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PROMISE AND CHALLENGES OF A METHOD FOR 5X5 SIGMA MATRIX MEASUREMENT IN A TRANSPORT LINE*

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Abstract

The Advanced Photon Source (APS) is upgrading the storage ring to a design that requires on-axis injection. Matching between the incoming beam and the ring is important in ensuring high injection efficiency. Toward this end, we have developed and tested a method for measuring all sigma matrix elements except those related to the time coordinate. We report on challenges encountered, based on simulation and real-world trials.

INTRODUCTION

Measuring beam size as a quadrupole is varied is a basic method of measuring beam emittance and beta functions. The method can be extended to scanning of multiple thick-lens quadrupoles [1] for two-plane measurements [2] and to allow measurement of the elements of the 4x4 sigma matrix [3]. This technique seems to be unworkable when there is dispersion in the transport line at the location of the quadrupoles being varied or of the screen used for beam size measurement, since the problem appears ill-defined [4].

Here we present a method of measuring the non-temporal sigma matrix elements in a transport line with dispersion. While the method works very well in simulation, real-world application has proven complex.

The transformation of sigma matrix Σ by a transport line follows [5]

$$\Sigma = R\hat{\Sigma}R^T, \tag{1}$$

where $\hat{\Sigma}$ is the starting sigma matrix, R is the transport matrix, and Σ is the final sigma matrix. If there is no vertical bending ($R_{36} = R_{46} = 0$), we can express the measurable elements of Σ as

$$\begin{aligned}\Sigma_{11} &= R_{11}^2\hat{\Sigma}_{11} + 2R_{11}(R_{12}\hat{\Sigma}_{12} + R_{16}\hat{\Sigma}_{16}) + R_{12}^2\hat{\Sigma}_{22} \\ &\quad +2R_{12}R_{16}\hat{\Sigma}_{26} + R_{16}^2\hat{\Sigma}_{66}\end{aligned} \tag{2}$$

$$\Sigma_{33} = R_{33}^2\hat{\Sigma}_{33} + 2R_{33}R_{34}\hat{\Sigma}_{34} + R_{34}^2\hat{\Sigma}_{44} \tag{3}$$

$$\begin{aligned}\Sigma_{13} &= R_{11}R_{33}\hat{\Sigma}_{13} + R_{11}R_{34}\hat{\Sigma}_{14} + R_{12}R_{33}\hat{\Sigma}_{23} \\ &\quad +R_{12}R_{34}\hat{\Sigma}_{24} + R_{16}R_{33}\hat{\Sigma}_{36} + R_{16}R_{34}\hat{\Sigma}_{46}\end{aligned} \tag{4}$$

In an extension of Ref. [1], these equations can be formed into a matrix equation for a series of measured values of $(\Sigma_{11}, \Sigma_{33}, \Sigma_{13})$ as a function of R_{ij}, with the latter being altered by the variation of several quadrupoles. This matrix equation can be solved for $\hat{\Sigma}$.

This method was implemented in the program `sdds5x5sigmaproc`, which is distributed with and uses data generated by `elegant` [6]. The program takes R data from `elegant` as quadrupoles are varied, along with corresponding measured or simulated beam size data. It computes Σ_{ij} for $i = 1, 2, 3, 4, 6$, including error estimates. Multi-screen measurements from a common starting point are available by simply ensuring that the R data and corresponding beam moments measurements are organized into the same order in the input files.

CONSIDERATIONS FOR APS

The Advanced Photon Source (APS) booster-to-storage-ring (BTS) transport line features a dogleg that consists of an extraction kicker, several booster lattice elements, two extraction septa, five quadrupoles, and two bending magnets. Downstream of the latter are more quadrupoles and a YAG scintillator screen with digital camera. The R matrix computations start just upstream of the kicker, so we get $\hat{\Sigma}$ at this location.

Although the dogleg quadrupoles are nominally set to suppress the dispersion, the beam coming from the APS booster [7] may (depending on the booster lattice) originate in a location of non-zero $\hat{\Sigma}_{i6}$. From the equations above, we see that we can't hope to determine $\hat{\Sigma}_{i6}$ unless the transport line has non-zero R_{16} at the location of the measurements. Without this, we cannot correctly deduce the emittance of the beam, since the inferred transverse beam moments (e.g., $\hat{\Sigma}_{11}$) include unknown $\hat{\Sigma}_{i6}$-related contributions. We can resolve this by varying quadrupoles before and after the final dipole of the dogleg. Varying quadrupoles before the final dipole will strongly vary the terms related to R_{16}. Varying quadrupoles downstream of the dogleg will ideally vary only the non-chromatic elements of R. In reality, we needn't strictly separate the variation of the quadrupoles, as long as quadrupoles in both regions are varied.

Any transport line that has a similar arrangement of quadrupoles before and after a bending magnet should be amenable to measurements using this method. One common configuration is a linac with a four-dipole bunch compressor containing dispersion-matching quadrupoles.

It is important to distinguish between two related quantities [4]: the dispersive matrix element R_{16} and the dispersion function η_x, which is directly related to the beam size. R_{16} can be measured by changing the beam energy at the entrance to the BTS by varying the extraction time from the linearly-ramping booster, while keeping the extraction magnets fixed in strength but timed to the bunch center. η_x depends on R_{16}, but also on the booster quadrupoles, the momentum offset (via the rf frequency), the slow bump, etc., none of which change the measured R_{16}. Hence, measuring "dispersion" in the BTS by varying the extraction energy from the booster does not allow determining the dispersive

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contribution to the beam size, unless we are certain that η_x is zero at the entrance to the extraction kicker.

SIMULATION-BASED TESTS

We simulated such measurements using `elegant` and `sdds5x5sigmaproc`, starting with beam parameters for the -0.5% off-momentum booster lattice used in normal APS operations. After various trials, we chose to vary two quadrupoles (BTS:AQ4 and BTS:AQ5) just upstream of the final dogleg dipoles and one (BTS:BQ1) just after the final dipole. There is one fixed-strength quadrupole between BTS:BQ1 and the screen. In a real measurement, we need to ensure that the beam sizes do not get too large (which can make the beam spot dim or larger than the screen) or too small (which can cause saturation or resolution issues). We also need to ensure that beam sizes don't become so large along the transport line that beam is scraped.

To understand the variation in beam sizes, we simulated a wide-ranging scan of the quadrupole strengths, then down selected the settings to ensure at least 99% beam transmission and to use only those configurations for which both beam sizes are within the 70$^{\text{th}}$ percentile of their respective distributions. The latter requirement serves to reduce the incidence of very large beam sizes, which can be experimentally problematic. These filters left us with a manageable 34 sets of quadrupole strengths for use in performing simulated measurements and eventual real measurements. The simulated rms beam sizes in x (y) range between 0.4 (0.15) and 1.6 (0.66) mm.

With no errors, perfect resolution, and linear tracking, the method reproduces the input beam parameters ($\epsilon_{x,y}, \beta_{x,y}, \alpha_{x,y}, \eta_x, \eta'_x$) to a few ppm. Including 2$^{\text{nd}}$-order in the tracking yields larger, but still negligible, errors, as Table 1 shows. The standard deviation of the measured rms beam size is typically 15 µm. We average 10 measurements at each configuration, which reduces the error in the mean to ~5 µm. When this is simulated, discrepancies increase, as shown in Table 2. Interestingly, although the errors in η_x and η'_x increase significantly, $\sigma_\delta = \sqrt{\hat{\Sigma}_{66}}$ is still determined accurately.

Table 1: Simulated Performance without Errors, SI Units

Quantity	Actual	Sim. Meas.	Frac. Error
ϵ_x	9×10^{-8}	9.01×10^{-8}	0.000823
β_x	9.66	9.65	0.000599
α_x	2.11	2.11	0.000515
η_x	0.758	0.758	0.000377
η'_x	-0.169	-0.169	2.71×10^{-5}
ϵ_y	3×10^{-9}	3×10^{-9}	4.5×10^{-6}
β_y	4.76	4.76	0.000104
α_y	-1.14	-1.14	0.000123
η_y	4.76×10^{-17}	-8.73×10^{-6}	1
η'_y	1.39×10^{-17}	-3.03×10^{-6}	1
σ_δ	0.00117	0.00117	0.000232

Table 2: Simulated Performance with Errors, SI Units

Quantity	Actual	Sim. Meas.	Frac. Error
ϵ_x	9×10^{-8}	9.06×10^{-8}	0.00629
β_x	9.66	9.74	0.00782
α_x	2.11	2.12	0.00496
η_x	0.758	0.74	0.0236
η'_x	-0.169	-0.167	0.0121
ϵ_y	3×10^{-9}	3.05×10^{-9}	0.0148
β_y	4.76	4.81	0.0118
α_y	-1.14	-1.17	0.0191
η_y	4.76×10^{-17}	-9.14×10^{-6}	1
η'_y	1.39×10^{-17}	-3.22×10^{-6}	1
σ_δ	0.00117	0.00118	0.01

REAL-WORLD MEASUREMENTS

Based on the promising results from simulated measurements, we conducted actual measurements, which proved challenging. Our goal is to reproduce the measured and inferred properties of the beam in the booster, which should be close to those listed in the tables above, since the booster lattice has been measured using LOCO [8, 9]. Among the practical issues are: (1) Hysteresis in the quadrupoles being varied, which can introduce errors in inferring K_1 from magnet current. We addressed this by cycling quadrupoles when the required change in strength was in the opposite direction to standardization. To make this efficient, the settings were sorted in order to minimize such changes. (2) Motion of the beam on the screen due to beam offsets in the varied quadrupoles. We addressed this with steering feedback on the BPM errors, using dipole correctors and the septum magnet as actuators. Since the variation of these elements varies the dispersion, we recorded and included the variable steering correction in computation of the R matrices. (3) Fixed and variable dispersion errors due to beam offsets in fixed and variable quadrupoles. We addressed this by measuring the beam position in all quadrupoles upstream of the screen, then including those positions as quadrupole offsets in the computation of the R matrices. (4) Possible calibration errors in beam position monitors (BPMs), which would introduce errors in assessment of the optics via response matrix measurements. We checked this by measuring R_{16} in the BTS, which is independent of corrector calibration. (5) Suspected systematic calibration errors of the quadrupoles, on the order of several percent [10]. To assess this, we performed R_{16} and response matrix measurements for comparison with the model, which led to the conclusion that the quadrupoles are 2.5% stronger than expected. (6) Variation in brightness of the beam spot, leading to noisy results due to dim spots or distorted spot shapes due to camera saturation. We addressed this via automatic gain adjustment on the camera. The lens aperture was generally fixed near the minimum value, to improve depth of field.

The experiments used a similar series of quadrupole settings as the simulations above. The corrector and quadrupole

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Table 3: Comparison of Results for Three Measurements

Quantity	Units	BTS Meas. 1	BTS Meas. 2	BTS Meas. 3	Booster LOCO
ϵ_x	nm	70.4 ± 1.6	73.2 ± 1.6	86.0 ± 1.0	87.2
β_x	m	10.26 ± 0.25	9.55 ± 0.22	9.05 ± 0.13	9.82
α_x		1.48 ± 0.04	1.42 ± 0.03	1.41 ± 0.02	2.16
η_x	m	0.447 ± 0.007	0.438 ± 0.005	0.482 ± 0.005	0.699
η'_x		-0.142 ± 0.002	-0.138 ± 0.001	-0.133 ± 0.001	-0.158
ϵ_y	nm	1.09 ± 0.01	1.11 ± 0.01	1.15 ± 0.01	0.50
β_y	m	5.88 ± 0.05	5.66 ± 0.05	6.38 ± 0.04	5.08
α_y		-1.30 ± 0.01	-1.35 ± 0.01	-1.32 ± 0.01	-0.99
η_y	mm	-3.071 ± 0.49	-12.685 ± 0.49	-12.686 ± 0.37	8.809
η'_y	10^{-3}	-1.34 ± 0.14	-1.56 ± 0.14	-1.84 ± 0.10	-1.49
σ_δ	%	0.127 ± 0.013	0.123 ± 0.014	0.122 ± 0.013	0.118

currents were recorded during the experiment, translated into appropriate form (e.g., kick angle and K_1), then loaded into `elegant` using the `load_parameters` command. The experimentally-determined 2.5% quadrupole strength error was included at this step, giving a series of R matrices.

The ten beam images collected for each configuration were digitized with a pixel size of 20.5 µm and a system resolution of 34 µm. The resolution was measured by placing the beam at the edge of the screen to create a "knife edge" illumination, to which we fit a cumulative distribution function of the standard normal distribution. The pixel size and resolution appear to have no significant effect on the results. Analysis of the images included background subtraction, noise suppression, and computation of the true rms sizes $\sqrt{\Sigma_{11}}$, Σ_{13}, and $\sqrt{\Sigma_{33}}$.

In total, three measurements were taken including all of the factors listed above, all within a single eight-hour shift. Measurements 1 and 2 had the lens aperture fixed near the minimum value, whereas for measurement 3, the aperture was opened somewhat to improve signal levels. Except for the aperture adjustment, the measurements are nominally identical and should return the same results.

Separately, LOCO fitting was performed on the booster near the end of the ramp, which provides strength and tilt values for all quadrupoles. With this data, we used `elegant` to compute the 6D equilibrium beam moments for the -0.5% off-momentum orbit. This provides the beam moments throughout the booster; we used the values at the beginning of the extraction system to compute the projected beam properties for comparison with the BTS measurements.

As Table 3 shows, there are some significant areas of agreement between the various measurements, but also significant discrepancies. Taking the Booster LOCO results as the reference, the best agreement on the emittance is from BTS measurement 3, which differs from the others by having a larger camera aperture. Measurement 3 also generally shows smaller error bars, which suggests that with the smaller aperture, we had insufficient signal. However, measurement 3 gives relatively worse agreement for η'_x and β_y. Across all measurements, the largest discrepancies are for η_x and η'_x, which is somewhat surprising given that the energy spread is determined quite accurately. The vertical emittance is off by a factor of two, but the value is quite small and is not thought to be reliably determined by the LOCO fitting procedure.

Inspection of the measured $(\Sigma_{11}, \Sigma_{13}, \Sigma_{33})$ values reveals that although the three BTS measurements were nominally identical, there is a tendency for the third measurement (with larger aperture) to deviate from the other two, particularly when the beam size is large. This is again suggestive of a signal intensity issue with the other measurements, which would tend to show up more when the beam spot is larger (and therefore more tenuous).

CONCLUSIONS

We outlined a method for measuring the non-temporal elements of the sigma matrix in a transport line. The method works very well on simulated data for the existing APS booster-to-storage-ring transport line. In practice, a host of issues must be considered and, even then, agreement with beam properties deduced from a LOCO model is uneven. There is some indication that improving the signal-to-noise ratio by opening the aperture is beneficial.

This technique has been applied in simulation to the transport line design [11] for the APS upgrade [12] , which features an x-y emittance exchange system [13, 14]. It is hoped that the ability to measure the 5x5 sigma matrix will enable confirming that the exchange is properly tuned, though of course there are other approaches (e.g., trajectory response matrices). Because of the much stronger quadrupoles, the effect of chromatic aberrations is much more pronounced. This can be mitigated to some extent using two imaging locations separated by a drift space.

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