ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-TUPLM15

ARBITRARY TRANSVERSE PROFILE SHAPING USING TRANSVERSE WIGGLERS

G. Ha†, M.E. Conde, J.G. Power, Argonne National Laboratory, Lemont, IL, USA

Abstract

Transverse wigglers provide a sinusoidal vertical magnetic field along the horizontal direction. This magnetic field imparts a sinusoidal modulation on the horizontal phase space. Since sine and cosine functions are the basis of periodic function space, the combination of multiple wigglers would be able to impart arbitrary correlation on the horizontal phase space. In this paper, we numerically demonstrate the application of this new method for arbitrary transverse profile shaping.

INTRODUCTION

Argonne Wakefield Accelerator (AWA) group demonstrated arbitrary longitudinal shaping capability of the emittance exchange (EEX) beamline in 2016 [1]. Several different transverse masks were used to shape the beam transversely, and the transmission through the mask was around 40%. The masking is one of the easiest ways to control the profile, but this low transmission would make a significant drop in the beam quality due to a higher charge requirement in the gun, and it can make thermal issues for high repetition rate or high intensity beams.

We recently proposed a scheme to generate a tunable bunch train using an EEX beamline with a transverse wiggler [2]. This wiggler provides a sinusoidal magnetic field which makes a sinusoidal modulation on the transverse phase space. If the beam passes the series of transverse wigglers with different periods and strengths, one can correlate the particle's horizontal position and momentum arbitrarily. This new method can open up a new way to control all longitudinal properties including arbitrary current profile shaping without charge loss. The following sections describe the related theoretical background and numerical demonstration of the method for shaping application.

PRINCIPLE OF ARBITRARY SHAPING USING TRANSVERSE WIGGLERS

Profile shaping using wigglers requires two steps. Firstly, an appropriate correlation function should be defined to generate a desired profile. This correlation function can be found from the relationship with beam parameters and beamline parameters.

Particle transport can be described by the matrix formalism. If the particle's initial coordinate is (x_0, x_0'), its final horizontal position can be written as,

$$x_f = R_{11}x_0 + R_{12}x_0'. \quad (1)$$

† gha@anl.gov

If one applies arbitrary correlation (f) to the initial horizontal phase space, particle's momentum term (x_0') should be replaced to $x_{0,old}' + f(x_0)$. Here, we ignore the particle's initial momentum to simplify the calculation and only consider newly added arbitrary correlation.

When the particle's final coordinate is (x_f, x_f') and profiles are expressed as N, the initial and final profiles have a relationship as below due to the charge conservation.

$$N_f(x_f)\mathrm{d}x_f = N_0(x_0)\mathrm{d}x_0. \quad (2)$$

By substituting the final coordinate (x_f) to Eq. (1), Eq. (2) can be rewritten to,

$$N_f(R_{11}x_0 + R_{12}f)\{R_{11} + R_{12}f'\} = N_0(x_0). \quad (3)$$

If there is a desired final profile, Eq. (3) shows the required correlation function to generate the desired profile in the given system (i.e. beam transport and initial profile are fixed).

The next step is to correlate x and x'. To generate an arbitrary correlation, we use the concept of Fourier expansion. The summation of cosine functions can approximate an arbitrary function. Here we use transverse wigglers to generate cosine modulation on the horizontal phase space.

The transverse wiggler provides the alternating magnetic field which can be described as,

$$B_y \cong -2B_r \cos\left(\frac{2\pi}{\lambda_w}x\right)\cosh\left(\frac{2\pi}{\lambda_w}y\right)\exp\left(-\frac{\pi}{\lambda_w}g\right), \quad (4)$$

where B_r is the residual induction of the magnet, λ_w is the magnetic period of the wiggler, and g is the gap of the wiggler. Cosine term in this field provides cosine correlation on the horizontal phase space which will be the building block for cosine series constructing the correlation function.

To determine how many wigglers we need and other wiggler parameters such as gap and length, both Fourier expansion and genetic algorithm based optimization are tried.

APPLICATION TO ARBITRARY PROFILE CONTROL

In this section, we provide two simple examples of wiggler based shaping. We derived required correlation functions for each example using Eq. (3). Then, Fourier expansion is applied to the first example to find the wiggler setting while genetic optimization is used for the second example. Particles are generated and correlation is numerically applied to the distribution. This correlated particle

ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-TUPLM15

distribution is linearly transported for 1 m drift to demonstrate the method.

Uniform profile from Gaussian profile

If the desired profile is a uniform profile with the width $2w$. The final profile can be defined as,

$$N_f(x_f) = \begin{cases} C & when\ |x_f| \le w \\ 0 & when\ |x_f| > w \end{cases}. \tag{5}$$

The correlation function to generate this uniform profile should satisfy the relationship given in Eq. (3). This equation can be rewritten as below using Eq. (5).

$$C\{R_{11} + R_{12}f'\} = N_0(x_0). \tag{6}$$

If we assume the Gaussian function as the initial profile, the correlation function in Eq. (6) becomes,

$$f(x_0) = \frac{1}{2CR_{12}} \mathrm{erf}\left(\frac{x_0}{\sigma_{x_0}}\right) - \frac{R_{11}}{R_{12}} x_0, \tag{7}$$

where erf is the error function and σ_{x_0} is the initial rms beam size. Fig. 1a shows the shape correlation function with $R_{11} = 1$, $R_{12} = 1$ m, and $\sigma_{x_0} = 1$ mm.

Figure 1b shows two horizontal profiles. The blue one is the initial Gaussian profile with rms size of 1 mm. This profile is evolved to the orange curve after particles with the correlation traverse 1 m long drift. The core part of the profile shows a flat area as expected, and there are spikes near the edge of the profile due to the folding on the phase space. This is typical behavior that we can see from the Gaussian to uniform profile conversion [3].

The correlation function from Eq. (7) works as expected, so we need to find a wiggler setting to approximate this correlation function. Fourier transform is applied to this correlation function to find Fourier coefficients. Due to the symmetric characteristic of the correlation function, only cosine coefficients are non-zero. Here the fundamental period is 17.7 mm and we only consider first two modes for the shaping. Coefficients for the first and third harmonic are -1.12E-3 and 1.04E-3 respectively. These numbers correspond to the gap of 18.4 mm and 6.3 mm for the 2 mm long transverse wigglers with Br of 1 T.

This small wiggler array provides a correlation function close to the ideal correlation function in Fig. 1a. As a result, the Gaussian profile changes to the orange curve in Fig. 1c. The core part of the profile shows uniform distribution clearly. Due to the discrepancy near the edge of the correlation function, the spike formation in the horizontal profile is different from ones in Fig. 1b. It starts to have rising and falling length and the peak of each spike is reduced by this lengthening.

Although we don't provide related figures in this paper, adding more wigglers corresponding to higher harmonics makes the profile from the wiggler almost identical to the one from the correlation function directly.

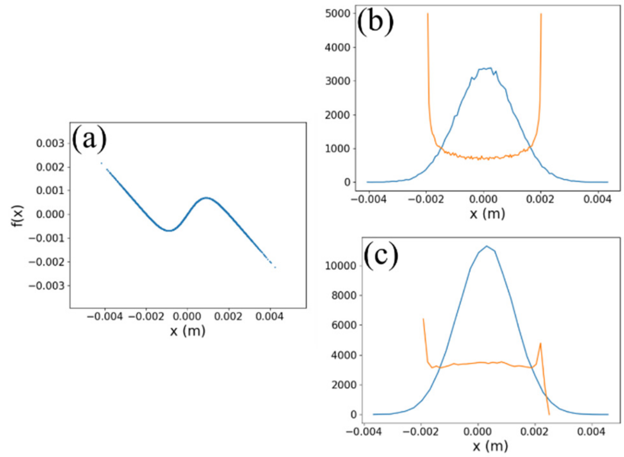

Figure 1: Numerical demonstration of the concept. Particles are generated and numerically tracked. (a) shows the correlation function calculated from the Eq. 7. (b) shows the initial Gaussian profile (blue) and shaped profile directly by the correlation function (orange). (c) shows the shaped profile from two wigglers (orange).

Triangle profile from quadratic profile

The second example is a triangle profile. Here we assume initially quadratic profile to avoid the spike that Gaussian profile generates as shown in Fig. 1b.

A triangle profile can be defined as,

$$N_f(x_f) = \begin{cases} -\frac{I_0}{2w}(x_f - C) + \frac{I_0}{2} & when\ |x_f - C| \le w \\ 0 & when\ |x_f - C| > w \end{cases}. \tag{8}$$

The relationship for the correlation function in Eq. (3) becomes a little bit more complicated as,

$$\left(-\frac{I_0}{2w}R_{11}x_0 - \frac{I_0}{2w}R_{12}f + \frac{I_0}{2}\right)\{R_{11} + R_{12}f'\} = N_0(x_0), \tag{9}$$

This ODE has a well-known form to solve. The correlation function can be written as,

$$f(x_0) = \frac{1}{R_{12}}\left\{w + C - R_{11}x_0 - \sqrt{2w^2 - \frac{4w}{I_0}\int N(s)ds}\right\}, \tag{10}$$

As mentioned earlier, $N(s)$ will be a quadratic function to represent the quadratic profile as shown in Fig. 2b. Then, Eq. (10) provides a simple shape given in Fig. 2a. We assumed 2 mm wide quadratic profile with zero offset term (i.e. C=0) for the figure.

ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-TUPLM15

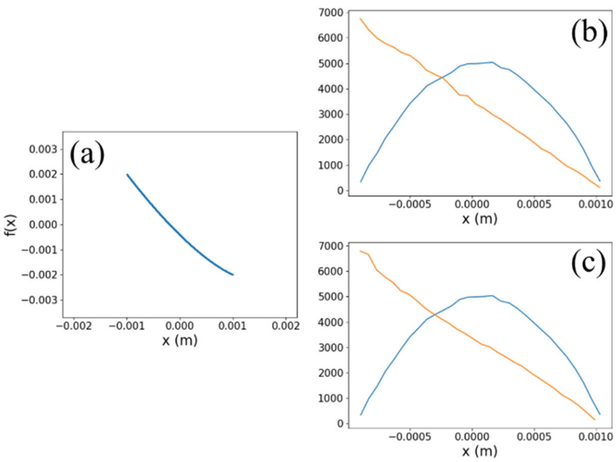

Figure 2: Numerical demonstration of the concept. Particles are generated and numerically tracked. (a) shows the correlation function calculated from the Eq. 10. (b) shows the initial quadratic profile (blue) and shaped profile directly by the correlation function (orange). (c) shows the shaped profile from two wigglers (orange).

As shown in Fig. 2b, the correlation function in Eq. (10) successfully converts a quadratic profile to a triangle profile with a hard edge on the one end. This is a promising result to significantly improve the transmission issue that masking methods have [1, 4]. This high shaping quality can be preserved if the conversion to longitudinal profile by EEX beamline is well-controlled [5, 6].

For the wiggler setting, we used genetic optimization for this case to find related wiggler parameters. Strength, period and phase are the optimization variables, and we used two wigglers only. The result is shown in Fig. 2c, and it provides a well-shaped triangle profile as the one from the correlation function. If we assume the length of the wigglers are 2 mm and Br of 1 T. The gap for the first and second wigglers are 14.1 mm and 6.3 mm. Periods are 40.0 mm and 5.6 mm. The phase will be interpreted as horizontal offset of the wiggler position, and they are +9.9 mm and -0.2 mm.

EFFECT FROM ALTERNATING HORIZONTAL MAGNETIC FIELD

While the wiggler provides a cosine correlation that we use for the correlation control, it also generates an alternating horizontal magnetic field along the horizontal direction. This field is expressed as,

$$B_x \cong 2B_r \sin\left(\frac{2\pi}{\lambda_w}x\right)\sinh\left(\frac{2\pi}{\lambda_w}y\right)\exp\left(-\frac{\pi}{\lambda_w}g\right). \qquad (11)$$

When the B-field in Eq. (11) exists, this field kicks the particle vertically and the direction depends on the particle's horizontal position (see Fig. 3). This behavior can significantly increase the momentum spread on the vertical phase space which means a significant emittance growth.

However, this field can be suppressed significantly by focusing particles tightly in the vertical direction when they pass wigglers. Since the beam requires a vertical focusing due to the gap of the wiggler, we automatically gain this suppression. Also, the required length for each wiggler is only 2 mm, so tight focusing can be easily achieved.

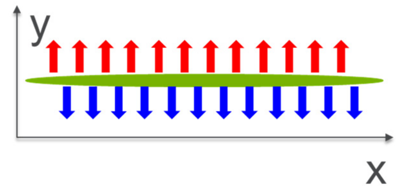

Figure 3: Conceptual figure showing the vertical momentum kick from transverse wigglers.

While we track the particle for the profile shaping, we also tracked the particle's vertical motion. The initial vertical beam size is 0.1 mm with a zero slope. Figure 4 shows the vertical phase space after both wigglers and 1 m long drift. Both cases show a few to tens of micro-radian changes on the phase space. This is small enough change to ignore in terms of emittance (nm scale).

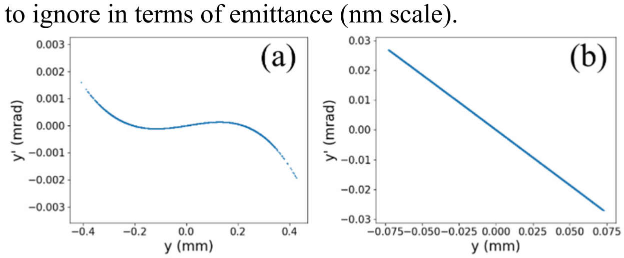

Figure 4: Vertical phase space after wigglers. Particle distribution started from zero-divergence with a finite size. (a) and (b) correspond to uniform and triangle generation cases.

SUMMARY

We introduced a new method to control the correlation between transverse position and momentum coordinates. Transverse wigglers are used to introduce sinusoidal correlations on the transverse phase space. This became a building block to approximate arbitrary correlation that we need for applications. Two shaping examples worked as expected with only two short wigglers (2 mm long).

ACKNOWLEDGMENT

This work is supported by the Department of Energy, Office of HEP and BES, under Contract No. DE-AC02-06CH11357. It is also supported by Argonne LDRD program under project number PRJ 1006621.

REFERENCES

[1] G. Ha *et al.*, "Precision control of the electron longitudinal bunch shape using an emittance-exchange beam line," *Phys. Rev. Lett.*, vol. 118, p. 104801, 2017.

[2] G. Ha, M. E. Conde, J. G. Power, J. H. Shao, and E. E. Wisniewski, "Tunable Bunch Train Generation Using Emittance Exchange Beamline With Transverse Wiggler," in *Proc. 10th Int. Particle Accelerator Conf. (IPAC'19)*, Melbourne, Australia, May 2019, pp. 1612-1614. doi:10.18429/JACoW-IPAC2019-TUPGW089

ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-TUPLM15

[3] Y. Yuri, N. Miyawaki, T. Kamiya, W. Yokota, K. Arakawa, and M. Fukuda, "Uniformization of the transverse beam profile by means of nonlinear focusing method," *Phys. Rev. ST Accel. Beams*, vol. 10, p. 104001, 2007.

[4] P. Muggli *et al.*, "Simple method for generating adjustable trains of picosecond electron bunches", *Phys. Rev. ST Accel. Beams*, vol. 13, p. 052803, 2010.

[5] D. Y. Shchegolkov and E. I. Simakov, "Design of an emittance exchanger for production of special shapes of the electron beam current," *Phys. Rev. ST Accel. Beams*, vol. 17, p. 041301, 2014.

[6] G. Ha, M. H. Cho, W. Gai, K.-J. Kim, W. Namkung, and J. G. Power, "Perturbation-minimized triangular bunch for high-transformer ratio using a double dogleg emittance exchange beam line", *Phys. Rev. Accel. Beams*m, vol. 19, p. 121301, 2016.