

# HELICAL TRANSMISSION LINE TEST STAND FOR NON-RELATIVISTIC BPM CALIBRATION \*

C. Richard<sup>†</sup>, Michigan State University, East Lansing, MI, USA  
S. Lidia, Facility for Rare Isotope Beams, East Lansing, MI, USA

## Abstract

Measurements of non-relativistic beams by coupling to the fields are affected by the properties of the non-relativistic fields. The authors propose calibrating for these effects with a test stand using a helical line which can propagate pulses at low velocities. Presented are simulations of a helical transmission line for such a test stand which propagates pulses at  $0.033c$ . A description of the helix geometry used to reduce dispersion is given as well as the geometry of the input network.

## INTRODUCTION

Measurements of beam properties using devices that couples to the fields generated by the beam, such as beam position monitors, are made easier if the beam is assumed to be relativistic. For slow beams, such as in the front end of heavy ion accelerators, the measured fields will be traveling non-relativistically,  $v < 0.1c$ , causing the measured results will be distorted. The measurements can be corrected using analytic solutions of the fields [1] and simulations [2]. However, the authors are unaware of any test stand used to calibrate beam-line devices for non-relativistic perturbations.

A test stand for this purpose will be strung through the device under test to replicate the structure and velocity of the bunch. Such test stands have been created using Goubau lines to replicate the fields from electron beams [3]. However, Goubau lines cannot propagate pulses slow enough to simulate non-relativistic beams. The authors propose using a helical transmission line. These lines can theoretically propagate pulses at arbitrarily low phase velocities based on the pitch of the helix [4]. In order to use helical transmission lines in a test stand, the impedance and dispersion must be characterized to ensure reasonable matching and pulse propagation.

## DISPERSION

In order to produce the specific pulse shape at the device under test, it is ideal to have a constant phase velocity so any pulse input into the transmission line will maintain the pulse shape throughout propagation. To calculate the phase velocity of a helical transmission line, the sheath helix model was used. This model approximates the helix as a thin cylinder where the current is forced to travel in a helical path along the surface with pitch angle,  $\psi$ . The boundary conditions at

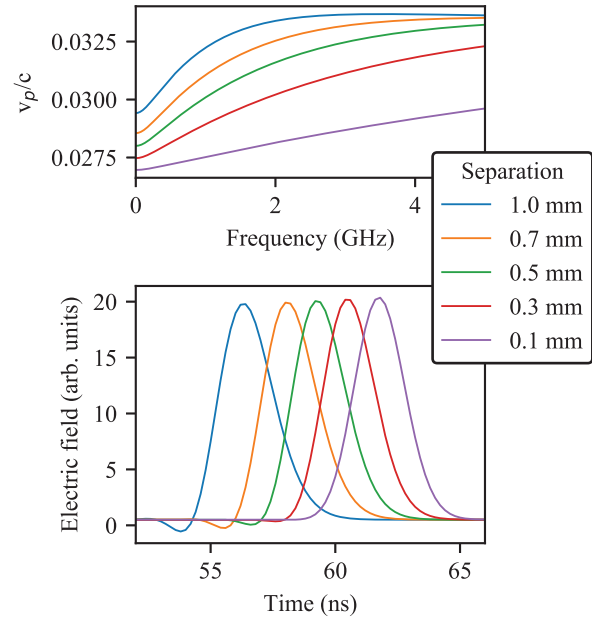


Figure 1: Top: phase velocities with varying separation ( $s$ ). All other dimensions are given in Table. 1. Bottom: the corresponding pulse deformation of a 1 ns rms pulse after propagating 0.5 m.

the sheath helix in polar coordinates are [4]

$$E_z^i = E_z^e \quad (1)$$

$$E_\theta^i = E_\theta^e \quad (2)$$

$$E_z^{i,e} = -E_\theta^{i,e} \cot(\psi) \quad (3)$$

$$H_z^i + H_\theta^i \cot(\psi) = H_z^e + H_\theta^e \cot(\psi). \quad (4)$$

where the superscripts  $i$  and  $e$  denote the interior and exterior regions of the helix. This model of the helical transmission line uses a sheath helix centered inside a conducting pipe. From these boundary conditions it was found the dispersion from a helix has large variation with frequency causing the pulse shape to deteriorate. For example, a 5 mm radius helix with  $\psi = 0.05$  rad will have the phase velocity reduced from  $0.085c$  to  $0.05c$  over 0.25 GHz. While the input pulse can be tailored to evolve under dispersion to the correct profile at the device under test [5], the generation of such pulses is complicated.

A more practical solution is to add a conducting rod inside of the helix to increase the capacitance of the system. Increasing the capacitance lowers the phase velocity at low frequency while leaving the high frequency limit unchanged

\* Work supported by the US Department of Energy, Office of Science, High Energy Physics under Cooperative Agreement award number DE-SC0018362 and Michigan State University.

<sup>†</sup> richard@nsl.msu.edu

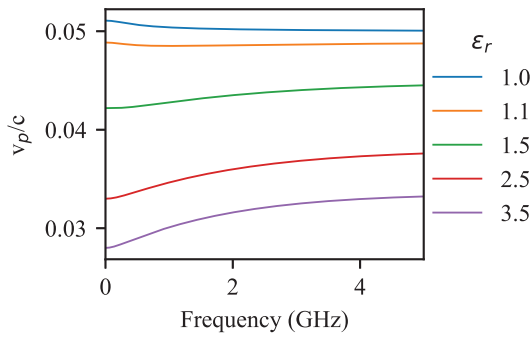


Figure 2: Phase velocity variations with different dielectric constants.

Table 1: Helix Parameters

Pipe radius	20.65 mm
Inner conductor radius	4.5 mm
Helix radius	5 mm
Separation, $s$	0.5 mm
Pitch angle	0.05 rad
Dielectric constant	3.5
Helix wire width	1 mm
Helix wire thickness	0.1 mm
phase velocity	0.033c

(Fig. 1 top). The low frequency limit linearly increases with separation  $s$  between the helix and inner conductor. Decreasing the separation maintains the same high frequency limit,  $v_p = c \cdot \sin(\psi)$ , but causes it to converge more slowly. The slow convergence results in less variation in  $v_p$  and so a narrow separation should be used when constructing a helical transmission line to limit the deformation of pulses (Fig. 1 bottom).

The helix must be supported by a dielectric layer around the inner conductor. In the limiting case with the dielectric constant  $\epsilon_r = 1$  between the inner conductor and the helix, the high frequency limit of the phase velocity is  $v_p = c \cdot \sin(\psi)$ . As the dielectric constant increases this drops approximately as  $\epsilon_r^{-1/3}$  for a fixed geometry while the low frequency limit drops as  $\epsilon_r^{-1/2}$  (Fig. 2). By using the correct dielectric constant, it is possible to align the high and low frequency limits. While this is an optimal solution, the reduced slope from using a small enough  $s$  makes it possible to propagate with little deformation for any  $\epsilon_r$ .

### Simulations

The phase velocity was measured with time domain simulations performed in CST Microwave Studio [6] for the geometry given in Table 1 (Fig. 3). The simulations were performed up to 2 GHz to excite lowest mode which does not have a cut off frequency. The next highest mode of a 5 mm sheath helix has a cut off frequency of  $\sim 10$  GHz and will not be excited [5].

A Gaussian pulse was input into the system and the radial electric field at the wall was measured using probes along the transmission line. The signals from each probe were transformed into the frequency domain to find the phase as a function of frequency. The phase difference between two probes was used to calculate phase velocity at frequency  $f$

$$v_p(f) = \frac{Lf}{(\phi_2 - \phi_1)} \quad (5)$$

where  $L$  is the separation between the probes and  $\phi_i$  is the phase at the  $i^{th}$  probe.

With a simulation up to 1 GHz, the measured phase velocity agrees with the sheath helix model within 3% up to 0.75 GHz. Above this the signal becomes dominated by noise and the phase velocity diverges (Fig. 4).

## IMPEDANCE

Once the dispersion relation is found, a complete description of the fields is known and the impedance can be calculated. For the three conductor geometry described above, two separate impedances can be defined: between the inner conductor and the helix, and between the helix and pipe. In general, these are similar except at low frequencies (Fig. 5).

To avoid reflections due to changes to the impedance caused by variations in construction, geometry parameters were varied in the analytic model to determine to which ones the system is most sensitive. As with dispersion, it is best to minimize the separation between the helix and the inner radius to reduce the impedance variations with frequency. More importantly, the separation between the helix and inner conductor must be constant along the transmission line to limit large variations in the impedance for even small changes (Fig. 5). For example, a 5% change in the separation can cause more than a 40% change in the low frequency limit of the impedance (Fig. 6). However, the radius of the inner conductor can vary as long as the helix radius also changes to keep the separation the same. Other geometry factors, such as the pitch and outer pipe radius, have a minimal impact on the impedance compared to the separation (Fig. 6).

### Simulations

The signal is coupled to the helical transmission line using a discrete port on a microstrip line with the bottom plate connected to the inner conductor and the upper strip connects to the helix (Fig. 3). The outer pipe is isolated at the input side. The microstrip impedance was set to the low frequency limit of the helical line impedance. The same microstrip is used at the end of the helix with an additional resistor connecting the helix and the pipe to match external fields at the output. Simulations were performed for the geometry in Table 1. Due to the relatively constant impedance up to 2 GHz a more complicated matching geometry was unnecessary.  $S_{1,1}$  was less than -30 dB at low frequency and below -15 dB up to 2 GHz.

The impedance of the helical line was measured using  $S_{1,1}$ . However, with the matching geometry described above,

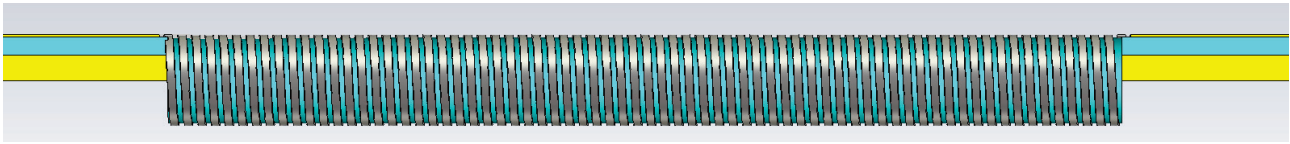


Figure 3: Example helix with microstrip matching on both ends.

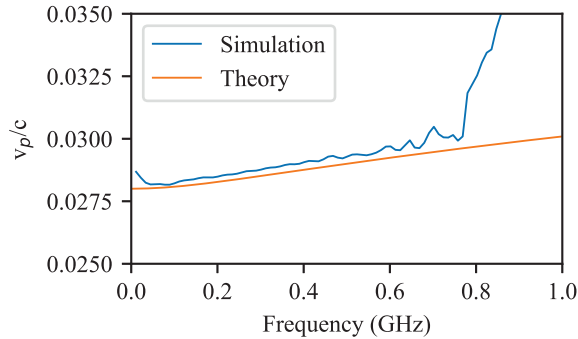


Figure 4: Comparison of phase velocity from simulation and analytic model using the parameters in Table 1. The probes are separated by 123 mm.

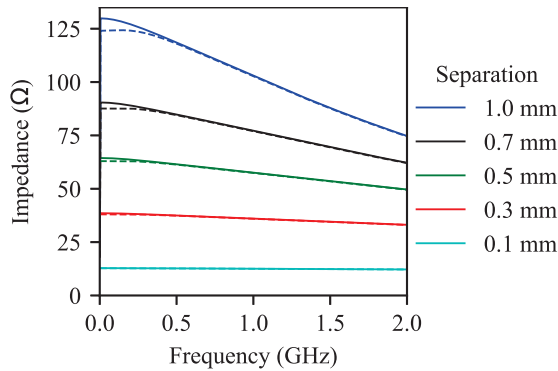


Figure 5: Impedance variations for different separation ( $s$ ) by changing the inner conductor radius. Solid lines represent the inner impedance and dashed line the outer impedance.

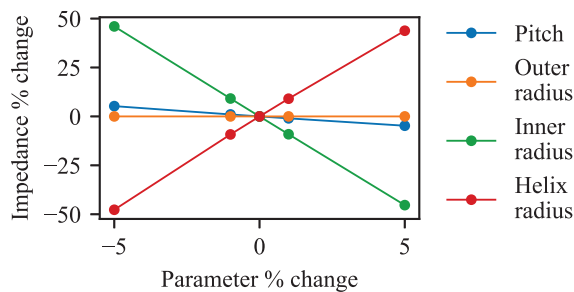


Figure 6: variation of inner impedance by changing the parameters in Table 1.

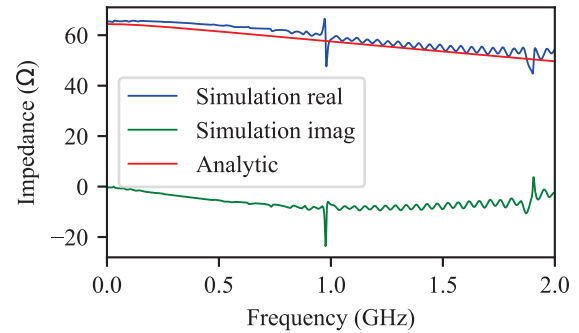


Figure 7: Comparison of the impedance from analytic model and simulation. The spike at 1 GHz is a resonance of the system length.

resonances were seen at frequencies where the length of the system was a harmonic of half wavelengths making it difficult to accurately measure the impedance. The resonances were damped out by changing the microstrip impedance to  $105 \Omega$  so as to be unequal to the helix impedance,  $64.5 \Omega$ , and matching with a resistive L-network. With this network the helix impedance is given by:

$$Z_{\text{helix}} = \left( R_{sh}^{-1} + \left[ Z_0 \frac{1 - S_{1,1}}{1 + S_{11}} + R \right]^{-1} \right)^{-1} \quad (6)$$

where  $Z_0$  is the impedance of the microstrip,  $R$  is the series resistance, and  $R_{sh}$  is the shunt resistance.

The real part of the impedance from simulations agrees with the analytic model within 3% up to 2 GHz (Fig. 7). The simulation also showed a small reactance that was <15% of the real impedance. This was not predicted by the analytic model using the sheath helix and it is believed is caused by the simulation using a finite width wire for the helix and not a sheath helix. While it is possible to design a reactive matching network to improve the match, current models do not account for the reactance. The mismatch using only the microstrip is deemed small enough for current studies.

## SUMMARY

The sheath helix approximation was used to study the impedance and phase velocity of helical pulse line. The results were compared to a computational model with good agreement up to 2 GHz. Variations in the phase velocity were reduced by adding a conducting rod inside the helix. The conducting rod also reduced the variations in the impedance allowing for matching using only a microstrip.

## REFERENCES

- [1] R. Shafer, "Beam position monitor sensitivity for low- $\beta$ beams," in *Proc. LINAC'94*, Tsukuba, Japan, Aug, 1994, pp. 905-907.
- [2] O. Yairet. al., "FRIB beam position monitor pick-up design," in *Proc. IBIC'17*, Monterey, CA, USA, Sep, 2014, paper TUP16, pp. 355-360.
- [3] F. Stulle, J. Bergoz, J. Musson, "Goubau line beam instrumentation testing, the benefits", in *Proc. IPAC'14*, Desden, Germany, July, 2014, paper THPME096, pp. 3462-3464
- [4] S. Sensiper, "Electromagnetic wave propagation on helicalconductors," Ph.D. thesis, MIT, 1954.
- [5] C. Richard and S. Lidia, "Simulating non-relativistic beams using helical pulse lines", in *Proc. IPAC'18*, Vancouver, BC, Canada, April 2018, paper WEPAL049, pp. 2288-2290.
- [6] CST Studio Suite, <https://www.cst.com>