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ADJOINT OPTIMIZATION APPLIED TO FLAT TO ROUND TRANSFORMERS*

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Abstract

We present the numerical optimization, using adjoint techniques, of Flat-to-Round (FTR) transformers operating in the strong self-field limit. FTRs transform an unmagnetized beam that has a high aspect ratio, elliptical spatial cross section, to a round beam in a solenoidal magnetic field. In its simplest form the flat to round conversion is accomplished with a triplet of quadrupoles, and a solenoid. FTR transformers have multiple applications in beam physics research, including manipulating electron beams to cool co-propagating hadron beams. Parameters that can be varied to optimize the FTR conversion are the positions and strengths of the four magnet elements, including the orientations and axial profiles of the quadrupoles and the axial profile and strength of the solenoidal magnetic field. The adjoint method we employ allows for optimization of the lattice with a minimum computational effort including self-fields. The present model is based on a moment description of the beam. However, the generalization to a particle description is possible. The optimized designs presented here will be tested in experiments under construction at the University of Maryland.

INTRODUCTION

In this paper we will illustrate the application of the adjoint optimization approach [1] to the design of Flat-to-Round (FTR) or Round-To-Flat (RTF) transformers as have been proposed for use in many applications [2–5].

The adjoint approach is described as follows. A set of moment equations are used to simulate the propagation of a beam through a system of magnets, which converts a beam with an elliptical cross section to one with a round cross section. A general figure of merit will be introduced that quantifies how successfully the shape conversion has been made. Subsequently, we will formally perturb this system by making small changes in the parameters defining the focusing forces. Then we will introduce an adjoint system of equations that will allow one to calculate compactly the changes in the figure of merit due to changes in the focusing parameters. Such an evaluation is then used in a gradient-based optimization scheme.

MOMENT EQUATIONS

In this section we present a system of equations that describes the evolution with distance of the second moments of a charged particle beam distribution in the presence of a combination of transverse forces. These forces include the Lorentz force of a spatially varying solenoidal (axial) magnetic field, the Lorentz force of a superposition of arbitrarily oriented quadrupole magnetic fields, and the electric and magnetic self-force due to the beam's charge and current densities.

The moments we consider are averages of products of all possible pairs of variables describing the transverse displacement of beam particles and the rate of change of the transverse displacement with distance. Due to the symmetry of this matrix only 10 elements are independent. Thus, our governing system consists of 10 moment evolution equations. We also choose to deal with differential equations for the continuous moments, as this allows us to introduce adjoint equations that include self-field effects and spatial profiles of focusing fields. Detailed derivations of these equations can be found [1]. Here we just present the equations as is:

$$\frac{d}{dz}\boldsymbol{Q} = \boldsymbol{P}\,, \tag{1}$$

$$\frac{d}{dz}\boldsymbol{P} = \boldsymbol{E} + \boldsymbol{O}\cdot\boldsymbol{Q}\,, \tag{2}$$

$$\frac{d}{dz}\boldsymbol{E} = \boldsymbol{O}\cdot\boldsymbol{P} + \boldsymbol{N}L\,, \tag{3}$$

$$\frac{d}{dz}L = -\boldsymbol{N}^{\dagger}\cdot\boldsymbol{Q} \tag{4}$$

Where the following variables are defined in terms of the beam moments:

$$\boldsymbol{Q} = \begin{pmatrix} Q_+ \\ Q_- \\ Q_x \end{pmatrix} = \begin{pmatrix} <x^2+y^2>/2 \\ <x^2-y^2>/2 \\ <xy> \end{pmatrix}, \tag{5}$$

$$\boldsymbol{P} = \frac{d}{dz}\boldsymbol{Q} = \begin{pmatrix} P_+ \\ P_- \\ P_x \end{pmatrix} = \begin{pmatrix} <xx'+yy'> \\ <xx'-yy'> \\ <yx'+xy'> \end{pmatrix}, \tag{6}$$

$$\boldsymbol{E} = \begin{pmatrix} E_+ \\ E_- \\ E_x \end{pmatrix} = \begin{pmatrix} <x'^2+y'^2> \\ <x'^2-y'^2> \\ 2<y'x'> \end{pmatrix}, \tag{7}$$

$$L = <xy'-yx'>\,. \tag{8}$$

The $\boldsymbol{O}$ and $\boldsymbol{N}$ matrices represent the magnetic field in the lattice elements as well as the effects of space charge forces

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and are continuous functions of longitudinal distance, z:

$$\boldsymbol{O} = \frac{k_\Omega^2}{2}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} + 2\,\Sigma_{\mathrm{quads}} K_q \begin{bmatrix}0 & c_q & -s_q\\ c_q & 0 & 0\\ -s_q & 0 & 1\end{bmatrix} + \frac{\Lambda}{Q_\Delta}\begin{bmatrix}1 & c_\alpha & s_\alpha\\ c_\alpha & 1 & 0\\ s_\alpha & 0 & 1\end{bmatrix}, \quad (9)$$

$$\boldsymbol{N} = 2\,\Sigma_{\mathrm{quads}} K_q \begin{pmatrix}0\\ s_q\\ c_q\end{pmatrix} - \frac{\Lambda}{Q_\Delta}\begin{pmatrix}0\\ s_\alpha\\ -c_\alpha\end{pmatrix}. \quad (10)$$

The first matrix in Eq. (9) is due to the solenoidal field, the second is due to the quadrupoles, and the third matrix is due to the self-fields. The first vector in Eq. (10) is due to the quadrupoles, and the second is due to the self-fields. In Eqs. (9) and (10) the following expressions and notation have been introduced. The quadrupole strength is defined as:

$$K_q = \frac{qB_q'(z)}{mc\gamma v_z} \quad (11)$$

and the strength of the self-fields is measured by the beam current parameter:

$$\Lambda = \frac{cqZ_0 I}{4\pi m v_z^3 \gamma_0^3}. \quad (12)$$

The remaining variables are terms that come into play due to the moment equations being rotated into the Larmor frame. See here for more details [1].

We compare the solution of the moment equations in Eqs. (1)–(4) with the moments calculated by the PIC code Warp [6] in Fig. 1. For this simulation we consider a symmetric triplet with parameters selected to convert a flat beam to a round one in the presence of space-charge. The space-charge is evaluated along the entire path of the beam. The solutions of the moment equations are solid lines, and the PIC values are shown as circles and crosses. As can be seen in Fig. 1, in the zero current case the beam becomes round at the end of the triplet, whereas in the presence of self-fields the roundness is spoiled.

OPTIMIZATIONS

We now imagine that Eqs. (1)–(4) have been solved for a given set of initial conditions, $(\boldsymbol{Q}, \boldsymbol{P}, \boldsymbol{E}, \mathrm{L})|_{z_i}$ and magnetic field parameters and profiles, which we label with a vector $\boldsymbol{a}$. An assessment of the configuration can be made on the basis of a figure of merit that depends on the final values of the moments,

$$F(\boldsymbol{Q}, \boldsymbol{P}, \boldsymbol{E}, L, \boldsymbol{a})|_{z_f}. \quad (13)$$

For a flat to round transition, the following FoM is used:

$$F = 1/2\Big[|\boldsymbol{P}^2| + k_0^2(Q_-^2 + Q_+^2) + k_0^{-2}(E_-^2 + E_+^2) + k_0^{-2}\Big(E_+ - \frac{1}{2}k_\Omega^2 Q_+ + \Lambda\Big)^2 + (2E_+Q_+ - L^2)^2\Big] \quad (14)$$

Where it is a sum of terms quadratic in the moments, each of which should be as small as possible in an FTR transformer that leads to a matched beam in the solenoid.

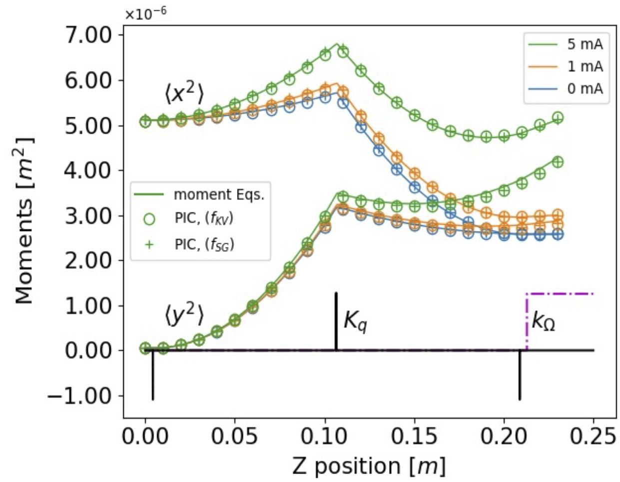

Figure 1: Comparison of solutions of the moment equations (solid lines) with solutions from the PIC code Warp (circular and cross symbols).

The quantity k_0 is introduced so that each term has the same units magnitude. The choice of each term is made as follows. We would like the beam in the solenoid to be round, $Q_- = Q_+$, and we would like all the nonzero moments to be independent of z. This implies for the spatial moments $d\boldsymbol{Q}/dz = \boldsymbol{P} = 0$. If we set the components of $d\boldsymbol{P}/dz = 0$, we find from Eq. (2) that $E_- = E_x = 0$ is required as well as $E_+ - k_\Omega^2 Q_+/2 + \Lambda = 0$ (radial force balance). Finally, the last term is designed to force the trajectories to be as laminar as possible and the rotation to be rigid in the solenoid.

Given the FoM in Eq. (13), derivatives can be taken with respect to the lattice parameters we wish to optimize. These derivatives of the FoM can be used to create a set of adjoint perturbation equations that allow us to calculate changes in the FoM due to perturbations in our lattice. Full details of this non-trivial derivation can be found here [1].

With the adjoint equations we can now optimize the parameters describing the magnetic fields in a FTR lattice by using the FoM in Eq. (13). We consider the FoM to be a function of the elements of a list of parameters $\boldsymbol{a}$. The elements of the list include the strengths, locations, and orientations of the quadrupoles as well as the strength and location of the solenoid. In the case of no self-fields, the strength and location of the solenoid, along with the beam energy, would

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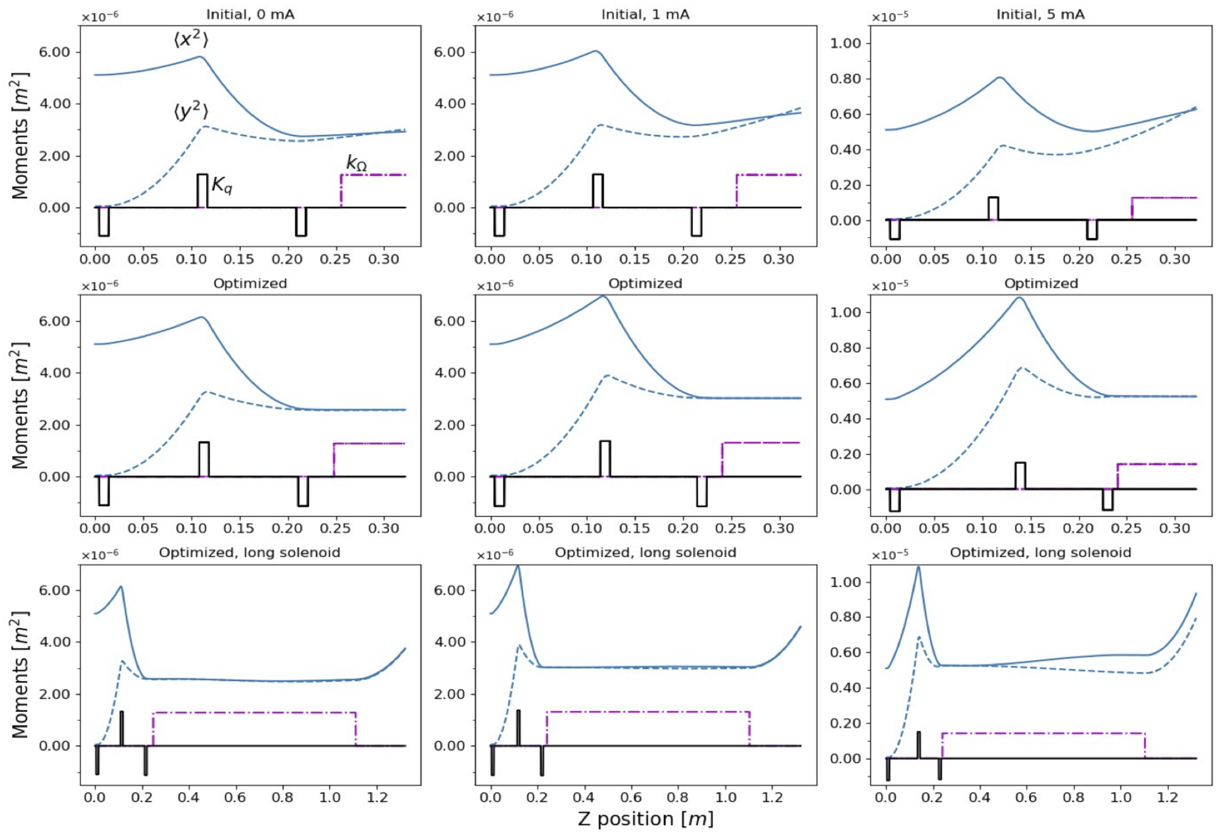

Figure 2: RMS beam size, $<x^2>, <y^2>$, plotted as a function of the longitudinal coordinate through a FTR transformer.

be enough to optimize the system. With self-fields the initial matching of the beam through the three quadrupoles no longer becomes trivial and requires the quadrupole parameters to be included in the optimization, resulting in a total of 11 parameters. We use a simple steepest descent algorithm in which we calculate the gradient of F in the space of parameters $\boldsymbol{a}$, $\nabla F(\boldsymbol{a})$ and then adjust the values of the parameters by moving from the current set of parameters along the line of steepest descent.

Figure 2 shows that by using the adjoint technique, we can find and optimize solutions with increasingly large self-field forces. Each panel shows the profiles of the $<x^2><y^2>$ moments of the beam as well as the location and strength of the quadrupole and solenoidal magnetic fields. The three columns in Fig. 2 correspond to three different beam currents, 0.0, 1.0, and 5.0 mA. The top row shows the spatial dependence of the moments with the initial magnet parameters selected based on the thin lens and no space charge approximation.

The middle row shows the same information after the magnetic fields have been optimized. The figure of merit is applied at $z = 0.3$ m, which is on the right of the panels in the first two rows. In these optimized cases the beam appears to be round and its moments are constant in the solenoid as demanded by the FoM.

The bottom row shows the evolution of the moments if propagation of the beam is continued through a longer solenoid, past the point at $z = 0.3$ m where the FoM is evaluated. In the 0 and 1 mA cases the beam parameters do not vary after this point and the beam is round indicating the FoM has been minimized to zero. However, in the 5 mA case it is seen that the beam is not perfectly round in the solenoid. This is a consequence of our having restricted the solenoid and third quadrupole to not overlap. When we run the optimizer with no restriction it finds a configuration where the beam is perfectly round in the solenoid. However, in this case the quadrupole and solenoid fields overlap. To avoid this overlap, we do not allow the start location of the solenoidal field to cross to the left of the third quadrupole.

CONCLUSION

In conclusion, we introduce a set of differential moment equations and used an adjoint approach to optimize a FTR lattice configuration in the presence of large space charge forces. The solution of the moment equations match those from a beam PIC code simulation.

The optimization is achieved by tuning a set of quadrupole and solenoid magnet parameters within the lattice. The adjoint approach and gradient descent algorithm are able to successfully optimize various transformer systems.

While the results shown here are based on a reduced model of the charged particle dynamics, namely a solution of the moment equations assuming linear restoring forces, the adjoint method can also be applied to a particle description (see [7, 8]).

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