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CALCULATION OF ACTION DIFFUSION WITH CRABBED COLLISION IN eRHIC*

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Abstract

To compensate the geometric luminosity loss due to the crossing angle in eRHIC, crab cavities are to be installed on both sides of the interaction point. When the proton bunch length is comparable to the wavelength of the crab cavities, protons in the bunch head and tail will not be perfectly tilted in the x-z plane. This may cause synchro-betatron resonance and even coherent beam-beam instability. In the article, we develop a simulation method to calculate the transverse action diffusion rate and study its dependence on the beam-beam related machine and beam parameters.

INTRODUCTION

The 2015 Nuclear Science Advisory Committee Long Rang Plan identified the need for an electron-ion collider (EIC) facility as a gluon microscope with capabilities beyond those of any existing accelerator complex. To reach the required high energy, high luminosity, and high polarization, the eRHIC design, based on the existing heavy ion and polarized proton collider RHIC, adopts a very small β-function at the interaction points (IPs), a high collision repetition rate, and a novel hadron cooling scheme.

The maximum beam-beam parameters for the electron and proton beams in eRHIC are targeted at $\xi_e = 0.1$ and $\xi_p = 0.015$, respectively. These choices of beam-beam parameters are based on the successful operational experiences of KEKB and RHIC. However, such high beam-beam parameters have never been demonstrated in the any previous proton-electron colliders. Especially, due to lack of the radiation damping, the long-term stability of protons with beam-beam interaction and crab cavities is one of the most important concerns we have to pay attention to.

In the present eRHIC design, a full crossing angle of 25 mrad at the interaction regions is adopted. To compensate the geometric luminosity due to the crossing angle, crab cavities are to be installed to tilt the proton and electron bunches by 12.5 mrad in the x-z plane at the IPs so that the two beams collide head-on in the head-on collision frame.

In the early weak-strong and strong-strong simulations, we observe proton beam size growth and luminosity degradation. Their change rates show strong dependences on the crab cavity frequency, the proton longitudinal and transverse tunes, the proton bunch length, and so on. All of them indicates that there is coupled motion between the transverse and longitudinal motions through beam-beam interaction.

Table 1: Beam-beam Interaction Related Machine and Beam Parameters Used in this Article

quantity	unit	proton	electron
Beam energy	GeV	275	10
Bunch intensity	10^{11}	1.05	3.0
β^* at IP	cm	(90, 5.9)	(63, 10.4)
Beam sizes at IP	μm	(112, 22.5)	
Bunch length	cm	7	1.9
Energy spread	10^{-4}	6.6	5.5
Transverse tunes		(0.31, 0.305)	(0.08, 0.06)
Longitudinal tune		0.01	0.069

In this article, instead of time-consuming direct massive calculation of beam size growth and luminosity degradation rates in million-particle and million-turn tracking [1, 2], we evaluate the so-called action diffusion rate in a relatively short-term tracking and with a much smaller number of macro-particles. This method had been previously used to SSC, LHC, and other colliders to determine the long-term stability of protons with beam-beam interaction. However, the direct connections between the action diffusion rate and the real emittance growth is not straightforward. We still need direct tracking to confirm in the end to confirm the findings from action diffusion rate calculation. In this article, we use the lattice and beam parameters described in the eRHIC design parameters v5.1 as shown in Table 1.

SIMULATION SETUP

In the action diffusion rate calculation, we use the weak-strong code SimTrack [3]. We assume that the electron bunch is rigid and the electron bunch is perfectly crabbed in the head-on collision frame. For each simulation case, we track 200 protons with identical initial transverse actions $J_{x,y}$. However, their phases in the phase space (x, p_x, y, p_y) are randomly assigned between 0 and 2π. We track these particles up to 100,000 turns.

In the code, the proton ring is represented by a 6×6 linear matrix. There is no coupling between the horizontal, vertical, and longitudinal planes. However, the particle's transverse tunes are adjusted turn-by-turn based on the settings of linear chromaticities and the particle's relative momentum deviation. The beam-beam interaction takes place at the interaction point (IP). The calculation of beam-beam

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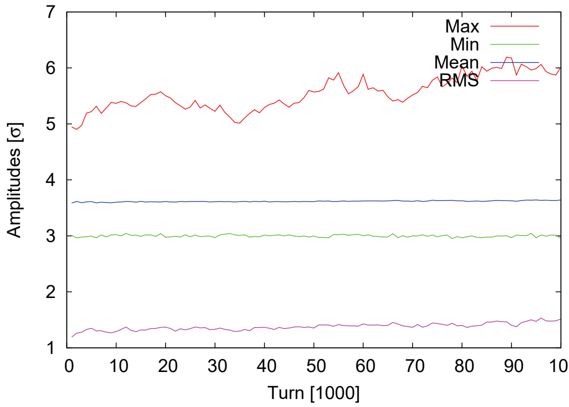

Figure 1: Example: evolutions of maximum, minimum, average, and RMS spread of horizontal actions.

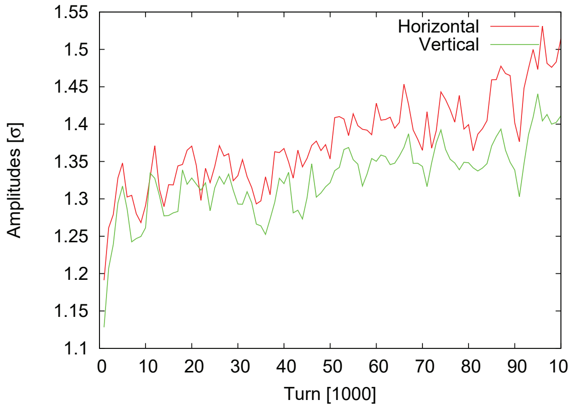

Figure 2: Example: evolutions of horizontal and vertical RMS action spreads.

interaction is based on the synchro-beam mapping according to Hirata [4]. The electron bunch is split into 3 slices longitudinally. The kicks from the proton crab cavities are included in the code. The crab cavity voltage is calculated based on the crossing angle, the crab cavity frequency, and the particle energy.

In the code, we calculate each test particle's transverse actions every turn. In 1000 turns or 1 tracking step, we record the maximum and the minimum transverse actions among the 200 test protons. We also calculate the average transverse actions and the RMS spread of transverse actions. We track all particle up to 100 tracking steps, or 100×1000 turns. From the last 80 steps, we calculate the change rates of average actions and RMS spread of actions. The change rate of RMS spread of actions are called action diffusion rate. In this article, we use σ to measure actions. For example, 2 σ corresponds 4 $(2J_{rms})$.

As an example, we calculate the action changes with the eRHIC design parameters v5.1. Action diffusion rates can be calculated with any initial transverse actions in the (N_X, N_y) plane, where $N_{x,y} = \sqrt{J_{x,y}/J_{x,y,0}}$. As usual, most of the time we track particles along the 45° direction. The radial transverse beam size is measured in unit of σ too, $N_t = \sqrt{N_x * N_x + N_y * N_y}$. In the 45° direction in the (N_X, N_y) plane, $N_t = \sqrt{(2)}N_{x,y}$, where $N_x = N_y$. To thoroughly study the action diffusion for the full phase space, we need to track in other phase angles in the (N_X, N_y) plane.

Figure 1 shows the evolutions of maximum, minimum, average horizontal actions, and RMS spread of horizontal actions of the 200 protons over 100,000 turns. The initial transverse actions for all those test particles is $N_t = 5$, and the longitudinal action is $N_l = 3$. The crab cavity frequency is 394 MHz. From the plot, the minimum and average actions keep relatively stable, while the maximum action keeps on increasing over the course.

Figure 2 shows the evolutions of horizontal and vertical RMS spread of actions of these 200 test particles. From the plot, both show steady increase during the tracking period. In the following, we will focus on the change rate of RMS action spread. We will fit the raw RMS action spreads from the last 80 steps with a linear function. As shown in Figure 2, the raw data of RMS action spreads are very noisy. Therefore, there will be a large error bar in the calculated action diffusion rate.

SIMULATION RESULTS

Head-on and Crabbed Collision

We first calculate and compare the action diffusion rates with head-on collision and with crabbed collision. In the crabbed collision, the full crossing angle is 25 mrad. The frequency of the proton crab cavities is 394 MHz. The initial transverse beam size is 5 σ and the longitudinal beam size is 3 σ_l. Figure 3 shows the results. From the plot, there is no much difference in the calculated RMS action growth rates between head-on and crabbed collision for particles with transverse beam size less than 4 σ_t. Beyond 4 σ_t, crabbed collision shows a much larger diffusion rate than the head-on collision case.

Dependence on Longitudinal Amplitude

Next we calculate the transverse action diffusion rates as a function of particle's longitudinal amplitude. In the study, we assume crabbed collision situation. The initial particle transverse amplitude is 5 σ_t. We scan the longitudinal amplitude from 0 to 3 σ_l with a step size of 1 σ_l. Figure 4 shows the results. From it, There is no difference in transverse diffusion rates up to 6 σ_t for particles with a longitudinal amplitude less than 2 σ_l. Particles with a longitudinal amplitude larger than 2 σ_l begin to show increased transverse diffusion rates when the transverse amplitudes are larger than 4 σ_t.

Dependence on Crab Cavity Frequency

The transverse action diffusion rates are related to the crossing angle and how the bunches are crabbed. In the design of eRHIC crab cavities, two frequencies 394 MHz and 197 MHz are considered. From the physics view of proton stability, 197 MHz crab cavities are preferred. However, 197 MHz crab cavities pose more technical and engineering

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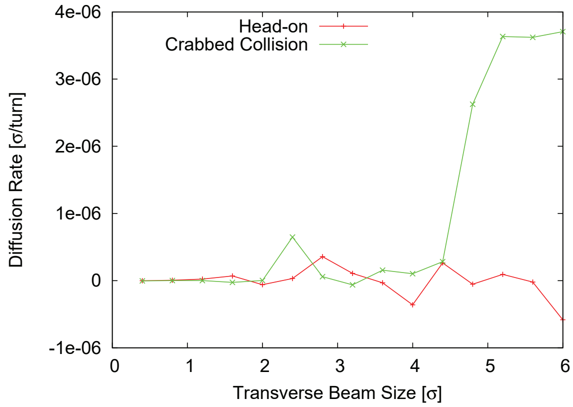

Figure 3: Comparison of diffusion rates with head-on and crabbed collisions.

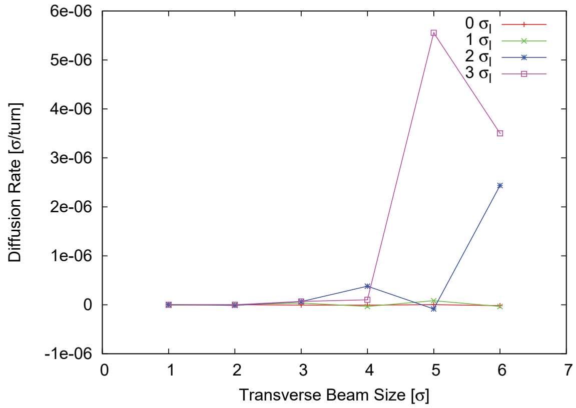

Figure 4: Comparison of diffusion rates with different longitudinal actions.

challenges than 394 MHz crab cavities. Figure 5 compares the calculated transverse action diffusion rates with 197 MHz and 394 MHz crab cavities. Clearly, 197 MHz gives a smaller diffusion rate than 394 MHz for large transverse amplitude particles.

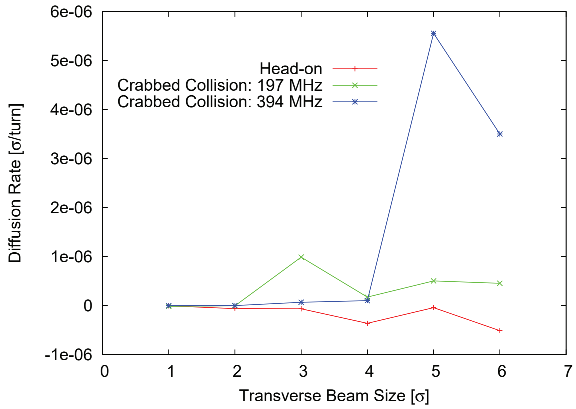

Figure 5: Comparison of diffusion rates with different crab cavity frequencies.

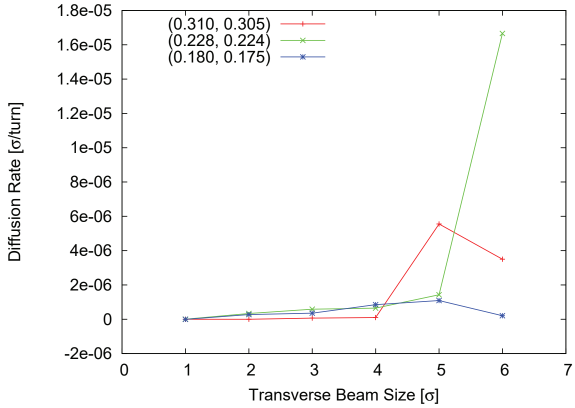

Figure 6: Comparison of diffusion rates with different proton tunes.

Proton Transverse Tune Scan

We also calculated the action diffusion rates in a proton transverse tune scan. We observed smaller action diffusion rates in some tune spaces along the diagonal line in the proton transverse tune space. Figure 6 compares the calculated diffusion rates with 3 proton working points. (0.310, 0.305) is the original eRHIC proton design tunes, which was used in the above calculations. (0.228, 0.224) is close to the tunes used in the RHIC ion operation. (0.180, 0.175) is close to the original design tunes for the RHIC polarized proton operation. From Figure 6 , both (0.228, 0.224) and (0.180, 0.175) show smaller action diffusion rates for particles with transverse amplitude larger than 4 σ_t.

SUMMARY

In this article, we calculated and compared the transverse action diffusion rates with head-on and crabbed collision with eRHIC machine and beam parameters v5.1. We also studied different crab cavity frequencies and proton transverse tunes. Although these calculations are only done for particles in the 45° direction in the transverse action space, the results reveal more or less physics behind the emittance growth and luminosity degradation rates calculated from the massive million-particle and million-turn tracking. Since it takes less time to calculate the diffusion rates, this method is less expensive for the scan studies of machine and beam parameters. Of course, the findings from the action diffusion rate calculation still need final million-particle and million-turn tracking to confirm.

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