ISBN: 978-3-95450-232-5 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2022-TUPA84

RECONSTRUCTING BEAM PARAMETERS FROM BETATRON RADIATION THROUGH MACHINE LEARNING AND MAXIMUM LIKELIHOOD ESTIMATION

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Abstract

The dense drive beam used in plasma wakefield acceleration generates a linear focusing force that causes electrons inside the witness beam to undergo betatron oscillations, giving rise to betatron radiation. Because information about the properties of the beam is encoded in the betatron radiation, measurements of the radiation such as those recorded by the UCLA-built Compton spectrometer can be used to reconstruct beam parameters. Two possible methods of extracting information about beam parameters from measurements of radiation are machine learning (ML), which is increasingly being implemented for different fields of beam diagnostics, and a statistical technique known as maximum likelihood estimation (MLE). We assess the ability of both machine learning and MLE methods to accurately extract beam parameters from measurements of betatron radiation.

INTRODUCTION

In plasma wakefield acceleration, a dense drive beam generates a linear focusing force by repelling the plasma electrons away from its path while leaving the much heavier plasma ions uniformly distributed. Subject to this focusing force, electrons inside the witness beam then undergo harmonic transverse betatron oscillations that give rise to betatron radiation. Because information about the beam is encoded in betatron radiation, measurements of the radiation can be used to reconstruct beam parameters, allowing devices which record information about betatron radiation, such as the UCLA-built Compton spectrometer, to be used for beam diagnostics. Machine learning (ML) has the potential to be applied to betatron radiation diagnostics, as ML methods are already implemented for some fields of beam diagnostics [1]. For example, the application of convolutional neural networks at FAST is able to produce a prediction for various downstream beam parameters from simulated datasets [2]. Another method of extracting information about beam parameters from measurements of radiation is maximum likelihood estimation (MLE), a statistical technique used to determine unknown parameters from a given distribution of observed data. The goal of this work is to assess the ability of both maximum likelihood estimation and machine learning as methods for accurately extracting a beam parameters from measurements of betatron radiation [3–5].

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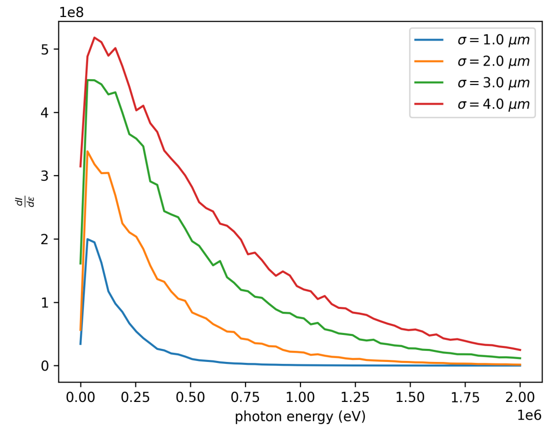

Figure 1: Examples of simulation-produced radiation spectra for spot sizes from 1-4 μm.

BEAM PARAMETER RECONSTRUCTION USING MLE

The method of maximum likelihood estimation involves a probability distribution function $f(x|\sigma)$, which specifies the probability of observing a data vector x given the parameter σ and is related to a likelihood function $L(\sigma|x)$ by $L(\sigma|x) = f(x|\sigma)$. Given a set of N observations of data vectors, the overall likelihood is the product of the likelihoods for each individual data vector [6], and the value of the parameter σ which is most likely to have produced the set of observed data is determined by maximizing the likelihood with respect to σ. To avoid possible problems with arithmetic underflow [7], log-likelihood was used instead of raw likelihood. The log-likelihood is given by

$$\ln L(\sigma|x_1, x_2, ..., x_N) = \sum_{n=1}^{N} \ln L(\sigma|x_n), \tag{1}$$

where the product of likelihoods has been converted into a sum of log-likelihoods.

The first task tackled by this work was to correctly identify a beam's spot size from its radiation spectrum using MLE. Several simulations of betatron radiation from beam particles in a plasma wakefield accelerator were run for beams of different spot sizes, using a simulation in which particles are sampled from a beam and tracked through idealized fields and betatron radiation was computed using Liénard–Wiechert potentials [8, 9]. The betatron radiation

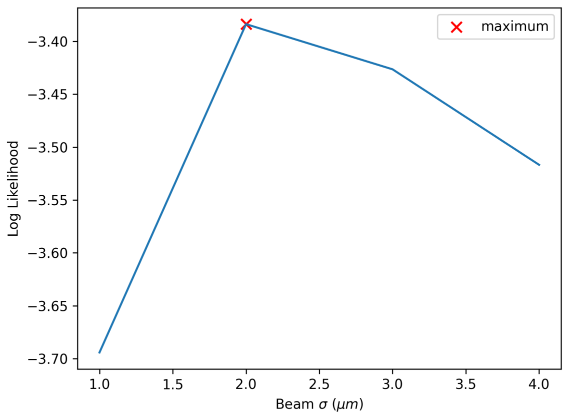

Figure 2: The log-likelihood function $lnL(\sigma|x)$ reaches a maximum at the test spot size, 2 μm.

spectra from these simulations, are shown in Fig. 1. Some "test" spot size was then arbitrarily chosen, which in this example was 2 μm, and an additional radiation spectrum was obtained for this spot size. Each of the spectra were then converted into a probability distribution, together forming a probability distribution function $f(x|\sigma)$, where x is photon energy and σ is spot size. The test spectrum was also converted to a probability distribution $f_{test}(x)$ for ease of comparison. Now, the likelihood that $f(x|\sigma)$ models $f_{test}(x)$ at different spot sizes for a test spectrum of discrete photon energies $x_1, x_2, ..., x_N$,

$$\ln L(\sigma|x_1, x_2, ..., x_N) = \sum_{j=1}^{J} f_{test}(x_j) \ln f(x_j|\sigma). \quad (2)$$

Figure 2 shows the log-likelihood function plotted and correctly identifying the test spot size of 2 μm.

Furthermore, this same MLE algorithm can also be expanded to analyze data for two-dimensional distributions, such as the double differential spectrum distributions and angular spectrum distributions shown in Fig. 3, with the expression in Eq. (2) now summed over all points in the 2D distributions. When tested, the MLE algorithm was able to correctly identify a test spot size of 1 μm using the 1D spectrum and both 2D distributions. This makes the 1D radiation spectrum a more attractive choice for use with the MLE algorithm (as well with machine learning) because it delivers similarly reliable results with less computation.

The MLE algorithm can also be expanded to identify different beam parameters. The results present in Fig. 2 were able to be replicated to identify beam energy, emittance, and beam charge by repeating the above methods and replacing spot size with the appropriate parameter.

To expand on the results in Fig. 2 and achieve more precise and accurate results with the same methods, 520 total simulations were run, each with a spot size chosen randomly between the values 0.1 and 9.0 μm. The 1D radiation spectra of 310 simulations were inputted into the MLE algorithm to

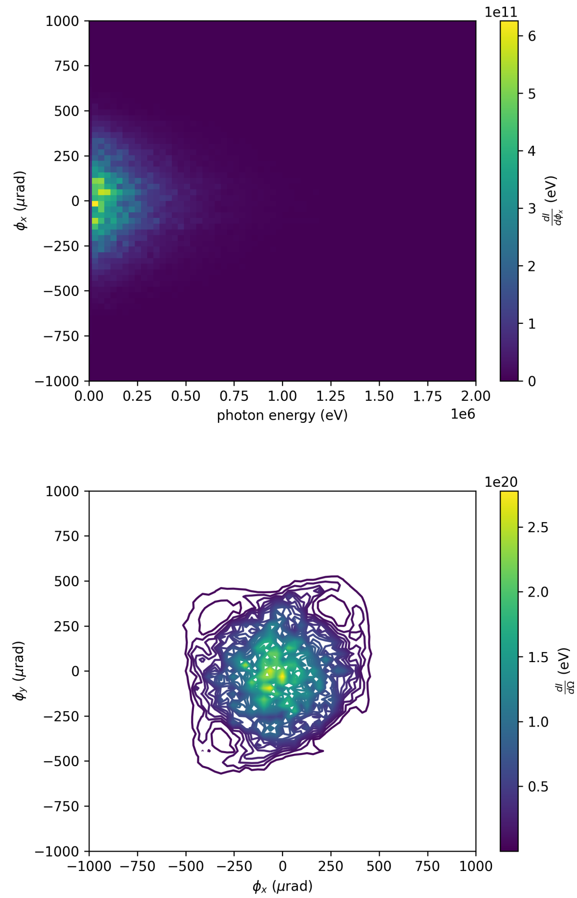

Figure 3: Two types of two-dimensional distributions plotted from same radiation data. Top: Double Differential Spectrum. Bottom: Angular Spectrum.

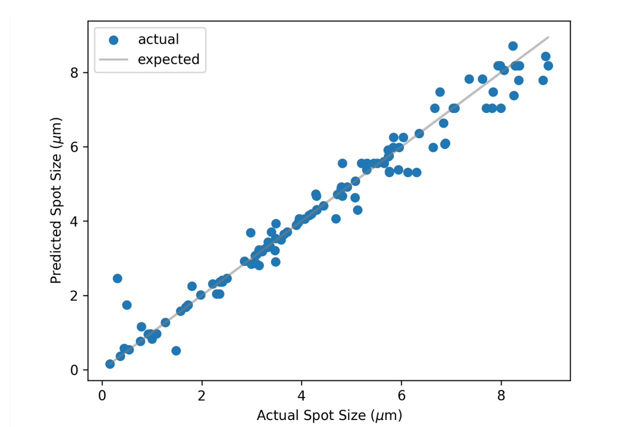

Figure 4: Overall spot size prediction results for 1D radiation spectrum MLE with 310 sets of reference data and 120 test cases.

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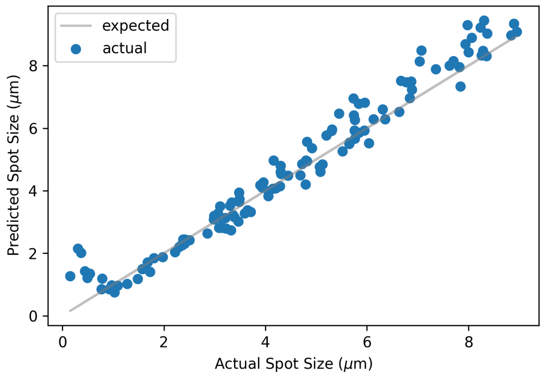

Figure 5: Spot size prediction results for 1D radiation spectrum ML with 400 sets of training data and 120 test cases (MSE=0.2638).

predict the spot sizes of the other 210 spectra, and the prediction results for the 210 test cases are displayed in Fig. 4. At a mean-squared error (MSE) of 0.186 μm^2, the prediction results appear accurate, except in the region below 1 μm, where a few predictions are significantly greater than the actual spot sizes.

BEAM PARAMETER RECONSTRUCTION USING MACHINE LEARNING

The MLE algorithm is limited in its prediction ability because it cannot predict parameter values not included in the values for which simulation data is already provided. Therefore, machine learning is also explored as another method of extracting beam parameters from betatron radiation data.

Radiation data in the form of the 1D spectra was used to train and test a densely connected neural network for predicting spot size. Simulations were run to generate 310 training data sets and 120 test cases data for different spot sizes ranging from 0.1 to 9.0 μm. The neural network contained one hidden layer, used ReLU activation functions, and was trained over 200 epochs.

The results for ML predictions displayed in Fig. 5, show a "tail" below ~ 1 μm, where the predictions all tend to be higher than the actual spot size values. The reason for the persisting inaccuracy at low sizes may be related to the K parameter values for the simulated small spot size beams. These K values can be calculated by $K = \gamma k_\beta \sigma_r$, where γ is the Lorentz factor, k_β is the wave number of the betatron oscillations, and σ_r is the beam spot size. Fig. 6 shows examples of radiation spectra and their associated K values while Fig. 7 shows K values for different test cases and the prediction error for each of those cases. The predictions are very inaccurate in the region from K=0 to K=5, and accuracy appears to decrease as K increases. It is likely that for K on the order of 1, the spectrum of the beam becomes impossible to distinguish from spectra of beams with higher spot sizes.

In addition to the ML model described above, another model was trained to predict spot sizes from images of the radiation spectra rather than from the spectrum data itself,

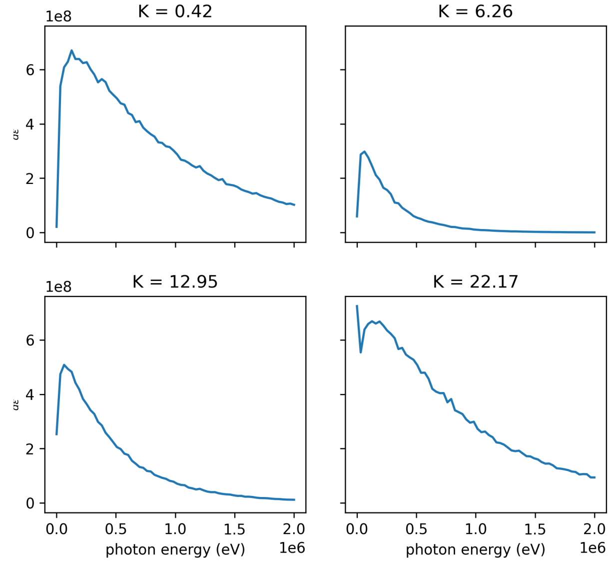

Figure 6: Examples of calculated K values for four different radiation spectra.

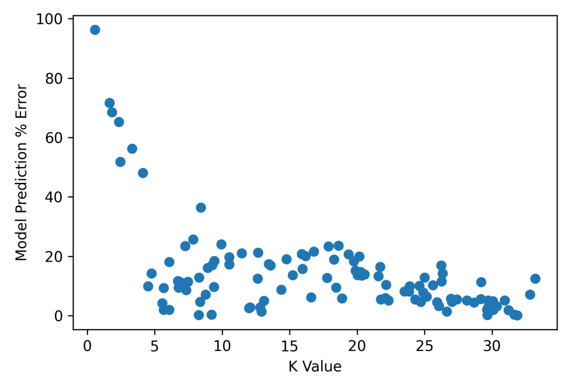

Figure 7: Error in ML model predictions for different K values of test data.

with similar levels of accuracy. Prediction of parameters from images allows for wider application of these ML beam diagnostics.

CONCLUSION

This work demonstrates that both MLE and ML can both effectively use betatron radiation data in order to wield betatron radiation as a tool for beam diagnostics, specifically in order to identify beam spot size, emittance, charge, and energy. While spot size is the most thoroughly tested of these parameters, both ML and MLE have difficulty accurately identifying small beam spot sizes.

ACKNOWLEDGEMENTS

This work was performed with support of the US Department of Energy, Division of High Energy Physics, under Contract No. DE-SC0009914.

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