

SIMULATION OF A SKEW PARAMETRIC RESONANCE IONIZATION COOLING CHANNEL

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Abstract

Skew Parametric-resonance Ionization Cooling (Skew-PIC) is designed for the final 6D cooling of a high-luminosity muon collider. Tracking of muons in such a channel has been modeled in MAD-X in previous studies. However, the ionization cooling process has to be simulated with a code that can handle matter dominated beam lines. In this paper we present the simulation of a Skew-PIC channel using G4beamline. We implemented the required magnetic field components into G4beamline and compared the tracking of muons by the two different codes. We optimized the cooling channel and present the muon cooling effect in the Skew-PIC channel for the first time.

INTRODUCTION

A muon collider has many advantages comparing to electron-positron colliders and hadron colliders: muons are 207 times heavier than electrons so the synchrotron radiation is small when the muons are accelerated in a ring; muons are leptons and their colliding interactions are cleaner than those in hadron colliders which involves many protons colliding over a wide range of energies.

Ionization cooling is the most effective method to cool the muon beam from the large initial emittance. In order to reach the required luminosity of $10^{34} \text{cm}^{-2} \text{sec}^{-1}$, the muons are cooled in a series of cooling stages and the final cooling stage requires an extremely strong magnetic field. The parametric resonance ionization cooling (PIC) method [1] is designed to avoid such a strong magnetic field while providing a significant cooling effect. It induces a parametric resonance in the cooling channel, and focuses the beam in both horizontal and vertical planes with the period of the channel's free oscillations.

The theory and analytic calculations of the PIC channel has been developed. A twin-helix cooling scheme [2] has been designed and simulated to practice the PIC channel. However, it is found that the non-linear resonance caused by the multipole fields introduces an instability to the beam. To overcome the non-linear resonance problem, the concept of Skew-PIC has been developed: the canonical betatron tunes are shifted from their resonant values and the point-to-point focusing needed for PIC is preserved.

Figure 1 shows an example of the Skew-PIC channel. The magnetic field in the channel consists of dipole, quadrupole and skew quadrupole components (in the coupled case), and

the muon dynamics in such a channel with high-order fields correction is simulated in in MAD-X [3]. However, the cooling effect of the Skew-PIC channel requires involvement of beam-matter interactions, which MAD-X can not handle. In this work we implemented the required magnetic fields into G4beamline [4], and compared the tracking of muons by G4beamline and MAD-X. We then present the cooling effect of the Skew-PIC channel and outline further improvement steps for future work.

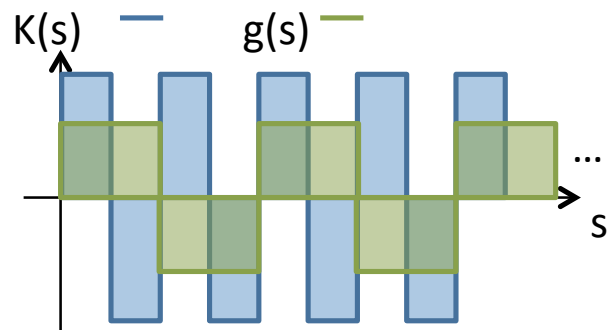


Figure 1: Curvature $K(s)$ and skew quadrupole coefficient $g(s)$ along the skew-PIC cooling channel's length s .

MULTIPOLE FIELDS IN G4BEAMLINE

Since Skew-PIC channel requires high-order fields in a bent magnet, we implement a magnetic field expansion in such a channel. In reference [5] this expansion is calculated in a bent solenoid up to the octupole order, and ready-to-use expressions for the magnetic field is given. We implemented this expansion into G4beamline and benchmarked the code with MAD-X simulations.

Figure 2 shows the trajectory of the reference particle in the uncoupled PIC channel simulated by MAD-X (upper plots) and G4beamline (lower plots). In MAD-X, the reference particle follows the centerline perfectly since MAD-X integrates the transportation matrix. However, the reference particle oscillates around the centerline in G4beamline: as the tracking distance becomes longer, the tracking error becomes greater. This is because G4beamline is tracking the particle step by step, and each step has a small error, which causes the overall error after a long distance. But this error (less than 1 mm in 160 m long channel) is acceptable.

Figure 3 shows the tracking of a gaussian distributed muon beam in the above PIC channel. The beam information is recorded at certain s positions along the channel. The

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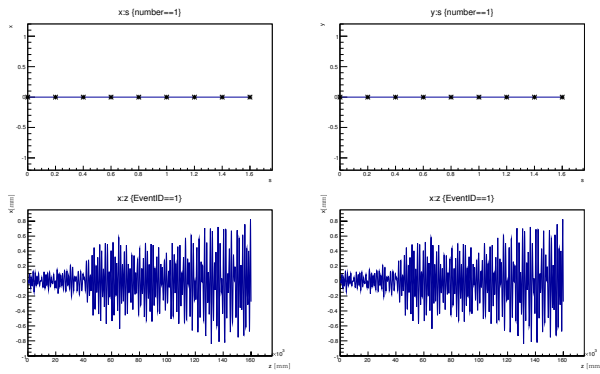


Figure 2: Tracking of the reference particle in the PIC channel by MAD-X (upper plots) and G4beamline (lower plots). Left plot: x versus s trajectory. Right: y versus s trajectory.

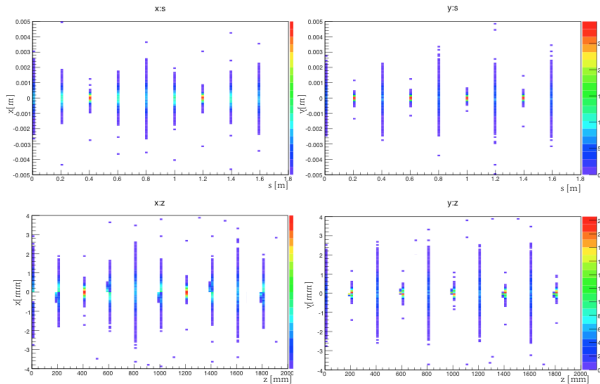


Figure 3: Tracking of a muon beam with a gaussian spatial distribution in the PIC channel by MAD-X (upper plots) and G4beamline (lower plots). Left plot: x versus s distribution. Right: y versus s distribution. Color represents the muon density.

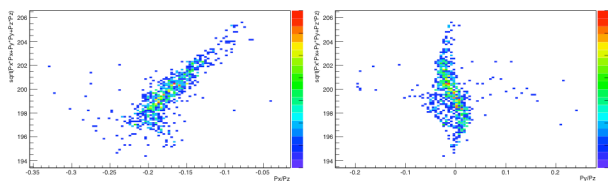


Figure 4: Muon momentum versus the angular spread in the horizontal (left) and vertical (right) plane.

upper plots are results from MAD-X, and the lower plots are simulated by G4beamline. The beam dynamics simulated by the two codes shows a perfect agreement in such a complex channel, which confirms the acceptance of the small tracking error.

COOLING SIMULATION

The beam dynamics in the Skew-PIC channel with high order fields corrections has been simulated and optimized in MAD-X [6]. In this paper we present the simulation of the beam cooling in G4beamline.

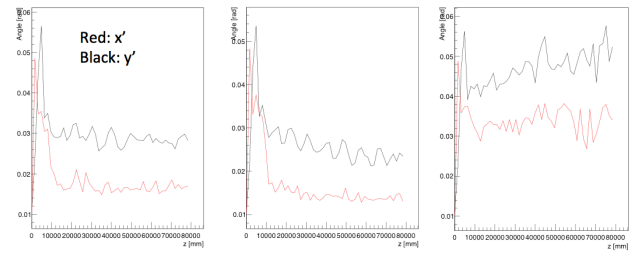


Figure 5: Angular spread in the horizontal (red curve) and vertical (black curve) plane along the Skew-PIC channel. Left: no cooling. Middle: cooling without stochastic processes. Right: stochastic processes on.

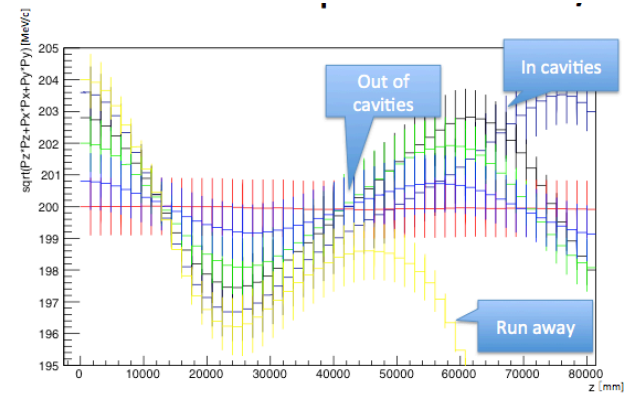


Figure 6: Momenta of six muons with initial momenta from 200 MeV/c to 204 MeV/c. The kinks on the curves represent the momentum change in the cavities.

An rf cavity is put at every focal point in the Skew-PIC channel. The beryllium window of the cavity is 2.5 mm thick each, and is used as the absorber. We start with a beam with no spatial spread and with an angular spread of 10 mrad. The momentum spread of the beam source is 2 MeV/c.

Because the bending of the magnet is only in the horizontal plane, the beam dispersion is also in this plane. Figure 4 shows a correlation between the muon momentum and the angular spread at the focal points. Muons with larger momentum deviation are dispersed more in the horizontal plane; and there is no such correlation in the vertical plane.

Figure 5 shows the angular spreads in the two planes along the 80 m long channel. Starting from a small number (10 mrad) the angular spreads prompt up and quickly reach the stable value. It is compressed by the cooling effect at the focal points when the stochastic processes are turned off. The multipole scattering process introduces a large angular dispersion at the absorbers.

The momentum spread of the beam is also an important source that causes the beam instability. Figure 6 shows the momenta of six representative muons with initial momenta from 200 MeV/c to 204 MeV/c. Muons with initial momenta deviation less than 4 MeV/c are accepted by the channel, other muons are not stable and will be lost eventually.

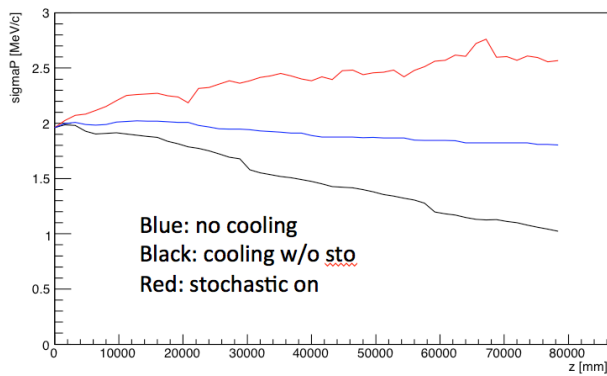


Figure 7: Change of momentum spread of muons along the channel. Blue curve: no rf cavities. Black curve: cooling without stochastic processes. Red curve: stochastic processes on.

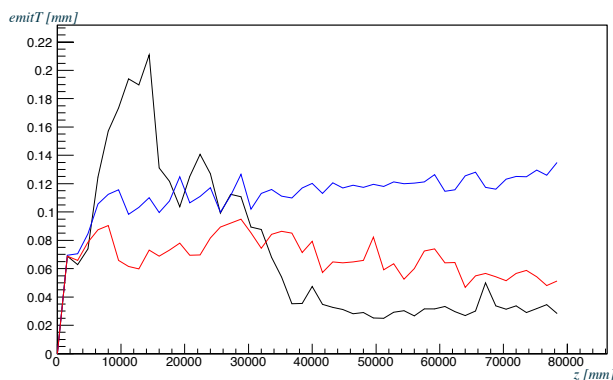


Figure 8: Transverse emittance of the muon beam along the simulated channel. Blue curve: no rf cavities. Red curve: cooling without stochastic processes. Black curve: stochastic process on.

Figure 7 shows the momentum spread of the muon beam along the channel. When the rf cavities are absent, the momentum spread is relatively stable at 2 MeV/c. The slight decrease comes from the particle loss with large deviation. When the cooling process is on, the muon momentum spread

is compressed quickly to 1 MeV/c. However, the fluctuation of energy loss process introduces more momentum spread.

Figure 8 shows the transverse emittance of the muon beam along the simulated Skew-PIC channel. The muon beam is cooled transversely by a factor of 2 in a 80 m long Skew-PIC channel when the stochastic processes are turned off. The transmission in this channel is about 92%. The multiple scattering and fluctuation of the energy loss cause a great instability to the beam and most of the muons are lost to the beam pipe.

SUMMARY AND OUTLOOK

The multipole fields that are needed by the Skew-PIC simulation has been implemented into G4beamline. The muon cooling in the Skew-PIC channel has been simulated. The cooling effect is obvious when the stochastic processes are absent. However the beam becomes unstable when the multiple scattering and energy loss fluctuation processes are turned on.

More study are in progress to increase the acceptance of the Skew-PIC channel. This includes tuning the coupling strength, adding high-order harmonics to the channel, optimizing the absorber thickness and the rf cavity strength. Also a global optimization will be performed with a genetic algorithm in the future study.

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