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COULOMB CRYSTALS IN STORAGE RINGS FOR QUANTUM INFORMATION SCIENCE*

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Abstract

Quantum computing is a burgeoning field which has seen significant advances in the last decade. However, we are still a long way from having a universal computer that can efficiently simulate quantum mechanical phenomena. Quantum computers, in principle, use a well-known quantum system (the computer) to predict the behavior of another quantum system (the system being simulated). In this paper we review recent work in developing systems for quantum information science (QIS) and the use of storage ring technologies to take QIS to large scales. We will discuss crystalline beams, the challenges faced in utilizing storage rings for QIS systems, the importance of quantum entanglement, and recent progress in tackling some of the challenges.

INTRODUCTION

Crystalline beams in storage rings have the potential to scale QIS systems well beyond current ion trap approaches [1, 2]. Crystalline ion chains have been created in ion trap systems and have proven to be useful as the computational basis for QIS applications [3, 4]. The same structures can be created in a storage ring, but the ions necessarily have a constant velocity and are rotating in a circular trap [5]. To be useful for QIS applications these crystals need to be cooled to ultra-low temperatures [6, 7].

Quantum information science is a growing field that promises to take computing into a new age of higher performance and larger scale computing as well as being capable of simulating quantum phenomena using quantum systems [8,9]. Computer scientists have worked out many additional problems that could be solved efficiently using quantum computers rather than conventional computers. This is illustrated in Figure 1, which shows an Euler diagram of the different complexity classes. Beyond simulating physics problems, there are problems that are uniquely suited and more quickly solved on quantum computers.

The outstanding issue in practical quantum computing today is scaling up the system while maintaining interconnectivity of the qubits and low error rates in qubit op-

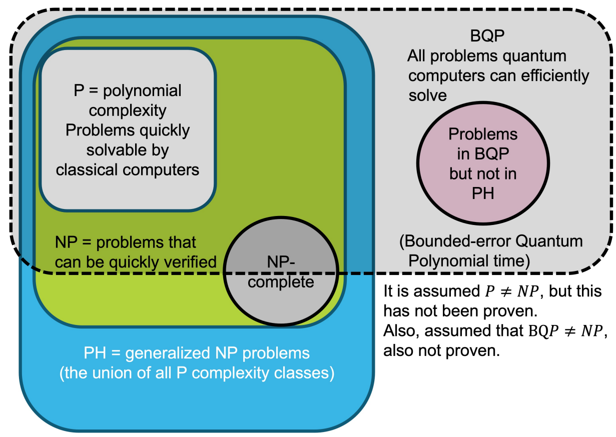

Figure 1: An Euler diagram illustrating the relationships of different complexity classes for classical and quantum computing. This is an active area of research, so this diagram is only meant to generally show how different classes of problems fit into either a classical or quantum computing universe [10].

erations to be able to implement error correction and fault-tolerant operations [11]. Trapped ion qubits offer long coherence times that allow error correction [12]. Error correction algorithms require large numbers of qubits. We can potentially create many thousands (or more) of qubits with long coherence states in a storage ring. A circular radio-frequency quadrupole (CRFQ), which is a large circular ion trap, acting as a Storage Ring Quantum Computer (SRQC), would be scalable and fault tolerant quantum information system. With computing demands potentially outpacing the supply of high-performance systems, quantum computing could bring innovation and scientific advances to nuclear and particle physics [13].

Trapped ions are isolated small quantum systems that have demonstrated low decoherence rates. Such systems can be controlled and measured using laser-induced manipulations of the ions. Stationary ions may be in the same inertial reference frame as the lasers and other systems used to control and measure eigenstates, but a single set of lasers can only operate on a small number of ions in the trap. Very long ion chains would not be easily addressable [14]. Moving ions,

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while in a different inertial reference frame, will pass by the system that control and measure the eigenstates.

Limits of Quantum Computing

While Feynman gave a compelling motivation to develop quantum computers, Quantum computing is now understood to be part of the natural evolution in computing to miniaturize and generalize the abstract process of representing complex problems in algorithms deployed using binary arithmetic. In classical computing certain problems can be solved because they are either computable in a relatively simple way in a relatively short amount of time (P and NP complexity) or because they can be approximated in a way that sufficiently bounds the problem. Quantum computing can not only simulate physical processes but can also solve problems that may be hard or even impossible to solve on a classical computer [10]. Feynman's conjecture that a quantum computer can simulate local quantum systems has been proven to be true [15], it has also been shown that classical algorithms can be solved more quickly on a quantum computer. Problems that are both impossible or hard to solve on a classical computer can now be solved, sometimes much more easily and quickly.

As shown in Figure 1, there are different types of problems that can or cannot be solved using classical algorithms. But are there problems that cannot be solved with a quantum algorithm? The easy answer is yes, well, probably. However, the real answer depends on whether we are asking a theoretical question (e.g., is it possible any computer that could be invented can solve the problem) or a practical question (e.g., given all the computational capability in the universe - such as using all the atoms known to exist). For our purposes, this really gets into what quantum algorithms can and cannot do. What a quantum computer does, above and beyond classical computers, is perform operations by exploiting the quantum principles of superposition and entanglement. A relatively small number of qubits can store, in superposition, a vast amount of information. If we have a 20 qubit computer, we can store 2^{20} values, or a little over one million. Until we measure the states, all values are held in superposition. Quantum algorithms exploit this property. The rules of quantum mechanics dictate that when we measure these states we will end up with just a single number. In addition, by entangling qubits, quantum circuits can be developed that perform operations simultaneously, since once the state of one qubit is measured the other is immediately known. Quantum computing has other features that get exploited to develop new and efficient algorithms. Since quantum processes are reversible, quantum circuits are reversible. And, since you cannot clone a quantum state (the measurement destroys the state), you can create unclonable encodings.

Ion Coulomb Crystals

Classical crystalline beams have been created in storage rings [16]. Ultracold crystalline beams, or ion Coulomb crystals (ICCs), have been created in ion traps. ICCs are cooled well below the Doppler cooling limit, but not deeply

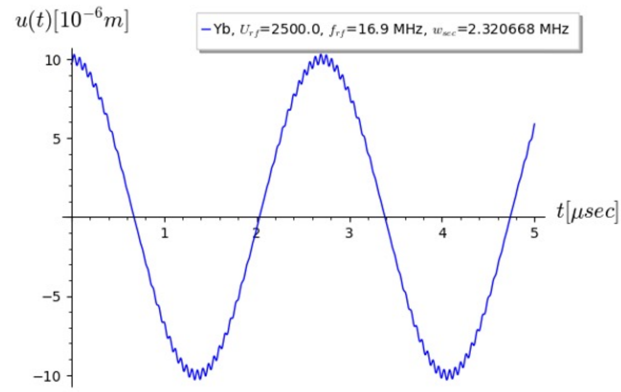

Figure 2: Secular motion is determined by the particles trajectory influenced by the rf focusing fields. Micromotion is a result of the trap rf frequency and the particle large amplitude away from the center of the symmetric fields.

into the Lamb-Dicke limit [17]. In this regime the ion-laser interaction can be well approximated as the carrier transition that does not affect the motion (simple spin flip) and two sideband transitions, the red sideband, and the blue sideband, that change the vibrational state by one quantum unit while flipping the spin [18]. The higher-order interactions (i.e., 2nd, 3rd and higher-order sidebands) are strongly suppressed. In this regime, thermal vibrations are small enough to distinguish the external quantum modes of the crystalline structure and to minimize any micromotion from the rf confining the ions in the center of the trap. The laser cooling systems can reduce the motional degrees of freedom of the ions but cannot diminish kinetic energy from the induced micromotion (see Figure 2). ICCs have been created in ion traps but not in a storage ring. (Note, ICCs have been created in ring shaped microchip traps [19].)

Ion trap systems exploit two quantum properties of the ions in the trap, external energy eigenstates, such as the axial center-of-mass motion of the string of ions in the trap, and the internal eigenstates of each ion in the string. When sufficiently cooled, the string of ions in the trap has properties that we can use to define a set of computational basis states operated on using laser excitation.

The primary method of establishing a qubit involves excitation and measurement of stable or metastable internal states of individual ions, such as the hyperfine states. The basic method, as described by Wineland et al., using $^9Be^+$ and then using the $^2S_{1/2}$ $(F = 2, m_F = 2)$ and $^2S_{1/2}$ $(F = 1, m_F = 1)$ hyperfine ground states (denoted $|\downarrow\rangle$ and $|\uparrow\rangle$, respectively), we can construct a practical qubit. Tuning a polarized laser beam to the $|\downarrow\rangle \rightarrow {}^2P_{3/2}$ transition and by observing the scattered photons we can resolve two distinct spin states. With this technique, per Wineland et al., the quantum states can be determined with almost 100% efficiency. Using Raman transitions and polarized light scattering, we can set and measure the hyperfine states.

In ion traps, a qubit is coherently manipulated by exciting it with a monochromatic travelling-wave laser beam. This

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can be described with a Hamiltonian as [20],

$$H = H^e + H^m + H^i, \tag{1}$$

where H^m describes the ion's motion, H^e the electronic structure of the ions, and H^i is the interaction of the ion with a laser. For a laser-cooled ion in a harmonic potential, H^m is the sum of three quantum harmonic oscillator Hamiltonians corresponding to the motional degrees of freedom,

$$H^m = \hbar\nu_j(a_j^\dagger a_j + 1/2), \tag{2}$$

where $\nu_j, \{j = x, y, z\}$ is the harmonic oscillator frequency, and $a_j^\dagger$ and a_j are the corresponding creation and annihilation operators of the quantum vibrational eigenstates, $\{|0\rangle, |1\rangle, |2\rangle, ...\}$. The electronic structure of the ion can be approximated using a two-level model of the excited state $|\uparrow\rangle$ and ground state $|\downarrow\rangle$. Then,

$$H^e = \frac{\hbar\omega_0}{2}\sigma_z, \tag{3}$$

where ω_0 is the atomic transition frequency, and σ_z is the Pauli spin matrix. Finally, the laser-ion interaction Hamiltonian is,

$$H^i = \hbar\Omega(\sigma_+ + \sigma_-)\cos(k\hat{x} - \omega_L t - \phi_L). \tag{4}$$

The frequency, ω_L, and phase, ϕ_L of the laser field as seen by the ion, described by the position operator, $\hat{x}$, along with the laser wave vector, k, and Rabi frequency, Ω, determine the strength of the interaction. The jump operators, σ_+ and σ_- induce the transitions between qubit states $|\downarrow\rangle$ and $|\uparrow\rangle$.

These interactions, whether with a stationary or moving ion, are unchanged. In the case of the moving ion, the time to perform a state transition is restricted to the laser width divided by the ions velocity, unless the laser field can be moved and follow in the same reference frame. One approach around this may be to spread the laser field out along the path of the ion or split the laser pulse repeatedly to spread the laser field along the path of the ions trajectory.

Related Work in Ion Traps

Trapped Ion quantum computers generally use Paul Traps or similar devices that use an electric field oscillating at rf frequencies to prevent ions from drifting out of the trap, while they are cooled and stopped (i.e., time averaged 3-vector velocity of each ion is 0) in the trap [21]. There are many designs and methods being developed to resolve issues that can lead to practical quantum computers using trapped ions [22]. These issues are usually defined by the DiVincenzo's criteria, which state that a system is scalable, have means for initializing qubits, allow operations within decoherence times, have methods for a universal set of operations, and allow qubits to be easily read [23]. What is today significant is the technologies are well enough advanced that companies are bringing to market ion trapped based quantum computing systems [24]. However, these systems remain small in scale and are far from universal.

STORAGE RINGS FOR QIS

A significant challenge in quantum computing is controlling quantum decoherence. However, research in ion traps has shown that quantum states in trapped ions can persist for very long times, even on the scale of minutes and in some cases, hours. The scaling of the number of qubits (N), while limited mostly to internal interactions, is related to the problem of decoherence. It has a unique temporal component we must consider, since we cannot operate on all ions in the crystalline beam simultaneously.

Inside the storage ring system, we can isolate groups of smaller numbers of ions from each other, using longitudinal rf potentials or by separating using velocity modulation in the cooling systems. We can then operate on these isolated sets of qubits independently. In the storage ring environment, there is potential for multiplexing as well as an ability to work on ions and groups of ions in parallel. A storage ring could contain thousands of these smaller individual crystals, producing 10's of thousands of qubits. The many small chains of ions could serve different purposes, depending on the algorithm being employed. For example, we can use some ion chains as quantum memory and some for other purposes, such as for systematic or error analysis. Having many ions and ion chains available opens many possibilities; simultaneous computations, quantum memory, error correction, and more.

Ion Traps for QIS

Trapped ions offer one of the most compelling systems for practical quantum computing. The basic requirements for universal quantum computing have all been demonstrated using trapped ions, and quantum algorithms using trapped ion-based qubit systems have been implemented. Trapped ions have advantages over other approaches. We have mentioned the advantage of long coherence times. Trapped ions allow for both single and two-qubit gates, implemented with very high fidelities (as high as 99.9999% [25]). This far surpasses other approaches. Two-qubit entangling gates have been demonstrated with very high fidelities (99.9% for hyperfine qubits [26] and 99.6% for optical qubits [27]). Preparing and measuring states is accomplished using well developed laser interactions. Measuring states can be done quickly, with 99.99% measurement fidelity in less than 200 μsec [28].

Trapped ions also have the unique property that all ions of a given species and isotope are identical. This means that the operations to address each ion will always be the same.

Circular RFQs

To create and maintain crystalline beams, the storage ring lattice needs to be specially designed to avoid linear resonances between phonon modes of the crystal and the lattice structure. To just create crystals, the azimuthal motion needs to be bounded from below, to avoid the negative mass instability. The basic idea is that for energies above the γ_{tr} point in the lattice, ions with a larger velocity take more time for a revolution owing to a longer path and they will lag behind.

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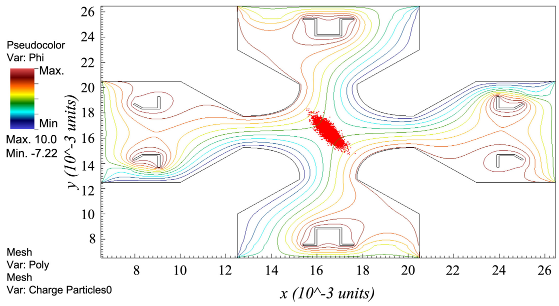

Figure 3: Visualization of a trapped beam of $^{24}Mg^{+}$ in the CRFQ using VSim.

This, with the repulsive Coulomb force that accelerates particles in front of others, causes longitudinal instability and the particles tend to self-bunch. A lattice could be designed with a negative momentum compaction factor or a very high γ_{tr}. However, for QIS applications there really is no need for higher energy ions. The use of circular radio-frequency quadrupoles, which are essentially Paul traps pulled into a circular structure, provides ideal conditions for creating crystalline beams.

In a storage ring the azimuthal motion of the ions is an angular precession at the frequency, $\omega_0 = \beta c/R$, where R is the radius of curvature of the ring. The alternating focusing period is $L = \beta\lambda$, where $\lambda = c/f_{\mathrm{rf}}$ is the rf field wavelength and β is the relativistic factor. Details for designing a CRFQ for quantum computing can be found in [1]. The trajectory of an ion in a CRFQ is shown in Figure 2, where both the secular and micromotion can be seen.

We use VSim [29, 30] to simulate the beam dynamics inside a model of the CRFQ. We drive an RF potential across the pole tips to produce an alternating electromagnetic field that keeps a beam of $^{24}Mg^{+}$ stable over time. Figure 3 shows the metallic boundaries defining the CRFQ, as well as control rods. The field lines and the transverse configuration space of the particles are also shown.

Cooling ions in a trap is well established [31,32]. Methods for 3D cooling, such as optical molasses or sympathetic cooling, are also well established [33]. However, cooling ions that are moving raises many challenges. Doppler laser cooling of moving ions was established in the PALLAS experiments [7, 16] and has been demonstrated at GSI using rf potentials as a counter force to a single laser system [34]. The amount of detuning required for high Doppler shift raises a challenge for relativistic beams [35]. More research and new ideas are needed to find ways to perform 3D cooling of moving ions, to get to the temperatures needed to use these ions for QIS applications.

The main difficulty with transverse cooling in a ring is the transverse velocity components are not accessible. To expose them requires adding some method of kicking the ions tangentially, as in sympathetic cooling, electron cooling, or even stochastic cooling, or finding a way to expose the transverse components to a longitudinal laser pulse. This would simulate cooling by optical molasses. To this end we are investigating the use of an electrostatic wiggler (both in a simple configuration or as a helical wiggler). This approach was described in [36] and continues to be an area of research. With the wiggler contained within the CRFQ lattice, particles will remain confined by the transverse rf focusing.

ENTANGLEMENT AND GATES

Entanglement involves combining the complex vectors describing the wavefunction states in our two systems. In our case, we exploit the hyperfine states of electrons in the upper energy level shells of atoms in an ICC. Our qubit states are the hyperfine spin states. If we had two particles with entangled hyperfine states, we could put one into a spin-up state, by exciting it with a laser pulse. The other ions spin state will then go into a spin-down state. We can observe this by measuring the states and seeing that if one is fluorescing when illuminated by laser light, the other one will always be dark. These are our canonical qubit $|1\rangle$ and $|0\rangle$ states for the ions in our ICC. The two ions are placed in such a state by performing operations that create an entangled wavefunction, such as $\psi = |01\rangle - |10\rangle$. Such a state does not factorize into a product of the single particle wavefunctions, so $\psi \neq \psi_1\psi_2$, meaning the two are entangled. In quantum computers, entangled states improve the processing speed of the quantum computation. This is necessary to achieve exponential speed-up over classical algorithms [37]. Methods to entangle ion trapped wavefunctions and creating simple gates are described in [1, 2, 36]. To entangle the internal eigenstates of two ions we use as our control bit an external eigenstate from the quantum motional modes of the ion chain. Through a series of laser excitation's, the CNOT gate operation is performed using this control bit to produce the entanglement [38].

Quantum circuits are interconnections [39] of quantum gates that can accomplish some desired overall function [40, 41]. There are well known quantum circuits for purposes such as teleportation, super dense coding and error correction. A gate may be expressed as an equivalent interconnection of more basic gates [42]. Sometimes gates entangle multiple qubits. Each logical qubit may be represented by several physical qubits (i.e ions and their states) [43].

MOTIONAL MODES

An ICC can be described, in a classical sense, as a string of charged masses acting as simple harmonic oscillators. Such a structure can be described in one dimension as masses coupled by springs, where the motion is small compared to the distances between the ions and so the spring coefficient, $m\omega_0^2$, is taken as constant. Here, ω_0 is the fundamental frequency for the chain of ions and is a function of the Coulomb and other potentials holding the ions in the chain. In this case, the motion is seen as modes in the axial motion of the ions. The quantized vibrational energy per mode n, is $E_n =< n|H_0|n >= (n + \frac{1}{2})\hbar\omega_0$. As the number of ions increase, the number of modes also increase. An ensem-

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ble of ions of size $N+1$ has $3N+3$ motional degrees of freedom, so as N become large, cooling and controlling the chain becomes very difficult. However, in a storage ring, we can imagine isolating two small ensemble of ions, cooling them independently, and then adiabatically merging the two chains, each already in a ground state. Tracking large numbers of ions may seem to be more an engineering problem. But given that we need to cool all ions down to stable ICCs of variable lengths, we need to locate quickly and precisely each ion. When two ions are entangled, we want to be able to identify them, especially in an environment where many entangled ions may exist in a single volume. The equilibrium positions of the ions z_n^0 ($x_n^0 = y_n^0 = 0$) are determined from the first derivative of the potential. Using the scaling parameter, u_n, then to determine the longitudinal equilibrium positions of the ions, we must satisfy

$$u_j - \sum_{n=1}^{j-1} \frac{1}{(u_j - u_n)^2} + \sum_{n=j+1}^{N} \frac{1}{(u_j - u_n)^2} = 0. \qquad (5)$$

The scaling parameter, $u_n = z_n^0/l$, where $l = [e^2/(4\pi\epsilon_0 m\omega_z^2]^{1/3}$. Equation (5) is a set of N coupled algebraic equations and can only be solved numerically. For large N, we need to determine the positions precisely. As N becomes large, the computation time becomes exceedingly long, scaling by $O(N^3)$. The calculation can be parallelized, but gets very CPU greedy as a function of N. On a GPU implementation the data transfers limit the wall clock time. We have looked at using machine learning and doing parametrized fittings to find solutions [44]. The times to find all u_j can be reduced, but finding the equilibrium positions remains challenging.

The Problem Finding Equilibrium Positions

Predetermining the equilibrium positions of many ions in the storage ring may be a bad approach, since there are processes that can cause ions to be lost or shifted momentarily in the chain. For example, an ion may interact with atoms in the residual gas, causing it to be lost or to be momentarily displaced until cooling can reduce its 3-vector velocity to fit back into the crystal. The entire crystal can be disrupted in the process. A more practical approach is to uniquely identify the ions that are currently being operated on in the trap, in real time. We are currently researching this approach, but an important component is to have precise timing systems. An advantage of a storage ring is it has a very well-defined clock, the revolution frequency of the ions in the trap.

SCALING TO A MILLION QUBITS

Ion trap researchers are looking at methods of scaling from single traps that can contain on the order of tens of qubits (i.e., ions) to systems that can scale to millions of qubits. One approach is to create many ion traps and treat them as registers, and then use optical switches to send the signals to a CCD pixel array (i.e., QCCD). In the storage ring approach, this kind of multiplexing and parallelizing of the information will take place automatically, as many lasers and sensors would be deployed to maximize the computational effort. This is also an active area of research.

CONCLUSION

The use of ICCs in storage rings can potentially allow scaling of QIS applications beyond what is available today or even in the near future. Since the SRQC is nothing more than an unbounded ion trap, the same technologies and techniques can be applied. The challenges in such an approach are in developing new methods to track ions with non-zero velocities, cooling large number of ions, and controlling the computational work as many lasers are applied to write and measure qubit states.

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