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ANALYSIS OF BEAM POSITION MONITOR REQUIREMENTS WITH BAYESIAN GAUSSIAN REGRESSION

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Abstract

With a Bayesian Gaussian regression approach, a systematic method for analyzing a storage ring's beam position monitor (BPM) system requirements has been developed. The ultimate performance of a ring-based accelerator, based on brightness or luminosity, is determined not only by global parameters, but also by local beam properties at some particular points of interest (POI). BPMs used for monitoring the beam properties, however, cannot be located at these points. Therefore, the underlying and fundamental purpose of a BPM system is to predict whether the beam properties at POIs reach their desired values. The prediction process can be viewed as a regression problem with BPM readings as the training data, but containing random noise. A Bayesian Gaussian regression approach can determine the probability distribution of the predictive errors, which can be used to conversely analyze the BPM system requirements. This approach is demonstrated by using turn-by-turn data to reconstruct a linear optics model, and predict the brightness degradation for a ring-based light source. The quality of BPMs was found to be more important than their quantity in mitigating predictive errors.

INTRODUCTION

The ultimate performance of a ring-based accelerator is determined not only by certain critical global parameters, such as beam emittance, but also by local properties of the beam at particular points of interest (POI). The capability of diagnosing and controlling local beam parameters at POIs, such as beam size and divergence, is crucial for a machine to achieve its design performance. Examples of POIs in a dedicated synchrotron light source ring include the undulator locations, from where high brightness X-rays are generated. In a collider, POIs are reserved for detectors in which the beam-beam luminosity is observed. However, beam diagnostics elements, such as beam position monitors (BPM) are generally placed outside of the POIs as the POIs are already occupied.

Using observational data at BPMs to indirectly predict the beam properties at POIs can be viewed as a regression problem and can be treated as a supervised learning process: BPM readings at given locations are used as a training dataset. Then a ring optics model with a set of quadrupole excitations as its arguments is selected as the hypothesis. From the dataset, an optics model needs to be generalized first. Based on the model, the unknown beam properties at POIs can be predicted. However, there exists some systematic error and random uncertainty in the BPMs' readings, and the quantity of BPMs is limited. Therefore, the parameters in the reconstructed optics model have inherent uncertainties, as do the final beam property predictions at the POIs. The precision and accuracy of the predictions at the POIs depend on the quantity of BPMs, their physical distribution pattern around the ring, and their calibration, resolution, etc. When a BPM system is designed for a storage ring, however, it is more important to consider the inverse problem: i.e. How are the BPM system technical requirements determined in order to observe whether the ring achieves its desired performance? In ref. [1] and this paper, we developed an approach to address this question with Bayesian Gaussian regression.

In statistics, a Bayesian Gaussian regression [2, 3] is a Bayesian approach to multivariate regression, i.e. regression where the predicted outcome is a vector of correlated random variables rather than a single scalar random variable. Every finite collection of the data has a normal distribution. The distribution of generalized arguments of the hypothesis is the joint distribution of all those random variables. Based on the hypothesis, a prediction can be made for any unknown dataset within a continuous domain. In our case, multiple BPMs' readings are normally distributed around their real values. The standard deviations of the Gaussian distributions are BPM's resolutions. A vector composed of quadrupoles' mis-settings is the argument to be generalized. The prediction at the POIs is the function of this vector. The continuous domain is the longitudinal coordinate s along a storage ring.

BRIGHTNESS PERFORMANCE AND BEAM DIAGNOSTICS

Consider a dedicated light source ring. Its ultimate performance is measured by the brightness of the X-rays generated by undulators. The brightness is determined by the transverse size of both the electron and photon beam and their angular divergence at their source points [4–7]. Therefore, the undulator brightness performance $\mathcal{B}$ depends on the ring's global emittance and the local transverse optics parameters,

$$\begin{aligned} \mathcal{B} &\propto \frac{1}{\Sigma_x\Sigma'_x\Sigma_y\Sigma'_y} \\ \Sigma_{x,y} &= \sqrt{\epsilon_{x,y}\beta_{x,y}+\eta^2_{x,y}\sigma^2_\delta+\sigma^2_{ph}} \\ \Sigma'_{x,y} &= \sqrt{\epsilon_{x,y}\gamma_{x,y}+\eta'^2_{x,y}\sigma^2_\delta+\sigma'^2_{ph}}. \end{aligned} \quad (1)$$

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Here $\epsilon_{x,y}$ are the electron beam emittances, which represent the equilibrium between the quantum excitation and the radiation damping around the whole ring. β, γ are the Twiss parameters [8], η, η' are the dispersion and its derivative at the undulators' locations, σ_δ is the electron beam energy spread $\sigma_{ph} = \frac{\sqrt{\lambda L_u}}{2\pi}$ and $\sigma'_{ph} = \frac{1}{2}\sqrt{\frac{\lambda}{L_u}}$ are the X-ray beam diffraction "waist size" and its natural angular divergence, respectively. The X-ray wavelength λ, is determined based on the requirements of the beam-line experiments, and L_u is the undulator periodic length. The emittance was found to be nearly constant with small β-beat. Therefore, monitoring and controlling the local POI's Twiss parameters is crucial.

The final goal of beam diagnostics is to provide sufficient, accurate observations to reconstruct an online accelerator model. Modern BPM electronics can provide the beam turn-by-turn (TbT) data, which is widely used for the beam optics characterization and the model reconstruction. Based on the model, we can predict the beam properties not only at the locations of monitors themselves, but more importantly at the POIs. The capability of indirect prediction of the Twiss parameters at POIs eventually defines the BPM system requirements on TbT data acquisition. Based on Eq. (1), how precisely one can predict the bias and the uncertainty of Twiss parameters β and η at locations of undulators is the key problem in designing a BPM system. Therefore, to specify the technical requirements of a BPM system, the following questions need to be addressed: in order to make an accurate and precise prediction of beam properties at POIs, how many BPMs are needed? How should the BPMs be allocated throughout the accelerator ring, and how precise should the BPM TbT reading be?

GAUSSIAN REGRESSION FOR MODEL RECONSTRUCTION AND PREDICTION

When circulating beam in a storage ring is disturbed, a BPM system can provide its TbT data at multiple longitudinal locations. TbT data of the BPMs can be represented as an optics model plus some random reading errors,

$$x(s)_i = A(i)\sqrt{\beta(s)}\cos\left[i \cdot 2\pi\nu + \phi(s)\right] + \varepsilon(s)_i, \qquad (2)$$

here i is the index of turns, $A(i)$ is a variable dependent on turn number, $\beta(s)$ is the envelope function of Twiss parameters at s location, ν is the betatron tune, ϕ is the betatron phase, and $\varepsilon(s)_i$ is the BPM reading noise [9–11], which generally has a normal distribution. Based on the accelerator optics model defined in Eq. (2), we can extract a set of optics Twiss parameters at all BPM locations [12–15]. Recently, Ref. [16] proposed using a Bayesian approach to infer the mean (aka expectation) and uncertainty of Twiss parameters at BPMs simultaneously. The mean values of β represent the most likely optics pattern. The random BPM reading error and the simplification of the optics model can result in some uncertainties, ε_β, in the inference process,

$$\beta = \beta(s, \boldsymbol{K}) + \varepsilon_\beta(s), \qquad (3)$$

here $\boldsymbol{K}$ is a vector composed of all normalized quadrupole focusing strengths, and ε_β is the inference uncertainty. Unless otherwise stated, bold symbols, such as "$\boldsymbol{X}$", are used to denote vectors and matrices throughout this paper. In accelerator physics, the deviation from the design model β_0 is often referred to as the β-beat. From the point of view of model reconstruction, the β-beat is due to quadrupole excitation errors and can be determined by

$$\Delta\beta = \beta(s, \boldsymbol{K}_0 + \Delta\boldsymbol{K}) - \beta_0(s, \boldsymbol{K}_0) \approx \boldsymbol{M}\Delta\boldsymbol{K}, \qquad (4)$$

where $\boldsymbol{K}_0$ represents the quadrupoles' nominal setting and β_0 is the nominal envelope function along s. $\boldsymbol{M}$ is the response matrix composed of elements $M_{i,j} = \frac{\partial\beta_{s_i}}{\partial K_j}$ observed by the BPMs. The dependency of β on $\boldsymbol{K}$ is not linear in a complete optics model. However, when quadrupole errors are small enough, the dependence can be approximated as a linear relation which significantly simplify our problem.

Given a set of measured optics parameters βs at multiple locations s from BPM TbT data, the posterior probability of the quadrupole error distribution $p(\Delta\boldsymbol{K}|\boldsymbol{\beta})$ can be given according to Bayes theorem [17],

$$\begin{aligned} p(\Delta\boldsymbol{K}|\boldsymbol{\beta}) &= \frac{p(\boldsymbol{\beta}|\Delta\boldsymbol{K})p(\Delta\boldsymbol{K})}{p(\boldsymbol{\beta})} \\ &\propto p(\boldsymbol{\beta}|\Delta\boldsymbol{K})p(\Delta\boldsymbol{K}). \end{aligned} \qquad (5)$$

Here $p(\boldsymbol{\beta}|\Delta\boldsymbol{K})$ is referred to as the likelihood function, which corresponds to the linear response matrix normalized by measurement resolution. The prior quadrupole excitation error distribution $p(\Delta\boldsymbol{K})$ can be determined by comparing them against the design optics model,

$$\begin{aligned} p(\Delta\boldsymbol{K}) &= \mathcal{N}(\Delta\boldsymbol{K}|0, \sigma^2_{\Delta K}) \\ &= \frac{1}{\sqrt{2\pi}\sigma_{\Delta K}}\exp\left[-\frac{\Delta\boldsymbol{K}^2}{2\sigma^2_{\Delta K}}\right], \end{aligned} \qquad (6)$$

with,

$$\sigma_{\Delta K} \sim \kappa|\Delta\beta| = \kappa|\bar{\beta} - \beta_0|. \qquad (7)$$

Here "$\sim$" describes a statistically proportional relationship between β-beats (in the unit of "m") and quadrupole strength error ΔK (in units of m^{-2}). The coefficient κ can be computed based on the optics model either analytically or numerically before carrying out any measurements.

To predict the uncertainty at POIs, the output of all possible posterior quadrupole error distributions must be averaged,

$$\begin{aligned} p(\Delta\boldsymbol{\beta}_*|\boldsymbol{s}_*, \Delta\boldsymbol{\beta}, \boldsymbol{s}) &= \int p(\Delta\boldsymbol{\beta}_*|\boldsymbol{s}_*, \Delta\boldsymbol{K})p(\Delta\boldsymbol{K}|\Delta\boldsymbol{\beta}, \boldsymbol{s})\mathrm{d}\boldsymbol{K} \\ &= \mathcal{N}(\boldsymbol{m}_*, \boldsymbol{\Sigma}^2_*). \end{aligned} \qquad (8)$$

Here $\Delta\boldsymbol{\beta}_*$ is the predicted result at POIs' locations $\boldsymbol{s}_*$ given the measured $\Delta\boldsymbol{\beta}$ at $\boldsymbol{s}$. The mean values and the variances of the predicted distributions at POIs are

$$\begin{aligned} \boldsymbol{m}_* &= \sigma_\beta^{-2}\boldsymbol{M}_*\boldsymbol{A}^{-1}\boldsymbol{M}^T\Delta\bar{\boldsymbol{\beta}} \\ \boldsymbol{\Sigma}^2_* &= \boldsymbol{M}_*\boldsymbol{A}^{-1}\boldsymbol{M}^T_*, \end{aligned} \qquad (9)$$

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$\boldsymbol{M}_*$ is the Jacobian matrix of the optics response to quadrupole errors observed at POIs. The difference between the mean value $\boldsymbol{m}_x$ and the real β at a POI is referred to as the predicted bias. By substituting the bias and uncertainty back into Eq. (1), we can estimate how accurate the brightness could be measured for given BPMs' resolutions. Based on the desired brightness resolution, we can determine the needed quantity and resolution of BPMs.

APPLICATION TO NSLS-II RING

We use the NSLS-II ring and its BPM system TbT data acquisition functionality to demonstrate this approach. The NSLS-II undulator (POIs) are located at non-dispersive straights. Typical photon energy from undulators is around 10 keV. The undulators' period length is 20 mm. The horizontal beam emittance is 0.9 $nm \cdot rad$ including the contribution from 3 damping wigglers. The emittance coupling ratio can be controlled to less than 1%. At its 15 short straight centers, the Twiss parameters are designed to be as low as $\beta_{x,y}$ = 1.80, 1.20 m, and $\alpha_{x,y}$ = 0 to generate the desired high brightness x-ray beam from the undulators.

The horizontal emittance growth with an optics distortion was found to be small. Degradation of an undulatorís brightness is mainly determined by its local optics distortion which can be evaluated with Eq. (1). Multi-pairs of simulated $\beta - \alpha$ were incorporated into the previously specified undulator parameters to observe the dependence of the X-ray brightness on the β-beat. A change of approximately 1% of the $\beta_{x,y}$ in the transverse plane can degrade the brightness by about 1%. Because multiple undulators are installed around the ring, the predicted performance needs to be evaluated at all POIs simultaneously.

First we studied the dependence of predictive errors on the quantity of BPMs. A comprehensive simulation was set up to compare the Gaussian regression predictive errors with the real errors. A linear optics simulation code was used to simulate the distorted optics due to a set of quadrupole errors. The β-beats observed at the BPMs were marked as the "real" values. On top of the real values, 0.5% random errors were added to simulate one-time measurement uncertainty seen by the BPMs. A posterior distribution of the quadrupole errors was obtained by reconstructing the optics model with the likelihood function, and the prior distribution (6) and (7). The predicted optics parameters with their uncertainties were then calculated based on another likelihood function between quadrupoles and the locations of undulators with Eq. (8).

Next, we studied the effect of β measurement resolution on the predictive errors. A similar analysis was carried out but with different β-resolution as illustrated in Fig. 1. By observing Fig. 1, several conclusions can be drawn: (1) The degradation of the β resolution reduced the accuracy of the generalized optics model. However, this can be improved by applying a more complicated optics model [16]. Thus, the BPM TbT resolution is the final limit on the resolution of β parameters. In order to accurately and precisely predict the beam properties at POIs, improving the resolution of BPMs is crucial. (2) After a certain point, the predicted performance is not improved significantly with the quantity of BPMs as seen in Fig. 1. The advantage of reduction of predictive errors will gradually level out once enough BPMs are used. Meaning that quantitatively, the improvement in error reduction will eventually become negligible compared to the cost of adding more BPMs. The higher the resolution each individual BPM has, the less number of BPMs are needed. There should be a compromise between the required quality and quantity of BPMs to achieve an expected predictive accuracy. (3) The quality (resolution) is much more important than the quantity of BPMs from the point of view of optics characterization. For example, at NSLS-II, in order to resolve 1% brightness degradation, at least 120 BPMs with a β resolution better than 1% are needed, or 90 BPMs with a 0.75% resolution, etc. Having more BPMs than is needed creates no obvious, significant improvement. Having 60 high precision (0.5% β-resolution) BPMs yields a better performance than having 180 low precision (1%) BPMs in this example.

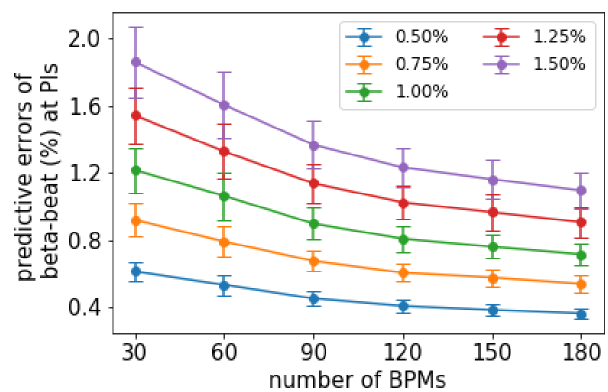

Figure 1: Predictive β-beat errors (including bias and uncertainties) at the locations of undulator (POIs). βs are observed with different number of BPMs and different resolutions. The resolution of β is the final limit on predictive errors. The higher the resolution each individual BPM has, the less number of BPMs are needed.

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