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FAST TWO-DIMENSIONAL CALCULATION OF COHERENT SYNCHROTRON RADIATION IN RELATIVISTIC BEAMS

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Abstract

Coherent Synchrotron Radiation(CSR) in a relavistic beam during compression can lead to longitudinal modulation of the bunch with wavelength smaller than bunch length and is regarded as one of the main sources of emittance growth in the bunch compressor. Current simulations containing CSR wake fields often utilize one-dimensional model assuming a line beam. Despite its good computation efficiency, 1D CSR model can be inaccurate in many cases because it ignores the so-called 'compression effect'. On the other hand, the existing 3D codes are often slow and have high demands on computational resources. In this paper we propose a new method for calculation of the three-dimensional CSR wakefields in relativistic beams with integrals of retarded potentials. It generalizes the 1D model and includes the transient effects at the entrance and the exit from the magnet. Within given magnetic lattice and initial beam distributions, the formalism reduces to 2D or 3D integration along the trajectory and therefore allows fast numerical calculations using 2D or 3D matrices.

INTRODUCTION

Coherent Synchrotron Radiation(CSR) is one of the main limits to improvements of the brightness of electron beam in storage rings, free electron laser (FEL) light source and high-energy colliders. In a bending magnets, the beam radiates electromagnetic field and experiences a radiation reaction force. 1D model for this forces has been developed [1–4] and implemented in code ELEGANT, which is widely used for the design of bunch compressor in the linac.

Although the 1D models are simple and easy to use, they miss an important part of the total force in relativistic beams moving in a curvilinear trajectory. The attention to this force was attracted by M. Dohlus [5] in 2002, who pointed out that if the beam is compressed (either longitudinally or transversely) the energy of its Coulomb field changes and this should result in a change of the kinetic energy of the beam particles. A force that is responsible for this change can be called the compression force. Note that this force is different from the radiation reaction force, because the compression is a reversible process, and if the beam is decompressed, this force changes sign. It cannot be associated with what is conventionally called the space charge force because the latter typically scales as $1/\gamma^2$ with γ the Lorentz factor. The compression-decompression effect occurs even in the limit $\gamma \rightarrow \infty$(hence, $v = c$), when the space charge force vanishes. [6]

In this paper, we calculated 2D longitudinal CSR wake based on the integral of retarded potential, which can be very easily extended to three dimensions. The coding is implemented with MATLAB, which enables the fast calculation of the rate of energy change along the beam with 2D matrix. Transient effects at the entrance of the dipole are shown as a benchmark. We also calculated the CSR wake inside a chicane in a configuration studied at the CSR workshop at DESY-Zeuthen in 2002. The differences in longitudinal rate of energy loss compared between 1D and 2D model are discussed .

FORMULAS FOR 3D CSR

Electron beam can be described by its time-dependent charge density $\rho(\boldsymbol{r},t)$ and velocity $\boldsymbol{v}(\boldsymbol{r},t)$, where $\boldsymbol{r}$ is the coordinate vector and t is the time. Energy loss per unit time and unit charge due to CSR along the beam can be given by

$$\mathcal{P} = q\, \boldsymbol{v}(\boldsymbol{r},t) \cdot \boldsymbol{E}(\boldsymbol{r},t) \tag{1}$$

where the electric field inside the beam can be derived by scalar and vector potential, which are the integrals over retarded space coordinate $\boldsymbol{r'}$ around the beam along the trajectory at proceeding time $t' < t$

$$\begin{aligned}
\phi(\boldsymbol{r},t) &= \int d^3r' \frac{\rho(\boldsymbol{r'}, t_{ret}(\boldsymbol{r},t))}{|\boldsymbol{r'}-\boldsymbol{r}|} \\
\boldsymbol{A}(\boldsymbol{r},t) &= \frac{1}{c}\int d^3r' \frac{\boldsymbol{v}(\boldsymbol{r'}, t_{ret}(\boldsymbol{r},t))\rho(\boldsymbol{r'}, t_{ret}(\boldsymbol{r},t))}{|\boldsymbol{r'}-\boldsymbol{r}|} \\
\boldsymbol{E}(\boldsymbol{r},t) &= -\nabla_r \phi(\boldsymbol{r},t) - \frac{1}{c}\frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t}
\end{aligned} \tag{2}$$

We made the following assumptions. A cold fluid approximation is applied that at point $\boldsymbol{r}$ the status of particles is determined by density $\rho(r,t)$ and velocity $v(r,t)$. Second, we assume that the spread in velocity due to the angular and energy spread is negligible, which is valid for highly relativistic beams. Third, we assume that the size of the bunch is much smaller that the external scale of the problem under study. With further simplification, the energy loss due along the beam can be given by [6, 7]

$$\begin{aligned}
\mathcal{P} = -c\, q \int \frac{d^3r'}{|\boldsymbol{r'}-\boldsymbol{r}|}&[\boldsymbol{\beta}(\boldsymbol{r},t) \\
&- (\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\boldsymbol{\beta}(\boldsymbol{r'},t_{ret}))\boldsymbol{\beta}(\boldsymbol{r'},t_{ret})] \cdot \partial_{\boldsymbol{r'}}\rho(\boldsymbol{r'},t_{ret})
\end{aligned} \tag{3}$$

As in [6], to calculate ρ and $\boldsymbol{v}$ we use use the formalism of the Vlasov equation. In 2D, we use the notation x for the horizontal particle offset relative to the nominal orbit, z

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is the longitudinal coordinate of the particle in the bunch, and s is the path length along the nominal orbit. Assuming Gaussian beam distribution and nonzero R_{16} and R_{26} in a more general case, ρ and $\boldsymbol{v}$ can be obtained in 2D by

$$\begin{aligned} \rho(x,z,s) &= \frac{eN_b}{2\pi\Sigma} exp(\frac{-Ax^2+Bxz+Cz^2}{2\beta_f\Sigma^2}) \\ \boldsymbol{v}(x,z,s) &= \frac{Ex+Fz}{\beta_f\Sigma^2} \end{aligned} \tag{4}$$

where A,B,C,E,F are parameters depending on twiss parameters and dispersion calculated from the given lattice and initial beam condition.

$$\begin{aligned} A =&(hR_{56}+1)^2[(\alpha^2+1)D^2\epsilon \\ &+2\alpha\beta DD'\epsilon+\beta(\beta D'^2\epsilon+\sigma_z^2)]+\beta R_{56}^2\sigma_\eta^2 \\ B =&-2(hR_{56}+1)\{\epsilon[(\alpha^2+1)D^3h+\alpha\beta D(2DD'h-1) \\ &+\beta^2D'(DD'h-1)]+\beta Dh\sigma_z^2\}-2\beta DR_{56}\sigma_\eta^2 \\ C =&\beta^2\epsilon+(\alpha^2+1)D^4h^2\epsilon+2\alpha\beta D^3D'h^2\epsilon \\ &+\beta D^2[h^2(\beta D'^2\epsilon+\sigma_z^2)-2\alpha h\epsilon+\sigma_\eta^2]-2\beta^2DD'h\epsilon \\ E =&(\alpha^2+1)D^3D'\sigma_\eta^2\epsilon-D^2\sigma_\eta^2\epsilon(-2\alpha\beta D'^2+\alpha^2R_{56}+R_{56}) \\ &+\beta DD'[\sigma_\eta^2(\beta D'^2\epsilon+\sigma_z^2)+(hR_{56}\epsilon+\epsilon)^2] \\ &+\beta\epsilon[R_{56}\sigma_\eta^2(\beta D'^2-\alpha R_{56})-\alpha(hR56\sigma_z+\sigma_z)^2] \\ F =&\sigma_\eta^2[(\alpha^2+1)D^3+\alpha\beta D(2DD'+R_{56}) \\ &+\beta^2D'(DD'+R_{56})] \\ &+\beta(hR_{56}+1)(h\sigma_z^2(\alpha D+\beta D')+D\epsilon) \\ \Sigma^2 =&\epsilon(hR_{56}+1)^2(\beta\sigma_z^2+D^2\epsilon) \\ &+\sigma_\eta^2/\beta\{\beta D^2\sigma_z^2+\epsilon[(\alpha^2+1)D^4 \\ &+2\alpha\beta D^2(DD'+R_{56})+\beta^2(DD'+R_{56})^2]\} \end{aligned} \tag{5}$$

TRANSIENT CSR WAKE OF A BEND MAGNET

To benchmark Eq. (3) against known solutions of 1D CSR problems, we consider the problem treated in [3,4]. In this problem we assume the magnetic a bending magnet of finite length L. OAutside of the magnet, the beam moves along straight lines tangential to the circular orbit at the points of entrance and exit, respectively. The bunch distribution and velocity are given by Eq. (4) and (5) with s the length along the orbit and x the horizontal coordinate perpendicular to the orbit. We have chosen the same set of parameters as in [4]: $R = 1.5m, \sigma_x = 50\mu m, Q = 1nC$ and $L = 25cm$. In the 1D model of [4] the parameter $\sigma_x = 0$; in our calculations we have chosen $\sigma_x = sigma_z$. We first calculated the wake inside the bunch when it enters the bend from the straight line. The plot of this wake at various distances from the magnet entrance edge is shown in Fig. 1 by solid lines. For comparison, the dashed green lines show the result of 1D model computed in [4].We found an excellent agreement of our theory with a 1D model of the CSR wake.

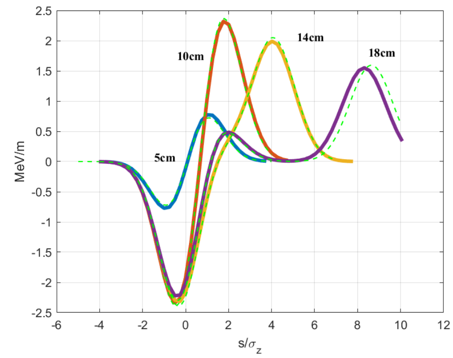

Figure 1: Longitudinal wake in the bunch as a function of distance from the entrance edge of the bend shown by a number near each curve. The longitudinal coordinate s in each case is measured from the center of the bunch.

CSR WAKE IN A BUNCH COMPRESSOR

A much more difficult problem is presented by a chicane bunch compressor consisting of four dipole magnets. To illustrate how Eq. (3) can be used in a situation when σ_z varies with s, we calculated the CSR wake in a configuration studied at the CSR workshop at DESY-Zeuthen in 2002 [8]. The four magnets have the length $L = 0.5m$ with the bending radius $R = 10.35m$ resulting in the momentum compaction factor $R_{56} = 2.5cm$. In our simulations, the beam with the energy of 5.0 GeV and Gaussian distributions in energy and coordinates is compressed from the initial rms length of 200μm to the final length of 150 μm with energy chirp $h = 10m^{-1}$ and to final length of 20 μm with energy chirp $h = 36m^{-1}$ as shown in Fig. 2. The beam charge is 1 nC and the slice energy spread is 10^{-4} We calculated the CSR wake in the middle of the second and third magnets as well as at the center of the chicane, see Fig. 3.

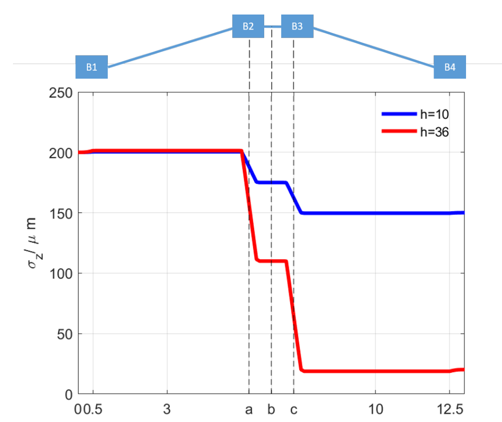

Figure 2: Variation of the bunch length σ_z(s) through the chicane (The positions of magnets are shown on the top of the figure).Blue and red lines show the bunch length with energy chirp $h = 10m^{-1}$ and $h = 36m^{-1}$ respectively. a,b and c show three positions in the chicane where the CSR wake was calculated.

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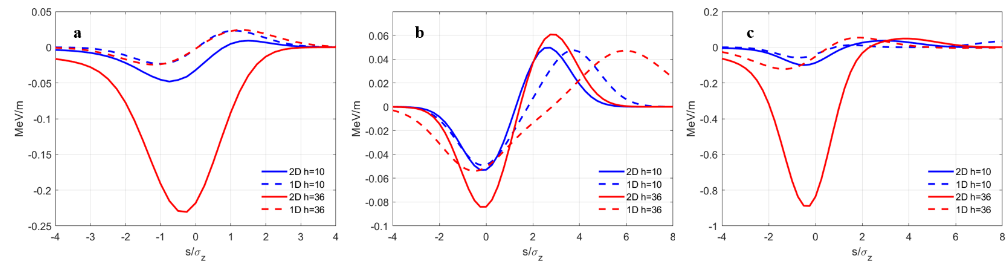

Figure 3: Wakes calculated in 1D model are shown by dashed lines and ones calculated in 2D model are shown by solid lines: a — in the middle of the second magnet (s = 5.75 m), b — at the center of the chicane (s = 6.5), and c — in the middle of the third magnet (s = 7.25 m). Blues lines shows the results with energy chirp $h = 10m^{-1}$ and red lines with $h = 36m^{-1}$. The coordinate s is normalized by the rms bunch length, $\sigma_z(s)$, at the location of the bunch.

The wake calculated in the 2D model is shown in Fig. 3 by solid lines. These wakes are compared with the wakes calculated with a 1D model described in [6](shown by dashed lines). The 2D wakes plotted are actually calculated along the axis of the tilted beam. The plots show a considerable difference between the 1D and 2D wakes for both $h = 10m^{-1}$ and $h = 36m^{-1}$. The more the beam is compressed and tilted in x-z plane, the bigger the difference is.

DISCUSSION

The simple 1D models [1,2,9], of the CSR wake have two important limitations when applied to bunch compressors. First, they assume a constant bunch length, and, second, they ignore the tilt of the bunch with an energy chirp when it passes through the region of large dispersion. In contrast to the previous theories, our 1D model takes the variation of the bunch length into account but it still misses the bunch tilt. This is the reason of noticeable discrepancy between our 1D and 2D calculations in Fig. 4. We note that with a relatively small energy chirp($h = 10m^{-1}$) in the beam, and hence a small compression, the effect of the beam tilt when it passes through the second and third magnets was minimized. For the compression factor of 10 originally studied in [8] (which corresponds to the energy chirp $h = 36m^{-1}$), the tilt is much stronger, and the 1D model fails miserably near the center of the chicane. It is likely that it works much better in the last magnet of the chicane where the bunch is already compressed longitudinally and the tilt gradually vanishes together with the dispersion. Our results indicate that one has to very careful when simulating the beam dynamics in chicanes using a 1D model of the CSR wakefields.

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