ISBN: 978-3-95450-223-3 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2019-TUYBB4

# ONLINE MODELLING AND OPTIMIZATION OF NONLINEAR INTEGRABLE SYSTEMS*

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### *Abstract*

Nonlinear integrable optics was recently proposed as a design approach to increase the limits on beam brightness and intensity imposed by fast collective instabilities. To study these systems experimentally, a new research electron and proton storage ring, the Integrable Optics Test Accelerator, was constructed and recently commissioned at Fermilab. Beam-based diagnostics and online modelling of nonlinear systems presents unique challenges - in this paper, we report on our efforts to develop optimization methods suited for such lattices. We explore the effectiveness of neural networks as fast online surrogate estimators, and integrate them into a beam-based tuning algorithm. We also develop a method of knob dimensionality reduction and subsequent robust multivariate optimization for maximizing key performance metrics under complicated lattice optics constraints.


## INTRODUCTION

High intensity accelerators with strong space charge effects often exhibit current-limiting collective instabilities. A novel mitigation approach proposed by Danilov and Nagaitsev [1] is to suppress these with nonlinear integrable optics (NIO) lattices, which produce strong amplitude-dependent tune-shifts and hence, via Landau damping, prevent resonantly coupling energy into the beam. Previously, such tune-shifts were achieved with standalone elements like octupoles [2] at the cost of dynamic aperture degradation [3], a disadvantage that NIO mitigates. However, NIO lattices impose a number of linear and nonlinear optics constraints that must be carefully met and maintained, making conventional tuning techniques insufficient or difficult to apply. In this paper, we present several exploratory efforts to design more suitable methods, and study applicability of recently proposed machine learning approaches [4, 5].

### *Integrable Optics*

An ideal strong-focusing lattice is a linear system that has no amplitude-dependent tune shifts and is fully integrable. Due to misalignments, field errors, and the need to correct chromaticity and induce tune spread, real accelerators have significant nonlinearities which break integrability. Their set of initial conditions with regular motion is limited to a finite region, called the dynamic aperture (DA) - preserving its size is critical for achieving good accelerator performance.

* This work was supported by the U.S. National Science Foundation under award PHY-1549132, the Center for Bright Beams. Fermi Research Alliance, LLC operates Fermilab under Contract DE-AC02-07CH11359 with the US Department of Energy.
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Mathematically, the Hamiltonian for transverse particle dynamics is

$$H = \frac{1}{2}\left(K_x(s)x^2 + K_y(s)y^2 + p_x^2 + p_y^2\right) + V(x, y, s)$$

with $K_{z=x,y}$ being the linear focusing strength, and $V(x, y, s)$ containing any nonlinear terms (in general dependent on time ($\equiv s$) and transverse $(x, y)$ position). DN approach is to seek solutions for $V$ that yield two invariants of motion and are implementable with conventional magnets. First invariant comes from appropriate time scaling of $V(x, y)$, such that it becomes a time-independent potential $U(x_N, y_N)$ in normalized coordinates. It is furthermore possible to derive a specific form of $U(x_N, y_N)$ that yields second invariant of motion $I$, which we omit for brevity. Such system is both nonlinear and fully integrable, with ideally infinite DA.

### *Practical Implementation*

Above derivation implicitly imposed several lattice constraints - such as the need to remove chromaticity, which in turn introduces unaccounted sextupolar nonlinearities. Within the nonlinear region, there should be no dispersion and $\beta$-functions must be equal. Finally, the rest of the ring must have phase advance be a multiple of $2\pi$ and have a first-order transport matrix of a thin, axially symmetric lens. For fully integrable case, these conditions must be met with high precision (i.e. 1% $\beta$-beat) to maintain integrability [6], but are relaxed by about an order of magnitude for a system with only 1 invariant [7]. Such a lattice, as implemented in IOTA, is shown in Fig. 1.

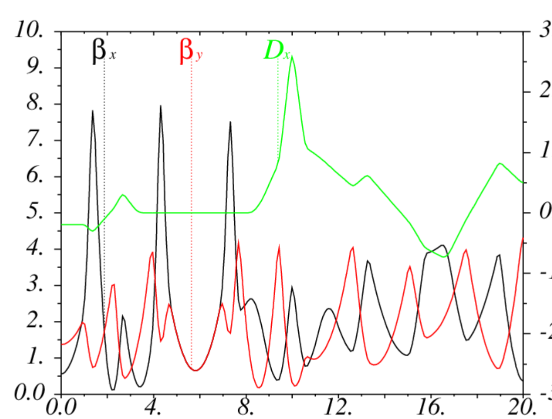


Figure 1: Half of IOTA lattice at working point $Q_{x,y}$=5.3. All units in meters, $\beta_{x,y}$ on the left, $D_x$ on the right. Lattice is mirror symmetric across 20m marker.

Realistically, due to field imperfections, magnet misalignments, and the unavoidable approximation of continuous

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DN potential with discrete elements, perfect integrability is never achieved. The goal of nonlinear lattice optimization is to minimize invariant deviations ($\Delta H/H$ and $\Delta I/I$) and bring the system as close as possible to being integrable, corresponding to maximums in DA and achievable tune spread.

## METHODS

For the following discussion, IOTA ring in 100MeV 1NL configuration with octupole single-invariant insert is used, since this setup was extensively characterized during run 1. However, proposed techniques are applicable to any NIO ring, since IOTA lattice is the hardest case with no superperiodicity or magnet family simplifications. All simulations are done with symplectic tracking in 'elegant', with PyTorch and scikit-learn libraries used for data analysis.

### *Linear Optics*

Linear optics correction is a standard procedure, typically done using linear optics from closed orbit (LOCO) approach, whereby lattice functions and alignment are determined from beam orbit responses. At IOTA, this is performed with an in-house DAQ and analysis tool '6dsim' [8]. For present study, we simply assume starting lattice state has similar random errors to what was observed during run 1.

### *Knob Selection*

Light sources typically contain only a few independently controllable sets of parameters, or 'knobs', since magnets by design are tied into families, and moreover the majority of adjustment margin is taken up by overriding requirements like low emittance [9]. In case of IOTA however, magnets are individually powered. Each mirror symmetric half has 19 normal and 10 skew quadrupoles, as well as 6 sextupoles and multiple correctors, in addition to 17 octupoles within the left and 18 DN magnet within the right inserts [10].

Such high knob count is excessive and highly degenerate, and we first seek to reduce it. A natural starting point is to tie elements into symmetric L/R pairs, such that any optimization affects both nonlinear regions. Further reduction can be achieved by calculating response matrices $\mathbf{R}$ for the parameters of interest (tune, chromaticity, etc.). For example, using singular value decomposition $\mathbf{R} = \mathbf{USV}^{\mathrm{T}}$, null space basis vectors (combinations that don't affect the parameter) can be obtained as those in $\mathbf{V}$ that have near-zero singular values. However, this approach is not suitable for tunes since it does not take into account the NIO constraints. Instead, we used multi-objective hybrid simplex optimization to derive a set of nominally orthogonal knobs - phase advance inside and outside the nonlinear regions, as well the longitudinal location of $\beta^*$ for a total of 3x2=6 knobs. These were verified to be near independent in the small region of interest studied, allowing for straightforward linear combinations. A full list of derived knobs is given in Table 1.

Table 1: Parameter Ranges and Knob Counts

| Parameter | Range | Knob count |
|---|---|---|
| $\mu_{x,y}$ (insert) | $0.3 \pm 0.02$ | 2 |
| $\mu_{x,y}$ (ring) | $5.0 \pm 0.02$ | 2 |
| $\beta^*$ (z) | $0\pm10$ cm | 2 |
| $\xi_{x,y}$ | $0 \pm 0.1$ | 2-6 |
| $\kappa$ (skew quads) | $0 - 0.01$ | 2-9 |
| $I_{NL}$ | $\pm5\%(\sim 0.1A)$ | 9-18 |
| H,V (correctors) | n/a | 28 |

### *Online Model*

Using above knobs, a sparse simulation set was created, with 11 points spanning the adjustment range of each parameter, and even more sparse cross-term sampling. Several figures of merit were extracted - DA area, minimum DA ellipse, and invariant jitter. Then, a surrogate model was trained which could achieve good agreement within this region of interest. It consisted of a simple, dense multi-layer perceptron with 5 hidden layers of 25 nodes, and ReLU activations. Input parameters were knob settings, normalized and scaled appropriately, with typical total count of $\sim O(10)$. Approximately 1000 samples, or 10% of data, were held back and used to verify the predictive accuracy.

### *Optimization*

Various optimization goals have been reported in literature, such as canceling lowest order resonant driving terms (RDTs) [11], reducing amplitude-dependent tune shifts [12], or optimizing DA directly [13]. Of these, only the latter is suitable for NIO due to the inherently nonlinear conditions, and is taken as the starting point for our approach.

Many multi-dimensional optimization strategies are available - gradient descent, genetic algorithms, particle swarm optimization, Nelder-Mead simplex, and others. All of these require large number of evaluations and are susceptible to noise in experimental data. Modifications of Powell's and simplex methods, called robust conjugate direction search (RCDS) [14] and robust simplex [15] respectively, were recently shown by X. Huang to improve noise robustness and improve convergence rate. However, they still require a somewhat large number of samples at each iteration. An NIO measurement involves at least several beam pings to determine DA and reconstruct invariants, each taking around 5-10 seconds due to slow damping time. With $5-7$ evaluations per knob, and at least 15 knobs (see Table 1), the required measurement times become unacceptable. Some recent work used global SVD [16] for a further dimensionality reduction, but it is not applicable here since the objective function is nonlinear, and in any case most degeneracy has already been eliminated with judicious knob choices.

One straightforward improvement is pre-setting optimal initial search directions/simplex values. In RCDS, these must satisfy the 'conjugate' condition, $\mathbf{u} \cdot \mathbf{H} \cdot \mathbf{v} = 0$, with Hessian matrix $H_{ij} = \partial^2 f/\partial x_i \partial x_j$ encoding relationship

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between objective function $f$ and the knobs $\mathbf{x} = x_1, x_2, \ldots$. However, once optimization is started, search directions are only modified based on measurements, without any use of *a priori* knowledge about the objective function landscape. This is especially disadvantageous for nonlinear systems, where initial search directions will not be universally optimal. Our proposed method, NN-RCDS, seeks to address this drawback by adding a neural network heuristic to improve search vectors and step sizes dynamically, based on model training and past history.

Specifically, for each iteration, we adjust initial bracketing step size and primary conjugate search direction based on weighted average of NN and naive algorithm values. To train the NN, which had similar MLP architecture as before, we used the above surrogate model to evaluate the invariant jitter cost function, thereby enabling fast simulated optimization runs, avoiding $\sim$ 10 CPU-minutes/point that would have been otherwise required. About 10k runs from random starting conditions were performed, and 10% of data was kept back for testing.

## RESULTS

### *Online Model*

Parameter ranges were chosen to match experimental region of interest from run 1, as detailed in Table 1. However, we held chromaticity fixed and pre-corrected orbit offsets and coupling, since these parameters have only a weak effect on the octupole NIO system. Two specific types of simulations were performed - dynamic aperture size search and particle invariant tracking. For all runs, random but optimistic misalignment errors were introduced to all dipoles and quadrupoles, and results averaged over 10 seeds.

The resulting surrogate model accuracy on test set is shown in Figs. 2 and 3. Note that performance was significantly better for invariant jitter, with DA determination quite noisy due to inherent resolution limits and stochastic nature of the DA search algorithm used for training set creation.

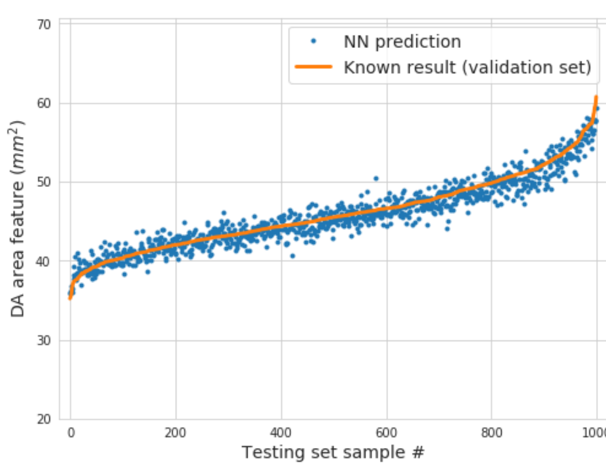


Figure 2: DA area NN performance.

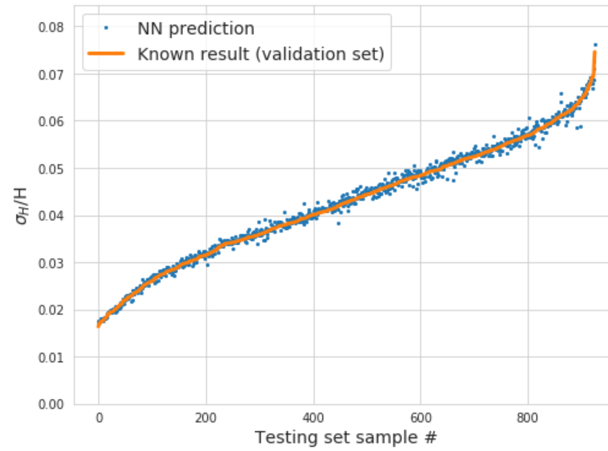


Figure 3: Invariant jitter NN performance.

### *Online Correction*

We evaluated NN-RCDS with separate NNs for step size scaling and direction prediction, finding MLP only suitable for the former task. Its performance in a typical run on 9D parameter space is shown in Fig. 4, with iteration 0 being first optimized step. Overall, convergence speed to same absolute error level is improved by about a factor of two, requiring ~500 evaluations to reach $10^{-2}$ MSE as compared to 1000 evaluations for naive RCDS.

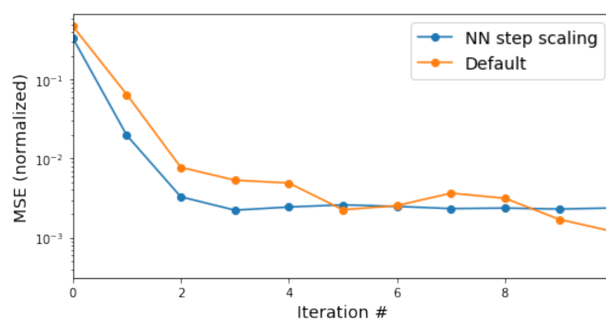


Figure 4: RCDS convergence comparison.

## SUMMARY AND FUTURE PLANS

We have presented an algorithm for online tuning of nonlinear integrable systems based on neural network augmented RCDS optimizer and a surrogate training model. Model accuracy was demonstrated to be very good for critical parameter of invariant jitter, but had significant noise in DA prediction due to training set deficiencies. NN-RCDS runs with surrogate model objective function and experimentally observed lattice parameters have shown factor of two faster convergence, while maintaining good noise rejection. We are exploring further improvements through use of recurrent neural networks, plan to extend our work with novel applications of convolutional generative networks for more complete, 2D (FMA, etc.) surrogate modeling and optimization.

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