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MODEL PARAMETERS DETERMINATION IN EIC STRONG-STRONG SIMULATION*

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Abstract

The emittance growth of ion beam is subject to numerical noises in the strong-strong beam-beam simulation for the Electron-Ion Collider (EIC). This paper discusses the impacts of model parameters: number of macroparticles, longitudinal slices, and transverse grids, on the beam size evolution in Particle-In-Cell (PIC) based strong-strong simulations. This study helps us better understand the causes of emittance growth in strong-strong beam-beam simulation.

INTRODUCTION

The beam-beam interaction is one of the most important phenomena to limit the luminosity in colliders. Beam-beam simulation is an essential tool to study beam-beam effects. Two models are often used in simulations: weak-strong and strong-strong. The weak-strong model is used to study the single particle dynamics, while the strong-strong model is used to study the coherent motion.

The particle-in-cell (PIC) approach is widely used in strong-strong simulation. It uses a computational grid to obtain the charge density distribution. The beam-beam force can be calculated from an arbitrary beam distribution by solving the 2D Poisson equation. Both beam distribution are updated during collision.

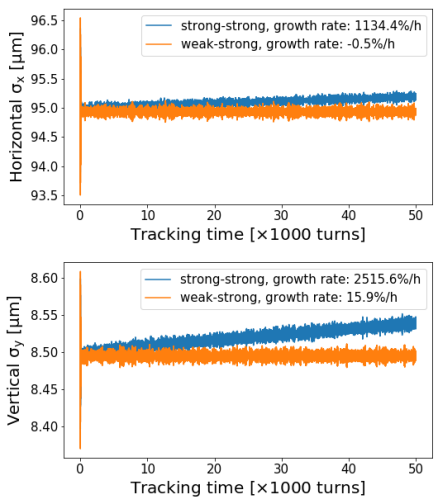

Figure 1: Strong-strong versus weak-strong simulation for EIC. The beam parameters can be found in [1], and the growth rate is linearly fitted from the last 60% tracking data.

Therefore, this kind of method is self-consistent. However, the PIC based strong-strong simulation is subject to numerical noise. The discrepancy between the weak-strong and strong-strong simulation for Electron-Ion Collider (EIC) has been found, as shown in Fig. 1. It is important to understand the difference in case there is some coherent mechanism shadowed by the large numerical noise.

In the following section, the strong-strong simulation is performed by the code BeamBeam3D [2].

SCALING LAW OF MACROPARTICLES

In PIC simulation, both bunches are represented by a number of macroparticles. In real beam, there are about 10^{11} charged particles per bunch. Due to the limitation of computation resources, only several million macroparticles are used in simulation. Therefore we are sampling the particle distribution at a rate about one in 10^5 particles. The sub-sampling causes artifical Monte Carlo noise.

The numerical errors are unavoidable in PIC simulation. In each time step, each macroparticle is interpolated on a finite grid. The Poisson equation is solved on that grid. The field is then interpolated according to the position of the macroparticle. The interpolations generate numerical errors.

The numerical noise will cause the particle diffusion in phase space. More macroparticles can reduce the impact of numerical errors. If the beam size or emittance growth is purely determined by numerical noise, it should obey the scaling law [3]

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}t} \propto \frac{1}{M} \tag{1}$$

where M is the number of macroparticles, and σ is beam size.

To understand the proton size growth in EIC strong-strong simulation, we scanned the number of electron macroparticles from 0.5 million to 4 million with a step of 0.5 million. All other parameters remain same. The scanned result is shown in Fig. 2. The horizontal and vertical growth rate are fitted with two different models:

$$g_{x,y} = \frac{A_{x,y}}{M^{B_{x,y}}} \quad \text{or} \quad g_{x,y} = \frac{A_{x,y}}{M} + B_{x,y} \tag{2}$$

where $g_{x,y}$ are horizontal or vertical growth rate which is calculated from the tracking data, $A_{x,y}$ and $B_{x,y}$ are fitting parameters. Two more examples of $M_e = 6$ million and $M_e = 8$ million are used to validate the fitting model. From Fig. 2, model 2 is a better guess.

The proton macroparticles are also scanned, and the corresponding results are shown in Fig. 3. We can see that the

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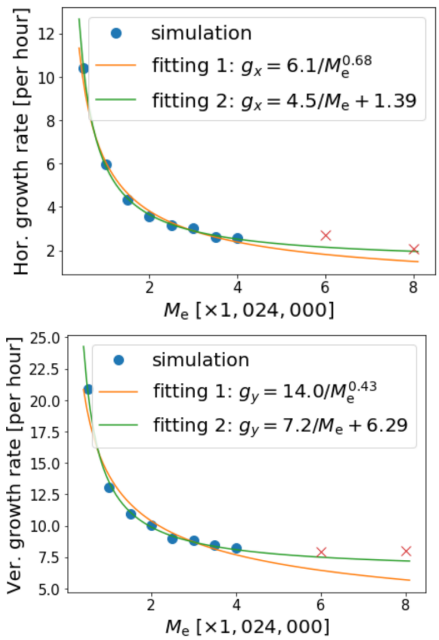

Figure 2: The proton beam size growth rate for different number of electron macroparticles. The fitting models are shown in Eq. (2). The blue circles are used to determine the fitting curves, and the red crosses are used for validation.

simulation results cannot fit into any model in Eq. (2). The reason would be the synchrotron radiation of the electron beam. To verify it, we replace the electron beam with a symmetrical "antiproton beam" — the particle properties are same in both beams except the opposite electric charge. Figure 4 presents the results of $p^- - p$ simulation. When the radiation damping and the quantum excitation are removed, both beam growth rates are well fitted to the model 2. It is interesting to notice that the other beam is "cooling" when we increase the number of macro particles of one beam. The damping and excitation prevent the emittance exchange between the two beams.

The emittance exchange is related to the number of particles. It may interfere with the real physical process. To minimize this effect, we should keep $M_e/M_p = N_e/N_p$ in strong-strong simulation, where N_e and N_p are real number of particles.

Although the fitting curves in Figs. 2-4 are consistent with $1/M$ scaling law, the non-zero intercept needs further explanation.

LONGITUDINAL SLICES

The number of longitudinal slices also impacts the numerical noise. Firstly, more slices make the beam-beam force more smooth along the longitudinal direction. In the strong-strong code, the integration is calculated by the drift-kick model. More slices will reduce the global numerical error. Secondly, the number of macroparticles per slice decreases as the number of slices increase. The Monte Carlo noise becomes more. Thirdly, the longitudinal beam-beam potential is linearly interpolated with the method by K. Ohmi [4]. The truncation error also decreases with more slices.

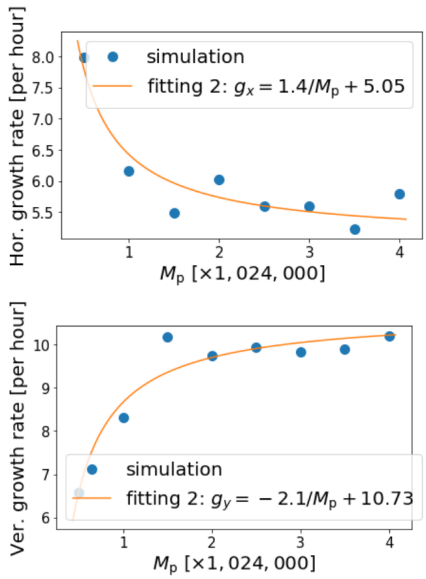

Figure 3: The proton beam size growth rate for different number of proton macroparticles. The horizontal and vertical growth rate are not well fitted to any model in Eq. (2).

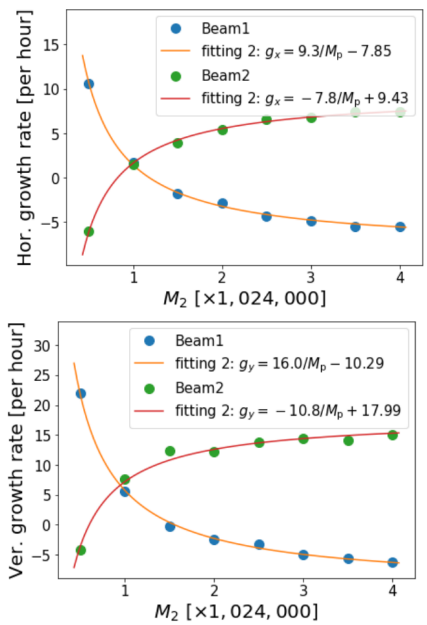

Figure 4: The proton beam size growth rate for different number of proton macroparticles. Beam 1 is composed of antiproton particles, while beam 2 is composed of proton particles. Other parameters are identical for both beams.

The total effect should be determined by simulations, as shown in Fig. 5. The coefficients $A_{x,y}$ and $B_{x,y}$ are defined by the 2nd fitting model in Eq. (2). Every point is fitted by 8 simulation jobs with the number of electron macroparticles from 0.5 million to 4 million. Figure 5 shows that B_y converges when the electron slice number is ≥ 21. More slices may be helpful, but the computation time, which roughly

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grows as the product of bunch beam slices number, is too long. There is no clear pattern for $A_{x,y}$ and B_x.

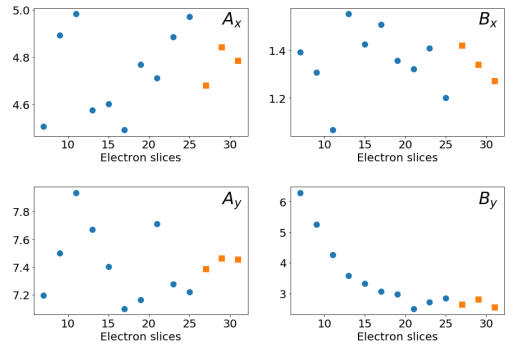

Figure 5: The fitting results for different number of electron slices. $A_{x,y}$ and $B_{x,y}$ are defined by the 2nd fitting model in Eq. (2). The blue circles are tracked by 50,000 turns, and the orange squares are tracked by 40,000 turns due to the limitation of computation time.

The dependance of growth rate on the proton slices is not as significant as the electron slices. We can choose it the same as the number of electron slices so that the macroparticle ratio in each slice is same as the beam intensity ratio.

PIC SOLVER

The converged intercept in Fig. 5 is $B_x \approx 140\%/\text{h}$, $B_y \approx 300\%/\text{h}$. The vertical growth rate is more than twice than the horizontal one. We should look into the 2D Poisson solver to find the reason.

Figures 6 and 7 present the relative error of PIC solver. The electron energy is artificially set to a huge number so that the electron beam is too "rigid" to be affected by the beam-beam interaction. Then we can get the exact beam-beam force by the analytic formula. Comparing both figures, the horizontal error is smaller and the histogram shape becomes much "Gaussian" when more electron macroparticles are used. However, the ratio of large relative error in vertical plane is not reduced. It explains why B_y is twice then B_x.

The difference between both planes comes from the flat beam collision scheme. The grid size is 128×128 in all our simulation. A large grid may help to reduce the large relative error in the vertical plane. But more macro particles are needed as the number of macroparticles per cell decreases. As a result, the computation time is too long.

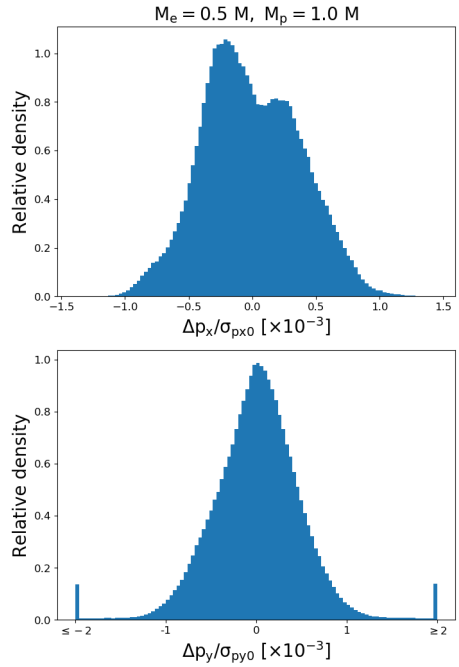

Figure 6: The histogram of relative error introduced by PIC solver, top: horizontal, bottom: vertical. The area under the histogram is normalized to 1. The grid size is 128×128. The number of electron macroparticles is $M_e = 0.5$ million.

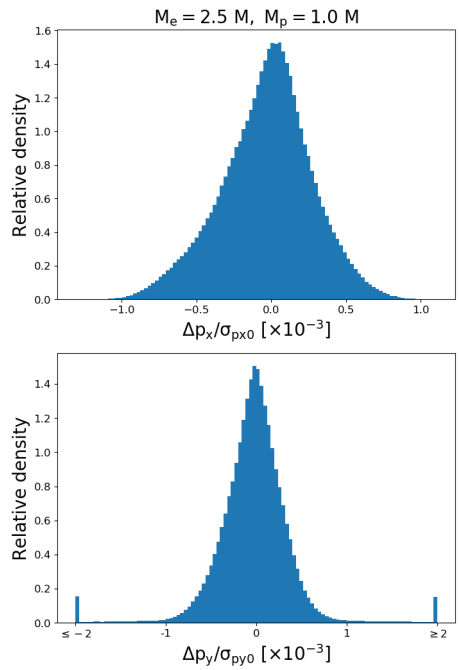

Figure 7: The histogram of relative error introduced by PIC solver, top: horizontal, bottom: vertical. The area under the histogram is normalized to 1. The grid size is 128×128. The number of electron macroparticles is $M_e = 2.5$ million.

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