical Shannon definition of information has been recently pointed out by Adami [1]. He claims that what Shannon called information and equated with entropy was really a measure of uncertainty, not information as it would be commonly understood. For example a coin that can have two defined states, heads or tails, has a defined amount of entropy, which is less entropy than a six-sided dice, which has six possible states. This is what Shannon called information, because a six-state system can hold more bits of information than a two-state system.

In that same article he offers a compelling re-definition of information as "that which allows you (who is in possession of that information) to make predictions with accuracy better than chance".

To my knowledge this new definition of information has yet to be applied to the case of Maxwell's Demon or that of stochastic cooling. The existing literature on Maxwell's Demon uses the standard Shannon version of information. It would seem that using the new definition of information wouldn't alter the ultimate implications for the second law of thermodynamics, however I would argue that it does alter the conclusions about the role of information in generating order. For example, Shannon's definition relates the rise of entropy with the rise of information. Information defined in this way takes on a "negative" meaning with respect to order. This I believe misses the role which the information plays in facilitating the generation of order. If we use the Adami understanding of information we can identify the inherent information present in the system. This inherent information is represented in the implied physical assumption that allows the demon to predict the future trajectories of the particles and thus sort them using the trapdoor, or even the fact that faster particles are those that make a system "warmer"and slower make a system "colder."

This information inherent in the system is overlooked using the standard definition of information. Further using this approach permits a better linkage to biological systems which use the information inherent in DNA to generate order. Adami argues that it is this "information" that is the real commodity of evolutionary biology. The better an organism is at modeling its environment the more chance it has of passing on its genes. The process of reflecting or adapting to the environment drives evolution.

COOLING AS AN APPLICATION OF ADAMI INFORMATION

We can now see how this new definition works very well with the stochastic cooling system. This is because the efficiency of the cooling depends directly on the ability of the system to predict or have information about the trajectories of the particles at the kicker site and to apply a correction. It is clear that maximum cooling can be achieved if we have perfect information about the trajectories of each particle and the ability to apply corrections to each particle.

To understand this better lets review the case of transverse cooling of coasting beam following [6]. If we assume the bandwidth for the system depicted in Fig. 2 is W, this results in a time resolution of T_s $1/(2W)$. A particle passing through the pick-up off-axis induces a short pulse of T_s length due to the finite bandwidth (W). This corresponds to $N_s = N/(2WT)$ particles per sample with N being the total number of particles in the beam. This signal is then processed and applied to the particle of the beam with a transverse position of x_i via the kicker. The resulting corrected position (xc_i) after the kick is given by,

$$\begin{aligned} xc_i &= x_i - \frac{g}{N_s}\sum_{k=1}^{N_s} x_k \\ &= x_i - g < x >_s \, . \end{aligned} \tag{1}$$

Here g is the system gain. Here we see that the single pass correction will be imperfect because the single for x_i particle is diluted with the other particles in the sample. We can see clearly here that in the limit of infinite bandwidth our sample will include only the same particle which we correct, thus achieving a perfect correction.

If we proceed further, squaring both sides and evaluating now $\Delta(x_i^2) = xc_i^2 - x_i^2$, we obtain,

$$\begin{aligned} \Delta(x_i^2) &= -2g\frac{x_i}{N_s}\sum_{k=1}^{N_s} x_k + \left(\frac{g}{N_s}\sum_{k=1}^{N_s} x_k\right)^2 \\ \Delta(x_i^2) &= -2g\left(\frac{x_i^2}{N_s} + \frac{x_i}{N_s}\sum_{k\neq i}^{N_s} x_k\right) + \frac{g^2}{N_s}x_{rms}^2 \\ \Delta(x_i^2) &= -2g\frac{x_i^2}{N_s} + \frac{g^2}{N_s}x_{rms}^2 . \end{aligned} \tag{2}$$

Here we use the fact that the average for a sample of a distribution goes to the average of the distribution which in this case is zero (i.e. $\sum_{k\neq i}^{N_s} x_k = 0$). Now averaging both sides we get an estimate for the cooling time,

$$\frac{1}{\tau} = -\frac{1}{T}\frac{\Delta(x_{rms}^2)}{x_{rms}^2} = \frac{2W}{N}(2g - g^2) \tag{3}$$

We can see here directly how the efficiency of cooling scales with 'information' that we have about a particles position via bandwidth. In this derivation there is also an implied perfect mixing per turn. More precise cooling rates can be derived with include imperfect mixing and noise effects.

Now considering a more formal definition of information given in [1]. Information for the 'ith' particle is defined,

$$\begin{aligned} I &= H_{max} - H(X) \\ I &= \log N_x + \sum_{j}^{N_x} p_j \log p_j \end{aligned} \tag{4}$$

Here H_{max} represents the maximum entropy and $H(x)$ embodies what you know about the probability distribution. Here an important and subtle point made by Adami [1] and Jaynes [7], needs to be clarified. The maximum entropy in fact represents the measurement resolution. If one reflects on the measurement of the entropy for the case of the coin, it becomes clear that N=2 state ascribed to the coin is arbitrary as one could chose to measure any number of atributes associated with the coin (i.e. planar orientation, temperature etc.). This is because entropy is an anthropomorphic concept and not a property of the physical system but a property of the particular experiment chosen to perform on it.

For our case N_x represents the total number of states. This represents the transverse resolution of our cooling system both the resolution of the pickup and the resolution of the kicker we can call it δx. This times the total possible positions gives us the maximum measurable beam size. Here we limit ourselves to the known maximum beam size which we call $\pm x_{max}$. This gives $N_x = \frac{2x_{max}}{\delta x}$. So if all we know about our system is that $p_i = 1/N_x$ or is uniform, then $I = 0$ or our information is zero.

With the first measurement the system now knows the position of x_i to within $< x >_s \pm x_{rms}/\sqrt{N_s}$ which makes the probability that our ith particle is in the jth state or position equal to,

$$p_j = \frac{e^{-(<x>_s - x_{max} + \delta x j)^2/(2x_{rms}^2)}}{x_{rms}\sqrt{2\pi}}\delta x. \tag{5}$$

Our information gain for the "ith" particle per turn can be calculated,

$$I = \log N_x + \sum_{j=1}^{N_x} p_j \log p_j \tag{6}$$

So we see as the sample size gets smaller or closer to single particle i our information becomes greater until $N_s = 1$ in which case the location of the jth particle is perfectly defined as either 1 for the position which contains the particle or zero for all other cases. This makes the $\sum_{j=1}^{N_x} p_j \log p_j = 0$, thus our maximum information is equal to the maximum potential entropy or $\log N_x$.

Another thing to point out is that if the particle distribution was static, that is there is no particle mixing between samples per turn, then each turn will sample the same distribution and return the exact same information, thus there would be no net gain in information about the particle distribution and thus no cooling.

Proceeding further one might try and define a minimum net information gain per turn for a given distribution which would permit the cooling process to overcome the additional uncertainty or entropy introduced from noise, inter-particle scattered or other sources.

From one point of view the process of mixing due to synchrotron motion might be viewed as driving a type of tomography which adds additional linearly independent equations to help resolve the individual particle's unique phase space coordinates. This might motivate other approaches to render more accurate information about the individual particles phase space trajectories using additional sampling and high precision lattice and longitudinal dynamics modeling.

REFERENCES

[1] Adami C. 2016 "What is information?" *Phil. Trans. R. Soc.* A 374:20150230.

[2] Simon Van Der Meer, "Stochastic Cooling and the Accumulation of Antiprotons" , Nobel lecture, 8 December, 1984 CERN, CH- 1211 Geneva 23, Switzerland.

[3] Shannon, C.E. (1948), "A Mathematical Theory of Communication", *Bell System Technical Journal*, 27, pp. 379–423 623–656, July October, 1948.

[4] Landauer, R., 1961, *IBM J. Res. Dev.* 5, 183

[5] M. D. Vidrighin *et. al., PRL* vol. 116, p. 050401, 2016.

[6] D. Mohl,"Stochastic Cooling," Conf. Proc. C 850916, 453, 1985.

[7] E. T. Jaynes, "Gibbs vs. Boltzmann entropies", *Am. J. Phys.* vol. 33, pp. 391–398, 1965, doi:10.1119/1.1971557.