SIMULATED MEASUREMENTS OF BEAM COOLING IN MUON IONIZATION COOLING EXPERIMENT*

Tanaz Angelina Mohayai†[1], Illinois Institute of Technology, Chicago, IL, USA
David Neuffer, Fermilab, Batavia, IL, USA
Chris Rogers, STFC Rutherford Appleton Laboratory, Oxfordshire, UK
Pavel Snopok[1], Illinois Institute of Technology, Chicago, IL, USA
for the MICE Collaboration
[1]also at Fermilab, Batavia, IL, USA

Abstract

Cooled muon beams are essential to high-luminosity Muon Collider and the production of high-flux and pure neutrino beams at the Neutrino Factory. When pions decay into muons, they form beams with large phase-space volumes. To optimize muon yield and fit the beam into cost-effective apertures, the beam phase-space volume needs to be reduced. Ionization cooling is the only technique that can reduce the beam phase-space volume within the short muon lifetime, and the international Muon Ionization Cooling Experiment (MICE) will be the first experiment to demonstrate this. A figure of merit for beam cooling is the transverse root-mean-square (RMS) emittance reduction. However, RMS emittance can be sensitive to non-linear effects in beam optics. This paper studies an alternative measure of cooling where a direct measurement of phase-space density and volume is made through the novel application of the Kernel Density Estimation (KDE) method.

INTRODUCTION

Ionization cooling in MICE Step IV (the current experimental configuration, Fig. 1) is achieved by passing the beam through a low-Z absorber where the beam's transverse momentum is reduced through energy loss, ultimately resulting in the desired reduction in the beam's phase-space volume. Two trackers are located upstream and downstream of the absorber, forming the cooling channel. Each tracker comprises five scintillating-fiber stations each with three doublet fiber layers. The Spectrometer Solenoids housing the trackers are each made of five superconducting coils, with two used for beam matching at the absorber (Match 1 and Match 2) and three for maintaining constant solenoidal fields in the tracking volumes (End 1, Center, and End 2) [1]. On September 2015, the Match 1 coil of the downstream Spectrometer Solenoid failed during magnet commissioning, and the current running configurations are designed to take this into account.

Phase-space density and volume are important concepts in describing the state of a system of particles. The phase-space density can be described as the probability density function (PDF) in phase space, where the PDF is the probability of finding a particle within a particular volume. With non-linear beam optics and beam distributions with a long tail, defining a parametric model for the distribution becomes challenging. The underlying PDF of such distributions can then be found using density estimation (DE) techniques. In non-parametric DE, no assumptions are made about the distribution parameters. KDE is an example of such a method which uses kernel functions to estimate the density.

KERNEL DENSITY ESTIMATION TECHNIQUE

In general, a six dimensional position-momentum phase space can be used to represent the individual muons in a MICE beam profile. In MICE Step IV, there is a particular interest in measuring the four dimensional transverse phase space. These coordinates are coupled in the presence of solenoidal fields. Muons in the beam distribution can be individually reconstructed in the trackers [2] obtaining position and kinetic momentum values for $\vec{X}_i = (x_i, p_{x_i}, y_i, p_{y_i})$, where i runs from 1 to n and n represents the total number of muons in the beam sample under study. When the Kernel Density Estimation (KDE) technique is applied in four dimensions to the MICE beam sample, Gaussian kernel functions in the form of multi-dimensional ellipses of variances $h = h_f\Sigma$ are centered at each muon. h and h_f are the bandwidth parameter and factor discussed in the next section, and Σ is the covariance matrix of the data set whose elements represent the amount of variances of each of the $\vec{X}_i$ coordinates. In MICE, dimension variable d is set to 4, representing the dimension of the transverse phase-space vector. The estimated density, $\hat{f}$, at an arbitrary point $\vec{x} = (x, p_x, y, p_y)$ in phase space is then determined by summing the contributions of the transverse coordinates $\vec{X}_i$ of all muons [3],

$$\hat{f}(\vec{x}) = \frac{|\Sigma|^{-1/2}}{n h_f^d \sqrt{(2\pi)^d}} \sum_{i=1}^{n} \exp\left[-\frac{(\vec{x}-\vec{X}_i)^T \Sigma^{-1} (\vec{x}-\vec{X}_i)}{2h_f^2}\right]. \quad (1)$$

A kernel function which acts as a weighting function, should satisfy a certain set of conditions for its sum to result in a PDF [4]: it should be non-negative and should integrate to 1. In addition, it should be symmetric about its center and its second moment should be finite [5]. The bandwidth parameter, h acts as the variance of the assigned

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† tmohayai@hawk.iit.edu

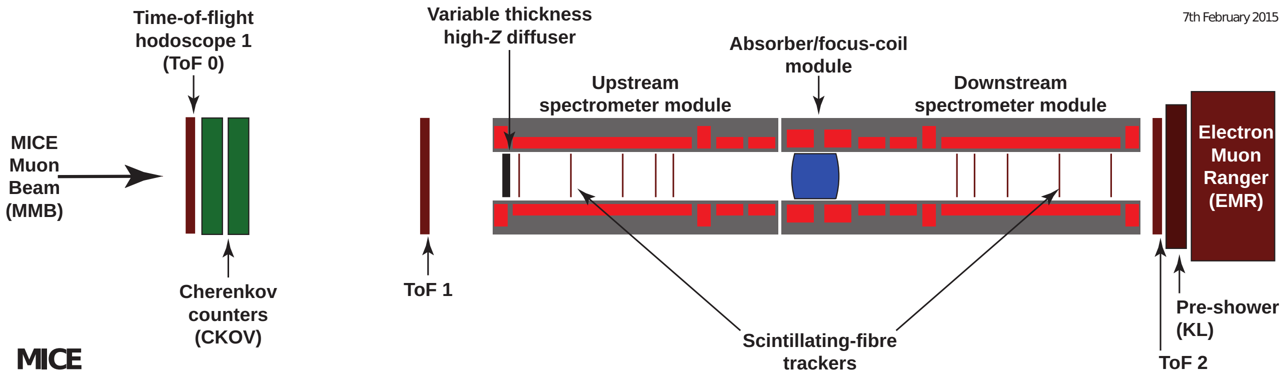

Figure 1: Schematic diagram of the Muon Ionization Cooling Experiment, MICE in its current Step IV configuration showing upstream and downstream spectrometers surrounding a cooling cell, and other particle ID detectors (ToF, Cherenkov, KL, and EMR).

kernel function and changing it has a strong effect on the estimated PDF; a large bandwidth parameter results in an over-smoothed density, while a small one results in an under-smoothed density. The bandwidth parameter is optimized such that the discrepancy between the estimated kernel density and the true PDF is the smallest. The criterion used to measure this discrepancy is the Asymptotic Mean Integrated Squared Error (AMISE) [4],

$$\text{AMISE}\left[\hat{f}\right] = \frac{1}{nh}C(K(\vec{x})) + \frac{h^4}{4}\mu(K(\vec{x}))C(f''), \quad (2)$$

where $C(K(\vec{x})) = \int K^2(\vec{x})dx$, $C(f'') = \int [f'']^2 dx$, $K(\vec{x})$ is the kernel function, and f'' represents the true PDF. $C(f'')$ represents the amount of roughness or curvature of the estimated distribution. The appropriate bandwidth parameter minimizes AMISE as defined in Eq. 2, and it is determined based on the relation

$$h = \left[\frac{C(K(\vec{x}))}{n\mu(K(\vec{x}))^2 C(f'')}\right]^{1/5}, \quad (3)$$

where $\mu(K(\vec{x})) = \int x^2 K(\vec{x})dx$ is the second moment of the kernel function. The larger the roughness $C(f'')$ of the distribution, the smaller the AMISE, which signifies that the appropriate bandwidth parameter, h should be small [4]. There are several methods for choosing the bandwidth based on the value of $C(f'')$. One method, called the rule of thumb computes $C(f'')$ for a known parametric family and multiplies the result by a scale parameter, determined from the data set [4, 6]. Scott's Rule [7] used in the `Scipy` module [8], uses the rule of thumb approach with a $C(f'')$ factor of $1.0n^{\frac{-1}{d+4}}$, and a scale parameter in the form of the data set covariance matrix in d-dimensions [9], $h = 1.0\Sigma n^{\frac{-1}{d+4}}$. Figure 2 illustrates the effect of varying the bandwidth parameter on the estimated PDF. The KDE technique is applied to the x coordinates of a subsample of 500 muons at the entrance of the upstream tracker (the distribution is described in the following section). The x distribution is approximately Gaussian and the use of the Scott's bandwidth parameter in the KDE routine approximately reveals a Gaussian distribution (displayed as the black curve in Fig. 2). When the Scott's bandwidth parameter is multiplied by a large factor, the estimated PDF is over-smoothed (green curve in Fig. 2). A smaller factor in the bandwidth parameter leads to a noisier estimated density (red curve in Fig. 2).

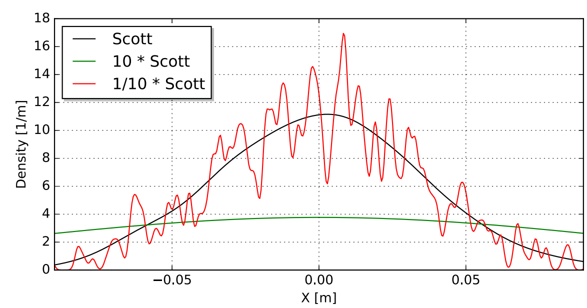

Figure 2: Plot of estimated density vs. x coordinates. Scott (black, solid) represents the use of Scott's bandwidth parameter in the `Scipy` module. The large bandwidth parameter, 10×Scott substantially over-smoothes the distribution. The small bandwidth parameter, (1/10)×Scott is noisier.

ANALYSIS RESULTS AND DISCUSSION

To generate an initial muon distribution, Monte Carlo Simulation (MCS) routines in MICE Analysis User Software (MAUS) [10] and Xboa [11] were used. The initial beam was chosen to be Gaussian in the transverse direction and was matched to the 3 T field of the upstream Spectrometer Solenoid. The input beam emittance and momentum were 6.0 π mm·rad and 200 MeV/c. The Xboa routine produces the initial distribution in the form of a BLTrack formatted beam file which is input into G4beamline [12], tracking 100,000 muons from upstream to downstream trackers in the MICE Step IV lattice (Fig. 1). The currents in the Spectrometer Solenoid and the Absorber Focus Coil modules are the default 4 T coil parameters in the MAUS geometry files [10, 13], scaled by 3/4 to produce the 3 T fields in

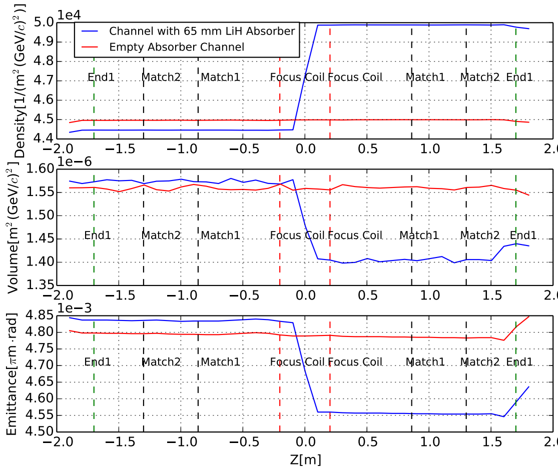

Figure 3: The preliminary density, volume and emittance evolution plots in the MICE Step IV channel. The red curves correspond to a channel with no absorber and the blue curves correspond to a channel with a 65 mm LiH absorber. The evolution curves remain constant in every region of an empty channel except at the downstream match coils.

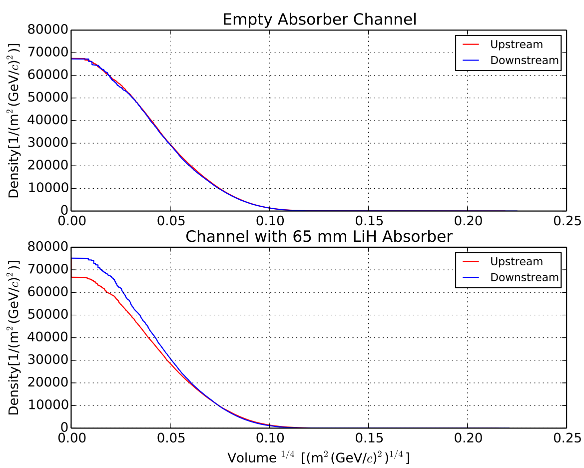

Figure 4: The preliminary density versus volume$^{1/4}$ plots. The horizontal axis represents the radius of the four-dimensional hyper-ellipsoid.

the modules. Unlike [13], a realistic MICE Step IV lattice is studied, where the fields in the downstream Match coils (Match 1 and Match 2) are set to zero to match the current running configurations. In addition, we only select muon tracks which make it to the downstream tracker. This selection results in a transmission efficiency of 85% in the cooling channel and 84% in the channel without an absorber.

Figure 3 displays the evolution plots of kernel density and volume, as well as the RMS emittance for channels with and without LiH absorbers. The information on each muon was obtained from G4beamline NTuple hits, recorded at every 0.1 m intervals in the MICE Step IV cooling channel. To obtain the density evolution plot, the Python `Scipy` package with Scott's bandwidth parameter was used [7, 8]. To compute the phase-space volume, a separate Monte Carlo (MC) method is used, where random points are thrown uniformly at a multi-dimensional rectangle in phase space, bounded by the minimum and maximum values of $\vec{X}_i$. Then, the volume of every contour of constant density is obtained by multiplying the volume of the rectangle by the ratio of the number of MC points within the contour over the total number of MC points [13]. The volume in Fig. 3 corresponds to the volume of a contour containing 9% of the total muons, representing the contour closest to 1σ of the beam. The Ecalc9 analysis routine in the ICOOL simulation package [14] was used to compute the transverse emittance shown in Fig. 3. The emittance and volume curves show cooling by 4% and 9%, respectively and the density increases by 12%. As expected by the Liouville theorem, the curves corresponding to an empty absorber lattice show conservation of these three quantities. The noise in the volume curve is statistical and is due to the randomness of the MC process used in the volume calculation. The spikes in the evolution curves at the downstream Match 2 coil are currently under investigation.

Figure 4 represents the densities at the locations of the upstream and downstream tracker planes closest to the absorber. In a channel with a 65 mm LiH absorber, the density at smaller volume$^{1/4}$ values increases as the beam passes through the absorber. In an empty channel, no change in density is observed. Volume$^{1/4}$ approximately represents the radius of the multi-dimensional ellipse, with zero representing the beam center. The density value at each radius represents the phase-space contours, and contours closer to the center have a higher density of muons.

CONCLUSIONS

The simulated MICE Step IV lattice studied in this paper has zero fields in the Match 2 and the inoperable Match 1 coils in the downstream Spectrometer Solenoids and shows cooling through phase-space density increase and phase-space volume decrease using the KDE technique. The same technique will be used to measure cooling using data from the experiment.

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